UNIVERSITY OF EDUCATION, WINNEBA

EFFECTS OF MULTIPLE REPRESENTATIONS-BASED INSTRUCTION ON JUNIOR HIGH SCHOOLSTUDENTS' ACHIEVEMENT IN LINEAR EQUATIONS IN ONE VARIABLE IN THE BIMBILLA MUNICIPALITY

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DECLARATION

Student's Declaration

I, MGNABIL PHILEMON TIJOTOB declared that this thesis, with the exception of quotations and references contained in published works which have all been identified and duly acknowledged, is entirely my own original work, and it has not been submitted, either in part or whole, for another degree elsewhere.

Signature: ………………………………...

Date: ……………………………………...

Supervisor's Declaration

I, hereby declared that the preparation and presentation of this work was supervised in accordance with the guidelines for supervision of thesis as laid down by the University of Education, Winneba.

SUPERVISOR'S NAME: DR. JOSEPH ISSAH NYALA

Signature: ………………………………

Date: ……………………………………

DEDICATION

To my dear father Yakun Mgnabil and mother Mgnabil N-muyeuni and my lovely sisters. God bless you all.

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TABLE OF CONTENTS

Contents **Pages**

CHAPTER TWO : REVIEW OF RELATED LITERATURE 15

CHAPTER THREE : RESEARCH METHODOLOGY 56

LIST OF TABLES

Tables **Pages**

LIST OF FIGURES

Figures **Pages**

ABSTRACT

The study investigated teachers' mode of representations in linear equations in one variable, what accounts for teachers' choice of representations and students' representations preferences and effects of multiple representations-based instruction on students' achievement in linear equations in one variable. A quasi-experimental design was used with a sample size of 86 mathematics teachers and 159 students (students in Junior High School 2) selected through purposive and convenient sampling techniques respectively in the Bimbilla Municipality in the Northern Region of Ghana. The research instruments included: linear equations' achievement test (pre-test and post-test), questionnaire, representations preference test (RPT) and interview guide. The data collected were analyzed quantitatively and qualitatively. It was found that most teachers (66.3%) used algebraic representation every time in teaching linear equations while few teachers used manipulatives, graphic and multiple representations due to reasons such as time, difficulty for students, lack of materials, lack of ideas, no relevance and many more. It was also found that most students 59(37.1%) had preferences for algebraic representation citing similar reasons of their preferences as what accounted for teachers' choice. The Analysis of Covariance (ANCOVA) revealed a statistically significant difference between students' scores in linear equations achievement test in favour of multiple representations-based instruction, $F\ (2,\ 155) = 10.20,\ P < 0.05,\ \eta^2 = 12.$ My major recommendation is that, multiple representations-based instruction should be used during instruction as well as improving teachers' knowledge on how to integrate unfamiliar representations easily in linear equations in one variable.

CHAPTER ONE

INTRODUCTION

1.0 Overview

This chapter discusses the background to the study, statement of the problem, purpose of the study, objectives of the study, research questions, and hypothesis, significant of the study, delimitations and organization of the study.

1.1 Background of the Study

Mathematics is important in many areas of life. Thus, a strong foundation in mathematics is a prerequisite for many careers and professions in today's growing technological society. Increasing evidence suggests that every country requires high levels of mathematical and technical skills for efficient development. According to Mishiwo (2007), mathematics is also defined as "the science of number and shape of which arithmetic and geometry are branches''. From these definitions, one could see that mathematics runs across every aspect of human life. Mathematics is important since it helps in the learning of other subjects and helps us to work with numbers, therefore making us know much about the solutions of linear equations in one variable, addition, subtraction, multiplication and division of numbers and also makes us think fast and accordingly. At all educational levels, the contributions of mathematics to understanding and application of many subjects are being recognized now more than ever before (Springer, 2007).

Mathematics enables students to achieve deeper understanding of scientific concepts by providing ways to quantify and explain scientific relationships and with a good background in mathematics, one has the chance of doing well in science and science related subjects. This indicates that without a proper understanding of the underlying principles in mathematics, the necessary skills and concepts in science and technology cannot be effectively acquired and applied by students (Charles-Ogan & Otikor, 2016). Mathematical achievement of Ghanaian children in recent times has been a subject of intense discussion among educators, policymakers and the public at large. Students' performance in mathematics has not been encouraging of late. Candidates are reported to exhibit a poor understanding of mathematical concepts and are unable to form appropriate mathematical models which could be tackled (WAEC, 2012). It has also been realized that many students have developed negative attitudes towards the study and learning of the subject as a result of the way and manner certain concepts are presented to them.

In addition to the low level of mathematics, many students fail to see everyday application of school mathematics; rather, they perceive it as something abstract (Bansila, James & Naidoo, 2010). Bruner (1964) suggested that in teaching mathematics, the presentation of concepts should be done first of all using concrete materials with gradual introduction of abstract symbols through the use of symbols and diagrams. According to Cockcroft (1982), Mathematics provides a means of communication, which is powerful, concise and unambiguous. To Nunes (1996), though Mathematics is seen as a school subject, it also forms an integral part of everyday life. For example, when sharing valuables with friends, when planning to spend their pocket money, when they argue about speed and distance dealing in different currencies as well as engaging in buying and selling. Nunes explained that though these acts are not directly seen as Mathematics, but in carrying these activities out one would inevitably use mathematical principles.

The unchangeable quantitative relationship that exists within this discipline across the world makes it unique and more important in every individual's life (Legner, 2013). According to Sherrod, Dwyer and Narayan (2009), countries that do not pay much attention to the study of mathematics will lag behind technological advancement. This is because, mathematics has been found to contribute immensely to technological improvement (Legner, 2013).

In spite of this advantage of mathematics, Ghana cannot benefit if it still adopts the traditional system of instruction. Pape, Bell and Yetkin (2003) all agree that the nature of mathematics instruction in the classroom is altered in order to allow students to appreciate the unified concepts, patterns, operations and relationships that exist in the real world. In view of this, appropriate classroom instruction should be adopted to enable students meet such challenges. The multiple representations-based instruction is one of such appropriate classroom instructional approaches considered in recent years in mathematics education (Boulton-Lewis, Cooper, Atweh, Pillay, Wilss & Mutch, 1997; Goldin & Shteingold, 2001; Moritz, 2000; Outhred & Saradelich, 1997; Panasuk, 2010; Swfford & Langrall, 2000). In the context of teaching and learning mathematics, multiple representations-based instruction is when various representations are used for teaching a concept or solving a problem instead of only one mathematical representation (Ainsworth, 1999). Also, multiple representation can be explained as providing the same information in more than one form of external mathematical representation (Goldin & Shteingold, 2001).

The multiple representations-based instruction is now widely accepted as fundamental to mathematics education in the classroom (Ainsworth, Bibby& Wood, 2002; Moseley, 2005; NCTM, 2000) due to the numerous benefits students get when they experience concepts in a variety of representations. According to Tripathi (2008), the use of multiple representations in teaching mathematics is a strong instrument that eases the understanding of mathematical concepts for students. Other researchers found that the use of multiple representations support abstraction of mathematical concepts and enhance students' learning and problem-solving (Cooper & Warren, 2011; Ross & Willson, 2012). Multiple representations-based instruction provides the means for students to construct and build their own understanding of the classroom instruction and this is the best means of teaching and learning (Goldin, 1990; Slavin, 2000; Nabie, 2009).

In terms of multiple representations in linear equations, several studies conducted in the classroom indicated significant improvement in students' achievement (Beryranevand, 2010; Doktoroglu, 2013; Cikla, 2004). Hong. Thomas and Kwon (1999) also found significant improvement in students' achievements in linear equations when they explore linear equations with three different representations namely: symbolic algebraic, graphic and tabular representations. Since mathematics concepts are learned through gradual building of mental images, it is important that we identify mathematical ideas in a set of different representations, manipulate the idea within a variety of representations and translate the idea from one to another (Owens & Clements, 1997). Consistent with this, Kaput (1991) found that, students create their internal representations when introduce to a variety of external representations. This according to Kaput, should form the focal point of any mathematics instruction in the classroom.

Multiple representations-based teaching approaches such as using graphs, pictures, manipulatives, tables, drawing and symbolic algebraic representations have been used widely by researchers and educators to teach mathematics concepts to increase students' mathematical competences (Flores, 2009). In this strategy, the teacher may model the mathematical concept using manipulatives such as blocks, counters, balance scales, algebra tiles, fraction bars and other concrete materials. Then, pictures, drawing or graphs are used to explore the same concept.

Representations involving symbols, variables or numbers is also used to teach the same concept. This mode of instruction has shown to be effective for students especially those who struggle with mathematics and those with learning disabilities (Avant & Heller, 2011; Cole & Washburn, 2010; Hudson & Miller, 2006; Witzel, 2005; Butler, Miller, Creshan, Babbitt & Pierce, 2003; Maccini & Ruhl, 2000).

However, even though multiple representations-based teaching has been proven to increased students understanding, it has not been utilized in most of the mathematics classrooms even where certain aspects of mathematics such as algebra and for that matter linear equations employs broad spectrum of multiple representations (Lubinski & Otto, 2002). Several other studies have indicated teachers' weakness integrating multiple representations in the teaching environment (Stein, Baxter & Leinhardt, 1990; Celik & Baki, 2007). In the light of this, most of the concepts are explored through single representation without using multiple representations. Findings from Hitt (2001) revealed that most mathematics facilitators focus on one mode of representation such as algebraic system of representation and ignore other representations. Consistent with this, Kieran (1992) reported that some teachers become so strict to students and force them to a single representation of solving linear equations which has been proven to be ineffective especially building students understanding and vision towards the concept of algebra.

According to NCTM (2000), "many students profit from hands-on collaborative learning that manipulatives afford" (p. 20). Collaborative learning enables students to come across ideas and questions of their group mates, check for their own understanding, and comprehend the concepts deeply (Mercier & Higgins, 2013). Using manipulatives to solve tasks in groups enhances learning in cooperative learning groups because using manipulatives motivate and entertain students

(Mulryan, 1994). Uribe Florez et al, (2010) stated that mathematical manipulatives offer students a way of understanding abstract mathematical concepts by enabling children to connect the concept to more informal concrete ideas. Researchers have found out that, young children who are unable to solve traditional linear equations in one variable problem are often capable of solving these problems when they are posed in the context with concrete objects (Ginsbury, 1989).

In single mode of representation in the classroom instruction, many researchers have lamented on its negative impact. According to Canterbury (2007), students' difficulties with understanding are as a result of traditional instruction that emphasize one mode of representation. Battista (1999) found that such mode of instruction has proven ineffective for generating a deep understanding of mathematics for all students. Again, Swafford and Langrall (2000) and Lowrie (2001) found that single representation of equations made little sense to students. Similarly, Fadel (2008) found that students who engaged in traditional approaches with single mode of representation did not perform compare to students engaged in multiple representations. In order to fill this gap, the process used in presenting concepts need to be differentiated by providing different modes of representations in the classroom instruction (Tomlison, 2000). Because, it is found from studies that students have different learning styles and multiple intelligence (Silver & Strong, 2003; Gardner, 1993).

Therefore, since single mode of representation does not help teaching and learning as found by (Fadel, 2008; Canterbury, 2007; Swafford & Langrall, 2000; Lowrie, 2001), students need to be exposed to different mode of representations such as algebra, manipulatives and graphic representations when teaching linear equations.

1.2 Statement of the Problem

The main rationale for teaching mathematics in Junior High School (J.H.S) as stated in the Junior High School Mathematics Teaching Syllabus in Ghana (Ministry of Education, 2012), is to enable all Ghanaian young students acquire the mathematical skills, insights, attitudes and values that they will need to be successful in their chosen careers and daily lives. It also builds on their knowledge and competencies developed at the Junior High School level. In Ghana a student is expected at the Junior High School level to move beyond and use mathematical ideas in investigating real life situations. The importance of linear equations is viewed by Huntley and Terrel (2014) as a hallmark for students' algebraic proficiency in mathematics. However, many students still struggle to develop symbolic and conceptual understanding of its concepts (Kilpatrick &Izsak, 2008; Poon & Leung, 2010). Evidence from different studies continue to confirm students' difficulties in dealing with equations of the form $ax + b = cx + d$(Nickson, 2000; Vlassis, 20002; Filloy, Rajano & Puig, 2007; Linsell, 2009).

Records of Junior High School Form Two Students performances in mathematics in their class test on linear equations in one variable with a total marks of 50 in 2018/2019 academic year at Makayili Junior High School showed that, only 6 students out of 65 students scored 19 marks out of total of 50 marks representing 25.68%, 8 students scored 17 marks representing 22.97%, 7 students scored 13 marks representing 17.57%, 9 students scored 11 marks representing 14.86%, 11 students scored 9 marks representing 12.16% and 24 students scored 5 marks representing 6.76% which means that, greater number of 24 students in the class performed poorly in the test and no student scored marks above 20, these shows that students have problems in solving linear equations in one variable and an intervention was putting in place in order to remedy the situation. Understanding and solving problems involving linear equations in one variable is one of the most important topics to be learned as a prerequisite to the study of algebra (Dugopolski, 2002). Unfortunately, pupils have challenges in learning linear equations in one variable (Richard, 2002).

Their performance at external examination is also not impressive. This is confirmed by the chief examiners report (WAEC, 2011), that most pupils could not answer questions on linear equations in one variable and those who attempted them showed little or no understanding of the principles of solving such equations. Studies have also shown that students are not able to solve mathematical problems in which linear equations in one variable are included (Brizuela and Schliemann, 2004).

The West African Examination Council Chief Examiners' Report (2001; 2002; 2006) pointed out that, majority of Junior High School students refrained from answering questions involving linear equations at the Basic Education Certificate Examination (BECE). Dogbe, Mereku and Quarcoo (2004) reported students' difficulty in linear equations by how they manipulate algebraic variables. Besides, reports from Anamuah- Mensah and Mereku (2005) indicated Ghanaians Junior High School two (2) students' abysmal mathematics achievement in algebra (including linear equations) among other content areas in the TIMSS-2003. According to them, the analysis of Ghanaian students' performance on the released items indicated that algebra (including linear equations) among other content areas was the candidates' weak area.

The mean percentage of the students making correct responses to the released items in algebra (including linear equations) was reported as 13.6% which reflected students' difficulties. This may be due to how algebra and for that matter linear equations are presented and taught to these students in various schools. Alio and

Harbor-Peters (2000) reported that non-exposure of students to appropriate representations and techniques by mathematics teachers are one of the factors responsible for the consistent poor performance in mathematics. Consistent with this, Mereku (2001) reported little attention given to other forms of representations contributing to low performance of students at the basic level of education.

It has also been found that most teachers have challenges integrating multiple representations in their teaching environment (Stein, Baxter & Leinhardt, 1990; Even, 1998; Celik & Baki, 2007). Therefore, most focus mainly on one mode of representation ignoring other representations which affect understanding (Hitt, 2001). Consistent with the research findings of Moseley and Brenner (1997), Swafford and Langrall (2000), Pape and Techoshanov (2001), Lowrie(2001), Canterbury (2007) and Ainsworth (2008) instructions that are based on single representation have been found not to be describing mathematical concepts fully and consequently affect students' understanding.

This suggests that an effective and appropriate classroom instruction should be adopted to help students understand and work skillfully with linear equations and other mathematics concepts. To do this, it is very important to conduct an empirical study to investigate effects of multiple-representations-based instruction on students' achievement in linear equations in the Bimbilla Municipality.

1.3 Purpose of the study

The purpose of the study was to investigate effects of multiple representations-based instruction on students' achievement in linear equations in one variable.

1.4 Objectives of the study

The specific objectives of this study focused on the following:

1. To investigate the mode of representations teachers use to teach linear equations in one variable in the Bimbilla Municipality.
2. To investigate what accounts for teachers' choice of representations in teaching linear equations in one variable in the Bimbilla Municipality.
3. To find out students' representation preferences in linear equations in one variable in the Bimbilla Municipality.
4. To investigate the effects of multiple representations-based instruction on students' achievement in linear equations in one variable in the Bimbilla Municipality.

1.5 Research Questions of the study

1. What mode of representations do teachers use to teach linear equations in one variable in the Bimbilla Municipality?
2. What accounts for teachers' choice of representations in teaching linear equations in one variable in the Bimbilla Municipality?
3. What forms of representations do students prefer to be taught linear equations in one variable in the Bimbilla Municipality?

1.6 Hypothesis of the study

The following research hypothesis was formulated to guide the study:

H_0: There is no statistically significant difference between students' scores in linear equations achievement test using multiple representations-based instruction and traditional instruction after controlling for students' age, gender, and pre-test scores.

1.7 Significance of the study

Significance for teachers: The relevance of this study to teachers cannot be over emphasized. The results from the study will reveal the appropriate representations teachers can use in the classroom to teach linear equations. This will add to teachers' knowledge in teaching linear equations in one variable.

Significance for students: The study will help boost student's confidence and interest in linear equations which will consequently influence their attitude positively towards mathematics and ground them as well in linear equations.

Significance for policy makers: The findings from this study will be of great benefit to bodies such as National Council for Curriculum and Assessment (NaCCA), Ministry of Education, Ghana Education Service, West African Examination Council (WAEC) and stakeholders who matter in education to embrace the concept of multiple representations-based instruction in teaching to meet the needs of students in Ghanaians schools. Similarly, the study will provide empirical evidence and data base (resource material) for stakeholders and future researchers who intend to research further into this area of study.

Significance for administrators: The findings from the study will go a long way to influence school administrators positively to appreciate the essence of encouraging and motivating teachers to utilize more representations in developing concepts in mathematics.

1.8 Delimitation

Linear equations cover a wide area of study such as equations in two and three variables. For the sake of this research, the study was restricted to effects of multiple representations-based instruction in linear equations in one variable. Also, the study was restricted to only Junior High School two (2) students in the forty-five (45) public

Junior High Schools as well as the mathematics teachers in these schools in the Bimbilla Municipality in the Northern Region of Ghana.

1.9 Limitation

Limitations are conditions which go beyond the researchers control and place some difficulties on the conclusions of the study (Best & Khan, 2005). Limitations are conditions or things a researcher does not have influence over and yet influence the generalizability of research results. The researcher would have extended the study to cover more basic schools, but because of financial problems and time constraints as well as lack of means of transport and limited resources, the study was limited to Junior High School Form Two Students in the Bimbilla municipality. Also, opportunity to secure instructional hours within the normal school hours was not possible so meeting hours with the students were scheduled from 2:00pm to 2:45pm. In fact, this made the students tired even though the researcher managed to sustain their interest.

1.10 Organization of the Study

The study was organized systematically into five different chapters. The first chapter which is the introduction consists of the background of the study, statement of the problem, purpose of the study, objectives of the study, research questions, and hypotheses of the study, significance of the study, delimitation, organization of the study and operational definitions of the study. Chapter two is the review of the related literature which highlights relevant views and ideas on the topic from other researchers, authorities and authors. Chapter three consists of the research design, population, sample and sampling techniques, research instruments, pilot study of the instruments, data collection procedures and data analysis. Chapter four deals with analysis of results and discussion while Chapter five focuses primarily on summary of major findings, conclusions, limitations, recommendations and suggestions for further studies.

1.11 Operational Definitions of Terms

Multiple representations-based instruction: Using blend of modes of representations such as algebraic, manipulatives, graphic, tables, etc to teach a concept or solve a problem.

Traditional or Regular instruction: Using one mode of representation such as algebraic representation to teach a mathematical concept (Smith, 2004). Therefore, traditional or regular instruction will be used interchangeably in this study.

Mode of representation: Mode of representation refers to particular representation which may be algebraic, manipulatives, graphic, tabular, spoken or written symbols use to teach mathematics concept.

Linear equation in one variable: This is first-degree equation in which variables have the highest exponent of one (Dugopolski, 2002). For instance, $2x + 4 = 6$, $3x + 2 = 10 + x$.

Manipulatives: Using concrete objects such as algebra tiles and other accessories to teach linear equations in one variable.

Graphical representations: Using graph to teach linear equations in one variable.

Algebraic representations: Using transposition and other related techniques in teaching linear equations without manipulatives and graphs.

Experimental group 1: Group who received multiple representations-based instruction with three different representations (algebraic, manipulatives and graphic).

Experimental group 2: Group who received multiple representations-based instruction with two different representations (manipulatives and algebraic).

Control group: Group who received traditional instruction with only algebraic representation.

Students' achievement: Students achievement refers to how students perform on the linear equations' achievement test.

CHAPTER TWO

REVIEW OF RELATED LITERATURE

2.0 Overview

This chapter discusses the review of related literature which highlights relevant views and ideas on the topic under study from other researchers, authorities and authors. The coverage of the review includes the following thematic areas: theoretical framework, the concept of representations, research studies on multiple representations, meaning of algebra, the concept of linear equations, understanding linear equations, representational modes for teaching linear equations, research on teachers' mode of representations in the classroom and what accounts for it, representation preferences of students and a summary of the chapter.

2.1 Theoretical Framework

The study is grounded on the multiple embodiment principle of Dienes (1960). The multiple embodiment principle states that by varying the contexts, situations and frames in which isomorphic structures occur, the learner is presented opportunities through which structural (conceptual) mathematical similarities can be abstracted (Dienes, 1960). In the same way, students' conceptual learning in the domain of mathematics is enhanced when students are exposed to a concept through a variety of physical representations (Dienes, 1960). For instance, when learning to solve simple linear equations say $4x + 2 = 10$, a student may be presented with how to use manipulatives (such as algebra tiles), graphs or algebraic representation. Similarly, different environments can be provided to make students see the structure of a concept from different perspectives to build a rich store of mental images (Resnick & Ford, 1981). When this is done, the student is presented opportunities through which structural mathematical similarities can be abstracted.

According to Dienes (1960), each learner sees the world differently, approaches differently and understands it differently. For these reasons, to make all students learn a concept with active participation, introduce them to multiple representations. Consistent with constructivists view, Slavin (2000) and Goldin (1 990), found that students construct knowledge by themselves through active participation in the classroom by providing interactive environment through multiple representations.

In support of Dienes' multiple embodiment principle, Bruner (1966) emphasized three categories of representations (enactive, iconic and symbolic) in the classroom. According to Bruner (1966), students need to be engaged in these representations to build a complete understanding of a mathematical concept. The enactive representation includes manipulatives and other concrete materials use to represent mathematics concept. The iconic involves using pictures, drawings and graphs to represent the same mathematics concept while the symbolic representation deals with algebraic symbols and numbers to represent the same mathematics concepts.

Janvier (1987) and Lesh, Post and Behr (1987a) also supported the Dienes' multiple embodiment principle by stating that learners should be able to identify a given mathematical idea across different representations, manipulate the idea within a variety of representations and translate the idea from one representation to another. In view of this, Janvier (1987) emphasized representations involving tables, graphs, formulation, verbal descriptions and objects in the classroom while Lesh, Post and Behr (1987b) emphasized representations involving real-world situations, manipulatives, pictures or diagrams, spoken and written symbols in the classroom.

These representations are shown diagrammatically in the Figure 2.1 and Figure 2.2

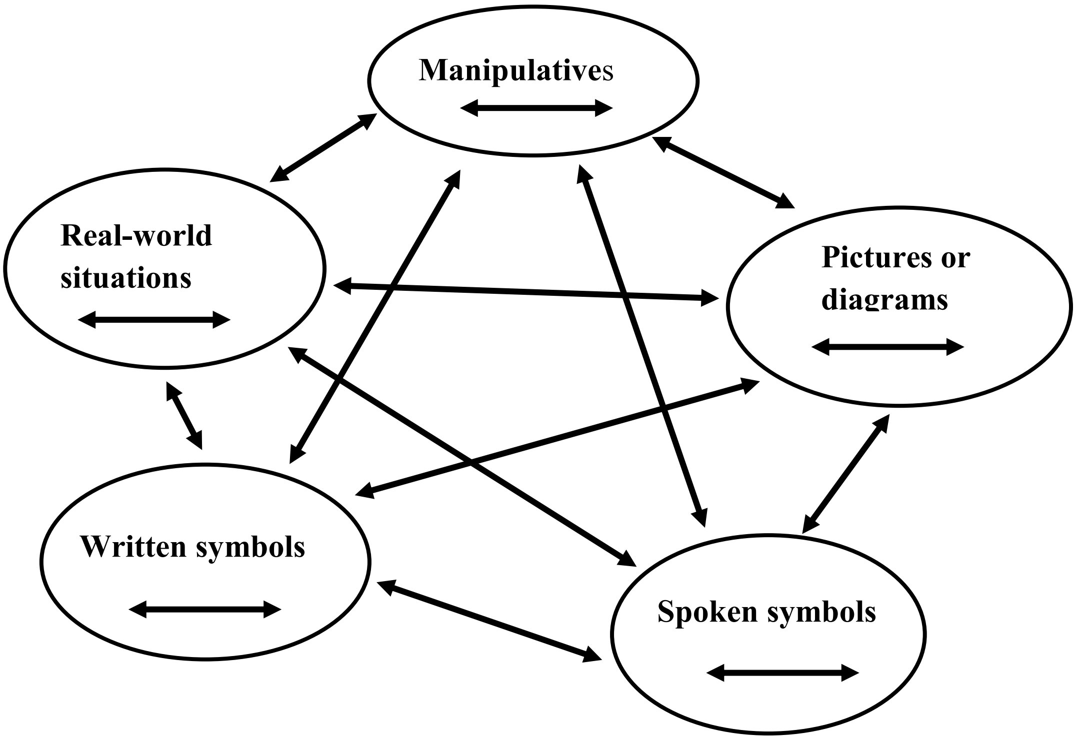

Figure 2.1: Lesh et al. model of translations among representations

The model in Figure 2.1 highlights translations among representations involving real-world situations, manipulatives, pictures or diagrams, spoken and written symbols. This re-echoes the need for multiple representations in the classroom instruction.

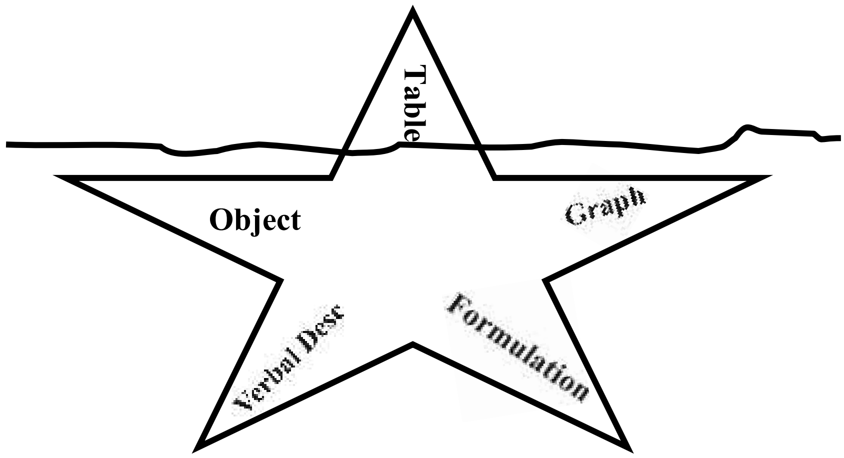

Figure 2.2: Model of representations by Javier

The model in figure 2.2 highlights representations involving tables, graphs, formulation, verbal descriptions and objects indicating the need to teach a concept by translating from one representation to another.

2.2 The Concept of Representations

The concept of multiple representations has been widely emphasized in the practices of education. A lot of studies have been carried out in this area of study and how it contributes to teaching and learning. Before reviewing literature on multiple representations extensively, it is important and crucial to review the concept of representations and its types.

2.2.1 Representations

The term representation has many meanings depending on the context it is used. In the context of teaching and learning, Goldin (2000) a constructivist, explains representations as systems including spoken and written symbols, pictures and graphs, manipulatives models and real-world situations. According to Goldin, one of these forms of representation must be used before a student can assimilate concept. Janvier (1987) explains representations as something written on paper, something existing in the form of physical objects or carefully constructed arrangement of ideas in one's mind. Janvier therefore classified representations into three components; written symbols, real objects and mental images.

Adding to others views, Lesh, Post and Behr (1987b) define representations as external embodiment of students' internal conceptualizations. They identified five modes of representation which includes; real-world situations, manipulatives, pictures or diagrams (static figural models), spoken symbols and written symbols. Kaput (1989) added by saying, representations are means by which individuals make sense of situations. The sense making can be through physical embodiments in the environment or mental configurations of that situation. In the view of Klein (2003) representations are signs from which students learn something. Seeger, Voight and Werschescio (1998) consider representations as any kind of mental state with a specific content, a picture, symbol or sign, a mental reproduction of a former mental state or something in place of something else.

2.2.2 External and Internal Representations

According to Pape and Tchoshanov (2001) representations can be grouped into external and internal representations. These two types of representations have been distinguished by researchers in one way or the other. Goldin and Janvier (1998) explain external representations as structured physical representations that can be seen as embodying mathematical ideas while internal representations deal with the individual's cognitive configurations. Thus, the external representations are the physical embodiments and the internal representations are what students form in their minds. Consistent with constructivists view is Cobb, Yackel and Wood (1992) and Goldin and Shteingold (2001) who explain internal representations as inside students' heads whereas external representations are situated in the students' environment.

Duffour-Janvier, Bednarz and Belanger (1987) also contributed to clarify external and internal representations from the semiotic view point. According to them, the term ''signified'' refers to internal representations that deal with external symbolic organizations that represent certain mathematical reality. In line with this distinction, Kaput (1994) stressed that clear distinction between signified and signifier should be established for achieving a permanent and meaningful learning and also the internal representations (signified) are mental configurations that should be created and developed by the person himself. They are not observable whereas external representations (signifier) are physical configurations and observable which include: pictures, graphs, symbols, manipulatives, equations or computer signs. Kaput emphasized that, the two concepts do not exist in isolation. Also, Janvier, Girardon and Morand (1993) indicated that external representations (signifier) act as stimuli on the senses whereas internal representations (signified) are regarded as cognitive or mental models, schemas, conceptions which are illusive and not directly observed.

Pape and Tchoshanov (2001) also made a significant contribution to the distinction between internal and external representations. They illustrated this in the context of numeracy where "five'' was used to show the relationship between internal and external representations in the form of circles. In the outer circle, drawings, manipulatives, written and verbal symbols were used to describe the concept of "five" which they described as external representations. In the inner circle, children's mental image of "five" was described which they called internal representations. This illustration throws more light on external representations as physical embodiment that can be manipulated whereas internal representations are seen as the mental construct of the external representations. They further emphasized that, learners should experience variety of external representations of mathematics concepts to create internal representations.

Zhang (1997) explains internal representations as mental images that deal with internal formulation of what we see around us. According to him, the two concepts of representations exist together. For instance, a child concept of "three" cannot be observed unless this is demonstrated with external representations. Once this is demonstrated, it means the child has already constructed in mind the concept of "three". Consistent with this, Hiebert and Carpenter (1992) acknowledged internal and external representations as closely related entities. To them, how students interact with external representations influence how the students represent the quantity internally. Thus, how a student deals with external representation tells how he or she has represented the information internally. This two-way interaction between external and internal representations is needed to help promote understanding and development of mathematical concept (Zhang, 1997).

In view of Kaput (1991), internal representations are mental structures and external representations being notation systems. To Kaput, the mental structures are means by which an individual organizes and manages the flow of experiences while the notation systems are material or linguistics artifacts. Kaput noted that one cannot learn something from notational systems when those systems are told separately from mental structures. This tells the co-existence between internal and external representations. However, for the sake of this research work, the focus is on external representations which will in turn affect internal representations since the two concepts co-exist.

2. 3 Multiple Representations

Having look at the relationships between external and internal representations, it is important to consider the concept of multiple representations. According to Ainsworth (1999), multiple representation is when various representations are used for teaching a concept or solving a problem instead of only one mathematical representation. Multiple representations can also be explained as providing the same information in more than one form of external mathematical representation (Goldin & Shteingold, 2001). Also, Owens and Clement (1997) explain multiple representations as identifying a mathematical idea in a set of different representations. Ozgun-Koca (1998) considers multiple representations as external mathematical embodiments that deal with providing the same information in more than one form of representation.

Thus, when a teacher uses different mode of representations to teach the same concept, then the teacher is said to have applied multiple representations. For instance, linear equations in one variable can be taught using algebraic, manipulatives and graphic representations.

The usefulness of employing this form of instruction (multiple representations) is acknowledged in the work of Dienes (1960), Bruner (1966) and other researchers. Bruner emphasized that even though some students might be quite ready for pure symbolic representation, it seems wise to present at least the iconic mode as well as so that learners would have mental images to fall back as incase their symbolic representations failed. This shows the essence of multiple representations. Because of this essence, the National Council of Teachers of Mathematics (2000) stressed the need to help students develop ways of interpreting and thinking about mathematics through multiple representations. Monk (2004) found that one of the purposeful activities' students can be engaged is by using multiple representations.

Researchers like Suh and Moyer (2007) found that by multiple representations, students can deepen their mathematical understanding. This is consistent with Herman's (2007) findings that, as students use different representations and become more comfortable with them their understanding of the concept deepens and by so doing, they can organize their ideas. This is because, understanding in mathematics depends on individual's ability to represent a mathematical idea in multiple ways (Cramer & Bezuk, 1991).

Several studies have shown that students have different learning styles and multiple intelligences (Silver & Strong, 2003; Gardner, 1993). Some of which are stronger than others and by using multiple representations in the classroom, students will have a chance to show off their strengths as well as develop those intelligences which are weaker.

Consistent with Gardner's view, Alagic (2003) states that using multiple representations provide a chance for students with a diverse ability level to be engaged and pointed out that, as representation forms vary, students with different levels of understanding will always have a type of representation they understand and this will move them to higher levels of understanding from that point.

Similarly, Spiro and Jehn (1990) found that multiple external representations encourage knowledge reconstruction since different representation can express the same idea in different ways. Cheng (2000) go as far to say that multiple representations are not only helpful but also an indispensable tool for conceptual learning in mathematics and science. The understanding students can gain by using multiple representations is broader and deeper than using single representation (Elia, Panaoura, Eracleous & Gagatsis, 2007). In view of this, Lesh, Post and Behr (1987b) emphasized for students to understand $\frac{1}{3}$ they should be able to recognized $\frac{1}{3}$ in a variety of different representations. In the same way, the concept of linear equations such as $x + 5 = 3x + 13,\ 3x - 2 = x + 4,\ 2(x + 3) = -3x - 4,\ \frac{1}{4}x - 4 = \frac{-1}{2}x - 1$ etc. can be understood when students experience them in a variety of different representations.

2.4 Research Studies on Multiple Representations

Several studies are conducted on multiple representations in linear equations and other concepts in mathematics. Even though other studies have reported students' difficulties in dealing with multiple representations and sometimes failing to benefit from it (Duval, 2006; Ainsworth, 2006; Sfard, 2000; Yerushalmy, 1991), yet there are broad base empirical evidences indicating significant improvement in students' achievements.

Beyranevand (2010) investigated the associations between students' achievement levels and abilities with respect to multiple representations. Participants were to recognize same linear relationships presented in different ways and also to solve linear equations with one unknown presented in multiple ways. Students ($n = 443$) from seventh and eighth grades participated in the study. They were asked to solve problems presented in verbal, pictorial and symbolic representations. Data were collected by means of tests, interview and survey and analyzed by means of multiple regression, correlation and chi-square. The results revealed that students who identified linear relations presented in different ways and solved linear equations with one unknown presented in multiple ways were significantly more likely to perform at higher level.

Doktoroglu (2013) also investigated the effects of teaching linear equations with a dynamic mathematics software on seventh grade students' achievement. The seventh-grade students (n= 60) explored linear equations with a dynamic mathematics software in three representations namely: tabular, algebraic and graphical. After various activities in the treatment group for three weeks, the researcher administered three mathematics achievement tests to assess students. Data gathered were analyzed by an analysis of covariance (ANCOVA). The results indicated a significant effect on students' achievement in linear equations. The researcher concluded with a reason that the experimental group students had a chance to examine three representations namely: tabular, algebraic and graphical.

Cikla (2004) investigated the effects of multiple representations-based instruction on seventh grade students' algebra performance, attitudes towards mathematics and representation preference. Seventh grade students ($n = 131$) from two public school participated in the study. Treatment groups experienced a series of activities on linear equations, functions and other aspects of algebra through multiple representations. Algebra achievement tests were administered to students. Data analyzed by means of multivariate covariance (MANCOVA) and chi-square indicated that, multiple representations-based instruction has a significance effects on algebra performance compare to the conventional teaching. Also, it was revealed that the experimental group used variety of representations for algebra problems. The researcher then recommended the use of multiple representations-based instruction for teaching mathematical concepts.

Hong, Thomas and Kwon (1999) carried out a study investigated students' understanding of linear equations via super calculator representations. A total of 35 students from a middle school were used. Students explored linear equations such as $2x - 5 = 3x - 9, x + 6 = -2x - 3$ etc. through three different representations such as symbolic, tabular and graphical representations. Data collected through pre-test and post-test indicated significant improvement of students in linear equations.

Rider (2004) researched into probable advantages of multiple representations on students' understanding and translations among graphical, tabular and symbolic representations of algebraic concepts. Eight (8) students were taught with traditional algebraic curriculum which emphasized symbolic representation and another eight (8) students taught with reform-based curriculum in which multiple representations were used. The results of this study revealed that multiple representations could be effective for helping students to improve their conceptual knowledge of algebraic and functional concepts. Also, Thompson and Senk (2001) investigated the effectiveness of advance algebra-based curriculum and instruction which emphasized representations such as pictures, graphs and concrete objects. The results indicated that students in the experimental classes did significant better on all instruments when compared with the control classes. Further, Girard (2002) investigated students understanding of limits and derivative concepts in terms of multiple representations. The two concepts were developed through numerical, tabular, graphical and algebraic representations. The students were later given tasks on conceptual understanding and representational methods of limit and derivatives. The results revealed that students who experienced multiple representations-based instruction demonstrated a multiple representational knowledge. It was also found that the algebraic mode of representation was most popular representation among the students even though tabular and graphical modes of representations were used by the students. The researcher then recommended multiple representations-based instruction to avoid excessive use of algebraic mode of representation.

2.5 Meaning of Algebra

The concepts of linear equations are embedded in algebra. Therefore, there is the need to discuss the meaning of algebra before concentrating on linear equations in one variable. The concept of algebra dates back to the Babylonians and these were the works of Hindus and Greeks which they normally preserved (Thompson, 1999). As the time grows, the Arabs took the work of the Greek and Hindus and expanded them. It is believed that algebra was derived from the book titled *ad-jabrw'al-muqabala,* the work of an Arabic mathematician named Muhammad Ibn Musa Al-Khwarizmi (Charles, 1994). The book was later called *al-jabr* and translated in Latin version as algebra. In the book, symbols were used to solve linear equations which gained popularity.

The work influenced Francois Viete (1540-1603), European mathematician who used Diaphantus ideas to develop symbolic algebra (Berlinghoff & Gouvea, 2002). Viete used letters to represent numbers. That is, vowels for unknown quantities and consonants for unknown numbers in equations which led the way to modern day algebra. Algebra was widely accepted by world mathematicians when Viete associated it with the Greek civilization (Berlinghoff & Gouvea, 2002). Robert Rocorde (1510- 1558), an English mathematician also contributed to the History of algebra. Recorde used the modern symbolism for equal sign in algebra books. The equal sign is widely used today in system of equations. However, it was Rene Descartes (1596-1650) who modernized algebra. In his book in titled *La Geometrie,* Descartes introduced modern day notation using lower case letters from the end of the alphabet for unknown quantities and lowercase letters from the beginning of the alphabet for known quantities. Following the historically development of algebra, a lot of researchers, authorities and authors have attempted to define or explain the concept in one way or the other. As explained by Mason (1996) algebra is derived from the problems of *al-jabr* which literally means adding or multiplying both sides of an equation by the same thing in order to eliminate negative or fractional terms which were paralleled by the problems of *al-muqabala* which also translates literally as subtracting the same thing from or dividing the same into both sides. This explanation only considers algebra as the process of solving equations. Sowell (1989) also describes algebra as a system consisting of a set together with operations that follow certain properties and techniques use in solving equations. Sowell pointed out that even after more than thousand years, solving equations and simplifying expressions continue to be the primary topics in an algebra course.

Wheeler (1996) describes algebra as a system (its presence is recognized by symbols), a calculus (it is used in computing numerical solutions to problems) and a representational system (it plays a major role in the mathematization of situations and experiences). Stacy and Chick (2004) added that algebra is a way of expressing generality, a study of symbol manipulation and equation solving, a study of functions, a way to solve certain classes of problems, a way to model real situations and a formal system involving set theory, logic and operations on entities. According to Kieran (2004), algebra can be understood as comprising activities such as generational, transformational and global or meta-level activities. The generational activities involve expressions and equations that are the objects of algebra. The transformational activities include collecting like terms, factorizing, substitution, expanding, adding and multiplying expressions, solving equations, simplifying expressions while the global or meta-level activities are those activities for which algebra is used as a tool and this includes: problem solving, modelling, noticing structure, studying changes, generalizing, analyzing relationships, justifying and predicting.

Bednarz, Kieran and Lee (1996) also view algebra as the study of regularities governing numerical relations, a conception that centers on generalization which can be widened by including the components of proof and validations. They continued to stress that algebra covers identifying and generalizing patterns by using symbols to express them in general terms. In the view of Laud (1995), algebra is a short hand of arithmetic where letters and symbols represent numbers. For instance, $3x - 4 = 8$, $4y + 6 = 9$ etc.

Usiskin (1998, cited in Egodawatte, 2011) characterized the meaning of algebra into four categories. The first one considers as generalized arithmetic where a variable is considered as a pattern generalizer. For instance, the arithmetic expressions such as $-3 \times 2 = -6, -1 \times 5 = -5$ could be generalized to give properties as $-x \times y = -xy$. The second characterization considers algebra as procedures for solving certain kinds of problems. For instance, $5x + 3 = 43$ could be solved by subtracting 3 from both sides of the equal sign and then dividing both sides of the equal sign by 5 to obtain $x = 8$. The variables in this category are either unknown or constant. Also, the third considers algebra as relationship among quantities where variables tend to vary. For example, for the area of a parallelogram of the form $A = b \times$ h, the variables A, b and h can take many values. Finally, the fourth category considers algebra as study of structures. For instance, when factorizing the problem $2x^2+6ax - 8x$the variable neither act as an unknown nor it is an argument. In the light of various subsets of algebra as discussed, the study is limited to only linear equations in one variable.

2.6 The Concept of Linear Equations

According to Berlighoff and Gouvea (2002), equations developed over several thousand years using the work of mathematicians from different civilizations. To them, equations first transpired between the first and third century in the Common Era.

The first use of equations was developed in Egyptians and Babylonians work. The evidence shows that Ancient Egyptians could solve simple linear equations using addition, multiplication and division. Babylonians also recorded mathematical ideas on tablets which eventually evolved into modern equations. They used many methods such as false position and parts of their hands to solve for unknowns and problems (Berlinghoff & Gouvea, 2002).

The ideas of equations were developed following the work of Ancient Egyptians and Babylonians, Diaphantus, a Greek mathematician used symbolic abbreviations to study number theory. Diaphantus wrote a book named *Arithmetica* containing 189 problems comprising Greek letters and symbols. The first time the formal equations were created (Bashmakova & Smirnova, 2000; Derbyshire, 2006). The work was a close representation of modern equations containing one unknown. Concepts like manipulating terms from one side of the equation to the other, distributing and factorizing were also demonstrated in *Arithmetica*. Diaphantus explicitly developed and used rules for multiplying negative and positive numbers (Bashmakova & Smirnova, 2000). Like the Egyptians, the Indians also used equations and words to develop their ideas. Their work often included several variables (Bashmakova & Smirnova, 2000). Then again, the concept of linear equations was used in the work of Muhammad Ibn Musa Al-Khwarizmi, an Arabic mathematician in the book titled *ad-jabr al-muqabala.* Al-Khwarizmi used geometric explanations and justifications to solve equations similar to those from Mesopotamian times. Much is not forgotten about the immense contributions of other Western European mathematicians, Francois Svietes (1540 – 1603), Rene Descartes (1596 – 1650) and Robert Recorde (1510 – 1558) in the development of linear equations (Berlinghoff & Gouvea, 2002; Kieran, 1992). In the 15th century, the universal use of mathematical symbols for solving equations was introduced. It took several years for the notation and symbols to evolve into those used today in textbooks and classrooms by all mathematicians.

Following the historical development of the concept of linear equations over the past to present, many researchers, authorities and authors have attempted to define or explain the concept. Dressler and Keenan (1998) define equations as sentence that describes two algebraic expressions as equal. To them, an equation may be: a true sentence such as 5 + 2 = 7, a false sentence such as 6 – 3 = 4 or an open sentence such as $x + 3 = 9$. The number that can replace the variable in an open sentence to make the sentence true is called a root or a solution of the equation. Consistent with Dressler and Keenan view, Calvin (1990) defines equation as a statement of equality of mathematical expressions. That is, mathematical expressions containing equal sign (=). Usiskin, Pereessini, Marchisoto and Stanley (2003) explain that equations define functions, express one variable in terms of the other or provide information about when a quantity is maximized or minimized. In the other, domain, Amissah (1998) identified reflective, symmetric, transitive and substitutive as axioms of equality for solving linear equations. If a = b, then the reflective axiom is assumed for all real numbers. For symmetric axiom, if a = b, then b = a.

The transitive axiom means if a = b and b = c, then a = c, and substitutive axiom implies if a = b, then a may replace b in any statement without affecting the truth or falsity of the statement.

In terms of linear equations, Prosser and Trigwell (1999) describes it as an algebraic statement in first-degree that is made up of three basic parts which include the equal sign (=), the expression to the left of the equal sign and the expression to the right of the equal sign. For instance, in the equation $3x + 2 = x + 12$, the three basic parts can be defined as: $3x + 2$ (the expression to the left of the equation sign), = (the equality sign) and $x + 12$ (the expression to the right of the equal sign). In the view of Singletary (1995) linear equations in one variable is the equation that can be written in the form $ax + b = c$ where a, b and c are real numbers and $a \neq 0$. Singletary explained further that linear equations in one variable can take the form $ax = b or ax + b = 0$ where a and b are real numbers and a $\neq$ 0. Dugopolski (2002) also defined linear equations in one variable as a first–degree equations where the variable have the highest exponent of one. For example, in the equation $x + 4 = 9$, the highest exponent of x is one (1) and this makes the equation linear.

Streter, Hutchison and Hoelzle (2001) explain linear equation in one variable as an equation which contains only one unknown quantity in the mathematical statement. The process of solving such equation is to find the value of the unknown quantity that makes the mathematical statement true. For instance, the equation $8x - 4 = 3x + 11$ contains only one unknown quantity which is represented by x and it is true when $x = 3$. Martin (1994) pointed out that, when you add or subtract the same number to or from both sides of an equation, the equation formed (second equation) is equivalent to the first one. In the same vein, when you multiply or divide both sides of linear equation by the same non-zero number, the second equation obtained is equivalent to the first one. For example, in the equation $x + 5 = 8$, when we subtract two (2) from both sides $(x + 5 - 2 = 8 - 2)$ to obtain$x + 3 = 6$, the equation formed is equivalent to the first one (i.e.$x + 3 = 6$).

To add to others, view, Laud (1995) defines linear equation as a statement where one algebraic expression in the first-degree equals another. Laud explained further that the equality of an equation is maintained if: (1) the same number is added to both sides of the equation. For example, if $3x = y$, then $n + 3x = n + y$.(2) the same number is subtracted from the both sides of the equation. For example, if $x = 3y$, then $x - 2 = 3y = 2$. (3) both sides are multiplying the same number. For example, $m = n$, then $am = an$. (4) both sides are divided by same number. For example, if $5a = b$, then $\frac{5a}{x} = \frac{b}{x}, x \neq 0$. (5) two different symbols are equal to the same symbol, then they are equal to each other. For example, if $y = x$and $y = z$then $x = z$.

2.7 Understanding Linear Equations

Capraro and Joffrion (2006) found that the successful students in algebraic equations were those with higher level of conceptual understanding. This can be achieved when students understand the concept of equal sign, variables, coefficients, constants, algebraic terms and expressions in equations.

2.7.1 The Concept of Equal Sign

The interpretation of equal sign serves as a major factor that influences students understanding of linear equations. Several researchers have ascribed to this assertion by lamenting on the lack of understanding of the equal sign as a pervasive problem associated with algebra (Kilpatrick, Swafford, & Fiundel, 2001; Knuth, Stephens, McNeil & Alibali, 2006; Rojano, 2002). If students do not have proper understanding of equality, they find it difficult to solve equations or find solutions of equations (Essien&Setati, 2006). That is why Keiran (2004) emphasized that to understand solving linear equations, focus should be on the meaning of the equal sign. Many students at the elementary level and the Junior High School interpret the equation sign (=) as a 'command' or 'do something sign' rather than a symbol denoting the relation between two equal quantities and due to this misconception failed to understand equations (Knuth, Stephens, McNiel & Alibali, 2006; Kieran, 1992).

In support of this, Knuth, Stephens, McNiel, and Alibali (2006) conducted a research on students' understanding of equal sign. In their study, middle school students ($n = 177$) completed a written assessment in which students responded to three questions related to equal sign and solving equations. The students' responses were coded as rational, operational, other and no response. In the first question, students were asked to describe the meaning of the equal sign in 3 + 4 = 7. Over 50%

of the students provided the meaning if equal sign as operational. In the other questions, students were asked to solve multi step equations in one variable such as $4m + 10 = 70$. The results indicated that students who defined equal sign as rational were more likely to solve the equations. Researchers concluded that many elementary and middle school students demonstrate inadequate understanding of equal sign and frequently viewing the symbol as an announcement of a result of an arithmetic operation rather than as a symbol of mathematical equivalence.

Kilpatrick, Swafford and Findell (2001) also found that, many students either conceptualize the equal sign as separation of problem and solution or as a left to right directional symbol for working out problems. Consistent with this, Usiskin, Pereesini, Marchisoto and Stanley (2003) found that when students were asked to find out what number would make the statement 7 + … = 10 + 5 true, many gave the answer as 3, seeing 10 as the results after addition ignoring the 5 on the right. In extended computations, they also found students working 13 + 45 + 7 as 13 + 45 = 58 + 7 = 65. These misconceptions affect students because, some equations include variables and constants on both sides of the equal sign. For instance, in equations such as $4x + 2 = 2x + 10, 3(x - 2) = -4x + 1$, solutions should not be realized from only left to right but both cases.

Adding to the views, Kieran, Booker, Filloy, Vergnaud and Wheeler (1990) identified two interpretations of the equal sign as symmetric and transitive. The symmetric indicates quantities on both sides of the equal sign are equal while the transitive indicates that a quantity on one side can be transferred to the other side using rules. They found that in elementary school the equal sign is used more to announce a result than to express a symmetric or a transitive relation. They gave example in their research work where students solved problem as 2.30 + 3.20 = 5.50 –

1.50 = 4. They argued that the symmetric property of the equal sign is violated because, 2.30 + 3. 20 ≠ 4 and added that students perceived equal the sign as a left-to-right directional signal rather than a structural property.

The understanding of the equal sign is crucial in linear equations. Therefore, emphasis should be placed on it to foster understanding. As part of this, Herscovics and Kieran (1980) recommended to teachers to begin with arithmetic identities such as $4 \times 3 + 5 = 2 \times 9 - 1$. To them, this method is a way to overcome students' limited understanding of the equal sign and extend their understanding to algebraic equations.

2.7.2 The Concept of Variables, Constants, Coefficients, Algebraic terms and expressions

Students understanding of variables, constants, coefficients and other algebraic terms in linear equations go a long way to influence the way a particular linear equation is manipulated or solved.

Many researchers acknowledged this fact in linear equations (Kuchemann, 1981; Poon & Leung, 2010; Dobe, Mereku & Quarcoo, 2004). Some students find it difficult to interpret letters or variables in an equation and others even ignore them at all. For instance, in the expression $3x + 4$, students can easily ignore the role of the variable and write$7x$.

These errors are consistent with the findings of Welder (2012) who found students simplifying $39x - 4$ as $35x$and $2yz - 2y$as z.

Similarity, Matz (1982) reported that when students were asked to simplify $3x + 5 = y + 3$, most of the students ended up with $x + 5 = y$ because, students considered $3x - 3$ as $x$thereby producing $x + 5 = y$ as their final solution. Evidences from different studies continue to confirm students' difficulty in dealing with equations of the form $ax + b = cx + d$due to improper manipulation of algebraic terms (Dogbe, Mereku&Quarcoo, 2004; Filloy, Rajano&Puig, 2007). According to Carry, Lewis and Bernard (1980), these misconceptions even occur with college students which affect equations solving.

Research findings of the Poon and Leugh (2010) also indicated students' misconception of coefficients and constants in linear equations. They studied students' understanding of 2 and 2x. The study revealed that students did not know whether 2 is coefficient or constant in both cases and thus affected the way they solved linear equations. These misconceptions therefore require multiple representations without necessarily resorting to only algebraic representation in linear equations in one variable. This is because, the more we know how to solve problems, the more mathematical tools we have (Sidney, 1993).

2.8 Representational modes for teaching Linear Equations

There are many representational modes for teaching linear equations in one variable. Examples include: algebraic, manipulatives, graphic and tabular representations.

2.8.1 Algebraic representation

Algebraic representation is one of the oldest means of teaching linear equations in one variable. It is more traditional and involve a lot of rules and procedures. Procedures such as addition or subtracting from the both sides of the equal sign, combining like terms, distributing or factoring, multiplying or dividing both sides by a variable or number are more with this representation. Star (2005) found these procedures as necessary for solving equations. Studies have also shown that many teachers and educators preferred algebraic representation for teaching mathematical concepts (Hitt, 2001; Herman, 2007; Monoyiou, Papageorgiou & Gagatsis, 2007).

Filloy and Rojano (1989) found this in their study with the balance model where at certain point in time most of the students detached from the model and went quickly to solve linear equations using algebraic representation. However, Sharp and Adams (2002) argued that if the algebraic representation is used through constructive procedures, students develop conceptual understanding in mathematics.

Researchers (Kieran, 1992; Linsell, 2008, 2009) have found different strategies (such as using number facts, counting techniques, cover up, undoing, transposing and balancing etc) teachers and students used to solve problems on linear equations algebraically. At times, solving linear equations involved finding all then numbers that may replace a variable in an equation to make such statement true (Austin & Volrath, 1989). To do this, you substitute a number to the algebraic expression on both sides of the equal sign and you check the outcome (Odili, 1990). For instance, to solve for $x$in $5x + 3 = 2x + 15$, we can start by substituting$x = 0$ in both sides [i.e., $5(0) + 3 = 3, 2(0) + 15 = 15$].The statement is not true since the Left-Hand Side is not equal to the Right-Hand Side. You then try with 1, 2, 3 or 4 until you realize the statement is true when $x = 4$.

However, Kieran (1992) cautioned that by trying various values of x ton satisfy the solution, the algebraic expressions are not operated upon but their numerical instantiations are dealt with. To Kieran, there is the need to shift from that approach to a structural approach where different sets of operations are carried out not on numbers but on algebraic expressions as in:

$$2x - 6 = x + 8,$$
$$2x - 6 + 6 = x + 8 + 6$$
$$2x = x + 14$$
$$2x - x = x - x + 14$$

$$x = 14$$

Kieran pointed out that the structural approach helps students grasp the structure of an equation and become successful in solving equations.

Similarly, Matz (1981) identified two kinds of processes involved in solving first-degree equations as deduction and reduction. The deduction involved performing the same operation on both sides of the equal sign while reduction deals with replacing one expression by another equivalent expressions as in:

$$3x + 7 = 2x$$
$$3x + 7 - 2x = 2x - 2x(deduction),$$
$$x + 7 = 0\ (reduction),$$
$$x + 7 - 7 = 0 - 7\ (deduction),$$
$$x = \ -7\ (reduction).$$

The process of solving linear equations has been identified as the balancing method (Kieran, 1992). The balance is retained when the same item is from or added to both sides of the equal sign. According to Kieran, students lack this idea and therefore do not make sense of equations. Also, other research works have shown that when applying the balancing method and the equation contains negatives it becomes more difficult for students to understand (Vlassis, 2002). In support of this Vlassis (2002) conductor a qualitative study on eighth grades students to explore their symbolic understanding of the minus sign. It was observed that most students were unable to solve problems such as $4 - x = 5$ because of the negative sign next to the variable.

Leung, Clarke, Holton and Park (2014) found that when using the balance method many teachers fail to explain certain mathematical essence to students. They found this in one of their classroom study where a teacher emphasized “do the same to both sides” when solving the equation $3x + 2 = 2x + 3$. the researchers observed that the teacher failed to explain mathematically why you subtract $2x$ from both sides $(3x - 2x + 2 = 2x - 2x + 3)$. Also, the inverse properties of operations were not explained to the students. The inverses of operations are used to undo each other. That is, addition (+) is used to undo subtraction (-) and divisions undo multiplication (Sidney, 1993). For instance, subtraction undo addition as in:

$$x + 4 = 6$$
$$x + 4 - (4) = 6 - 4$$
$$x = 2$$

Also, addition undo subtraction as in:

$$x - 2 = 5$$
$$x - 2 + (2) = 5 + (2)$$
$$x = 7$$

Again, divisions undo multiplication as in:

$$3x = 9$$
$$\frac{3x}{3} = \frac{9}{3}$$
$$x = 3$$

Lastly, multiplication undo division as in:

$$\frac{x}{4} = 5$$
$$4 \times \frac{x}{4} = 4 \times 5$$
$$x = 20.$$

As suggested by Lima and Tall (2008), students who do not make the mathematical connections involving operations are less likely to be able to justify when and how to remove terms when solving linear equations. Solving equations by transposing terms from one side of the equation to the other side and changing its sign is frequently used by teachers and students. Consistent with this, Lima and Tall (2008) investigated students' conceptions of equations and the methods they used to solve equations. It was found that solving equations simply involved moving symbols around. Cortes and Pfaff (2000) also found that the principles used by the 17-year old student to solve equations in their study were all based on "movement" of symbols from one side to the other side of the equal sign with an additional changes of signs as if the symbols were physical entities that could be moved from one side to the other side of the equal sign. For instance, the equation $2x - 6 = -x - 12$ can be solved by transposing the terms as in:

$$2x - 6 = -x - 12,$$
$$2x + x = -12 + 6,$$
$$3x = -6$$
$$\frac{3x}{3} = \frac{-6}{3}$$
$$x = -2$$

However, Li (2007) cautioned that even though this approach is more efficient and much faster, it is less transparent. Another algebraic representation for solving linear equations by teachers and students is the "cover up'' (Kieran &Drijvers, 2006). In this, you cover up certain part of the equation when solving. For example, in the equation $2x + ? = 5x$. We now think of what can be added to $2x$ to obtain $5x$. When the number has been thought to be $3x$, uncover the 9 and equate them i.e. $3x = 9$.Cover the $x$in $3x$ which becomes $3 \times ? = 9$. $3 \times 3 = 9$, therefore, x = 3 (Kieran, 1992).

2.8.2 Representation Involving Manipulatives

According to Puchner, Taylor, O'Donnell and Fick (2010), manipulatives are concrete tools used to create an external representation of mathematical ideas. For instance, the concept of linear equations can be taught using algebra tiles for pupils to understand the idea of adding or subtracting a variable from both sides of the equal sign. Manipulatives can also be described as any material or object from the real world that learners can hold, touch and move around. They are materials that appeal to the several senses of the learner, arouse and sustain learners' interest and ensure active participation of learners (Heddens, 1997 cited in Larbi & Okyere, 2014).

Manipulatives such as base-ten blocks, algebra tiles, color tiles pattern blocks, balance scales, attribute blocks, Cuisenaire rods, Unifix cubes, fraction bars and counters have been used to solve many mathematical problems and being appreciated by many researchers in education (Boggan, Harper &Whitmire 2010).

Caglagan and Olive (2010) also conducted a qualitative study on eighth grades students ($n = 24$) who solved linear equations using cups to represent variables and tiles for constants. After classroom activities, the data collected through video tape and interview showed that the manipulatives helped students to see that $2x$ and 2 are different mathematical concepts. The students again realized that constants and variables cannot be combined because they are unlike terms. Again, Vlassis (2002) conducted a study to determine the effects of a balance model in solving linear equations where students ($n = 40$)of low ability levels participated in the study. The results indicated higher performance of students after eight months of treatment with the balance model. It was also revealed that students could easily understand equations. Similarly, Sherman and Bisanz (2009) carried out a quantitative study to investigate how students ($n = 48$) solve problems such as $5 + 2 = 4 + ...$ Using manipulatives and symbolic representations. The data collected from the classroom activities showed significant difference in favor of manipulatives used. The researchers concluded that students' performance was due to the physical representation of equality on both sides of the equation using the manipulatives. Studies of Kurumeh, Chiawa and Ibrahim (2010) have also shown a positive influence on students' performance when manipulative was used to establish the concept of equations.

Yıldız (2012) conducted a qualitative case study to investigate the views of middle school teachers and students about the use of manipulatives in teaching and learning mathematics. In this study, algebra tiles, base-ten blocks, fraction bars, pattern blocks, geoboards and four-pan balance were used as manipulatives. Participants were four middle school mathematics teachers in a private school and their 6^{th}, 7^{th}, and 8^{th} grade students. Data were collected through one-to-one interviews, observations and analysis of annual plan, daily plan, notebooks of students, and the field notes.

According to the findings of the study, most of the middle school students expressed that they desire to learn mathematics by using manipulatives and they stated that in this way they both played and learned. In addition, students claimed that using manipulatives enabled them to have positive attitudes toward mathematics and learn the concepts much better.

Padmore (2017) study examined the use of manipulative materials in teaching Mathematics among junior high school teachers in the Wa municipality of Ghana with a sample of 94 teachers and 10 head teachers. His study results on benefits of manipulative use are as follows; (1) manipulative materials improve pupils understanding and help them to construct their own knowledge of the subject easily (2) it saves a lot of time and allows teachers to cover more topics easily, motivates pupils and helps bring on boards their needs to be met (3) help pupils not to shy away from mathematics but are able to relate real world situation to mathematical symbolisms, allowing pupils to work cooperatively in solving problems, mathematics ideas and concepts and (4) makes mathematics fun and easy way for teachers to introduce concepts. Also, his study results on challenges of manipulative use are as follows; (1) inadequate supply of manipulative materials to teachers (2) lack of continuous professional training on manipulative use (3) inadequate user guides for teachers on the use of manipulative materials (4) High cost in preparing and purchasing manipulative materials. (5) Teacher's little knowledge as to the use of manipulative materials (6) too much work load on teachers and (7) large class size affected teachers not to use manipulative materials in teaching mathematics.

The study conducted in Nigeria by Aburime (2007) indicated a significance differences between the experimental groups and the control groups which favored the experimental groups. Aburime's study used 287 high school students in a 10-week mathematics manipulatives study. Akpalu, Adaboh and Boateng (2018) study examined the effects of algebraic tiles on Senior High School (SHS) students' conceptual understanding of a system of two linear equations. Their study used achievement test as the main instrument during the data collection. Their study also used simple random sampling technique in selecting 70 students equally to experimental and control groups. Per the result, there was a statistically significant in the post-test scores in favor of the experimental group who were taught using the algebraic tiles for four weeks.

Rosli, Goldsby and Capraro, (2015) contend that, the use of algebra tiles during mathematics lessons support students' acquisition of symbols and mathematical language. Boakye (2018) study explored the use of mathematics manipulatives in teaching three upper public primary schools. The study used nine (9) mathematics teachers and two-hundred (200) pupils as the sample. Structured questionnaires were the main instrument used to collect the data. The results of the study revealed that the use of manipulatives yielded positive results by helping teachers to clarify mathematical concepts, makes mathematics teaching very interesting and practical for the pupils. His study also reported that manipulatives helps pupils to increase their performance in solving mathematical problems. Furthermore, findings on his study on challenges of manipulatives use by teachers are as follows; inadequacy of several manipulatives made specifically for mathematics, difficulty in preparing or finding the right type of manipulatives to match some complex topics, the use of over complex manipulatives thus lead to wasting away the lesson, and large class sizes making distribution and control of the use of some manipulatives difficult.

McIntosh (2012) stated that the use of manipulative is highly effective in teaching mathematics and manipulative materials are valuable tools to help students of any academic level understands mathematics. According to McIntosh (2012), "it is clear that even with minimal exposure, students of all intelligence levels can benefit greatly from the use of manipulative materials" (p. 6). According to Brookie (2014), manipulative materials are interactive and adaptable in which teachers can use to help students of any academic levels. Having manipulative materials available for them brings about the understanding of the concepts and allow students to devise their own solution strategies, promote thinking and create confidence in learning mathematics and the use of manipulative materials help pupils to build on what they already know and pupils' strengths and weaknesses developed at their younger age (Brookie, 2014). Using algebra tiles in mathematics increases the students' confidence to complete difficult mathematics problems.

In the 2013 version of Turkish Middle Grades Mathematics Curriculum, the curricular context in which this study took place, objectives related to linear equations in one variable learning area take part in 6^{th} grade level for the first time and students are introduced to linear equations in one variable, variable term, constant term, and coefficient concepts (MoNE, 2013). If the students do not learn basic linear equations in one variable concepts at this grade level conceptually and symbolically, they may not understand the other linear equations concepts in coming years. Research showed that when the students successfully completed linear equations in one variable course which they took in middle schools, they got higher performance on mathematics tests and they understood advanced mathematics much easier (Wang & Goldschmidt, 2003). Therefore, exploring the effects of manipulative usage on 6^{th} grade students' linear equations in one variable achievement might provide information on the strength of this achievement for future mathematics achievement. Algebra tiles are a versatile manipulative that can be used by students to represent algebraic concepts beginning with integer arithmetic, continuing with activities involving linear expressions and equations, and ending with factoring and equation-solving for quadratics. They enable PSTs to state the rules of algebra from their own experiences (Okpube, 2016). Algebra tiles can easily be made by cutting the cardboards (Karakırık & Aydın, 2011). Similarly, pioneers such as Dienes, Bruner, Froebel, Montessori and the and National Council of Teachers of Mathematics (NCTM, 2010) acknowledged the relevance of using manipulatives at all levels of education based on the premise that students need physical referents to develop abstract mathematical concepts.

In terms of linear equations, Filloy and Sutherland (1996) found the need to model linear equations in concrete context so that students become endowed with meaning. Berman and Friederwitzer (1989) used envelops marked with letters to signify an unknown quantity to solve linear equations. At the end of this activity, it was reported that students could understand algebraic concepts when concrete models were used. Borenson (1994) created Hands-on –Equations Learning System, a desktop set of manipulatives consisting of a balance blue and white pawns and red and green number cubes for solving equations. The results of a quantitative study of Borenson and Barber (2008) of this manipulative indicated a statistically significant difference between the pretest and posttest scores for the participants. It was found that students were impressed to solve algebraic linear equations in a game-like manner.

In the same way, algebra tiles can be used to model linear equations such as $-4x + 6 = 10,\ 3(x - 2) = -4x + 1,\ 3x - 6 = x + 8,\ \frac{1}{2}x + 5 = 2x + 3$ etc. Algebra tiles have different shapes and colours representing variables and constants. For instance, positive variables(x) can be represented by yellow, green and blue bars and positive constants represented by yellow, green or blue squares while negative variables $(-x)$ represented by red bars and negative constants represented by red squares. Sharp (1995) reported that algebra tiles increase students' mental imagery resulting in an easier acquisition of learning. Also, the National Council of Teachers of Mathematics (2000) stressed that the use of algebra tiles in teaching linear equations take away the abstractness of the concept.

Sobol (1998) found that using algebra tiles had significant effect on 7th, 8th, and 9th grade students' learning of algebraic concept of zero and operations with integers and the polynomials. Use of algebra tiles increased treatment group students' understanding in mathematics learning process compared to control group in Larbi's (2011) experimental study. Saraswati et al. (2016) found that algebra tiles helped students find the formal solution of linear equation in one variable. Using algebra tiles have also been found to assist students when they make geometric connection to factoring polynomials (Schlosser, 2010). In the same way, while teaching solving quadratic equations by completing a square, using algebra tiles helped students build connections between algebraic and geometric concepts (Vinogradova, 2007). In addition, high school students expressed meaningful and easy learning through algebra tiles in Sharp (1995) study although there was not any difference between the test scores of students who used algebra tiles while factoring and those who did not.

Akyüz and Hangül (2013) conducted a research study to investigate and eliminate 6th grade students' misconceptions about first degree equations with one unknown. Participants were 25 sixth grade students in a public school in Balıkesir in the spring semester of the 2011-2012 academic year. Researchers implemented a test including 20 open-ended items to detect the misconceptions, and conducted interviews with the students. After that, activity-based instruction was given to students for eight hours and then posttest was given. During the activity-based instruction, algebra tiles, colored papers, balance, and seesaw and model plane were used. Researchers found that activity-based instruction was effective in overcoming students' misconceptions. In addition, they suggested that algebra instruction should first begin with concrete materials, and then move towards symbols in order to make students understand algebra concepts better.

Even though manipulative improve students' achievement, its drawbacks cannot be ignored. Caglayan and Olive (2010) found in their research study that students experienced difficulties in linking the physical activities of the manipulatives and the mental operations necessary for solving equations. Consistent with this, Kaput (1989) found that sometimes the connection between the action on the manipulative and the action on the symbolic notation are unclear. Because of the fact that the cognitive load imposed when working with the manipulative is too great for students. Manipulative use involved a lot of procedures and activities which students need to master all of them in order to comprehend the mathematical concepts behind it. Based on this, Borenson and Barber (2008) recommended that to make connection when solving linear equations with manipulatives, there should be written records of steps on paper to help students. McNiel and Utal (2009) also suggested that teachers must help students see connections between manipulatives and abstract concept behind it explicitly.

Clement and McMillan (1996) found that at times physical actions with manipulatives suggest different mental action from what the teacher wishes students to learn. Some students think and conceptualize different thing instead of the normal concept the teacher want them to develop when using manipulatives. In view of this, Baroody (1989) cautioned that manipulatives alone are not enough to guarantee success. Therefore, there is the need to combine manipulatives with other representations to teach mathematical concepts.

2.8.3 Graphical Representations

Graphical representation has been one of the interested representations for teaching concepts in recent years (Bardini, Pierce & Stacey, 2004; Marcus, Huntley, Kaham& Miller, 2007). This is due to the numerous advantages of using this mode of representation. Monk (2003) illustrated how graphical representations are important for mathematics power. According to him, using graphs help students help students to analyze and explore concepts. Also, students can construct new entities and build shared understanding through graphs. Tsamir and Almong (2001) found students more successful when solving equations and inequalities using graphical representation.

However, this mode of representation for teaching concepts has received relatively little attention in the mathematics curriculum (Demana, Schoen & Waits, 1993; Leinhardt, Zaslavsky & Stein, 1990). Tossavainnen (2009) reported that only few mathematics teachers have applied graphic approach as a principal tool for finding solutions of linear equations until the existence of modern computers and mathematical software's. Again, Proulx, Beisiegal, Miranda and Simmt (2009) indicated that graphical approaches are set aside making students dependent on algebraic techniques. This contributes to students' limited understanding of graphs (Blume& Heckman, 2000; Swafford & Brown, 1989).

Therefore, to enhance students understanding of equations without necessarily subtracting to get the variable terms on the left and the constant on the right (Vlassis, 2002; Star & Seifert, 2006), such equations can be presented in linear relation form $y = mx + b$ (Beatty, 2010; Li, 2007). For instance, the equation $2x + 6 = 8$ can be solved graphically by creating two equations: $y = 2x + 6$ and $y = 8$. The two functions are then graphed on the same coordinate's axes. The intersection of the two lines traced to x-axis determines the solution of the original equation (Li, 2007). In the same way, equations like $2x - 3 = 5x, 4x - 2 = x + 4$ can be graphed by creating two functions to determine the value of x.

Davis (2007) reported students' difficulty when graphing the linear relationship between the number of scoops of ice cream and the amount of money it cost. These difficulties may be attributed to algebraic representations dominating the mathematical curriculum (Arnold, 1992).

Others also believed that students who learn to solve linear equations only by a set of memorized rules tend to develop an incomplete understanding of solving of equations (Capraro & Joffrion, 2006). Therefore, in order to curtail this, Ainsworth, Bibby and Wood (1997) recommended that if better understanding is the desired outcome of the teaching curriculum, then unfamiliar representations should be presented alongside familiar ones. This suggests the necessity of graphical representation to be explored alongside the dominated representations in the curriculum to teach linear equations in one variable.

2.9 Research on Teachers' mode of Representations in the Classroom and what accounts for it

There have been little research studies that investigate directly mode of representations used by classroom teachers in solving or teaching linear equations in one variable. However, a good number of studies have investigated representations used by classroom teachers in solving mathematical problems and hence such corresponding research studies can be relevant to this study.

Gagatsis and Shiakalli (2004) conducted a study to examine teachers' use of verbal, graphical and algebraic representations in solving problems of functions. Also, how teachers move from one representation to another was examined. One hundred and ninety-five (195) teachers were sampled for the study and data collected through test. The findings of the study revealed that graphical representation was least used by the teachers. Also, it was reported that the teachers who used graphical representation had low success.

Bal (2014) investigated the types of representations classroom teachers use for routine and non-routine problems, factors that affect how the types of representations were chosen and problems they experienced while using the representations. A total of 100 participants were used for the study. The representations were classified as verbal, graphical, algebraic (symbolic) and numeric. Problem-solving test on multiple representations as well as semi-structured interview were organized to collect data. After the analysis, it was found that most of the teachers used algebraic representation representing 90% while graphical and other representations were used by few of the teachers representing 10%. Half the number of the participants described their reason for using algebraic representation as habitual and others as understandable. Some also indicated that other representations cannot be used to solve the problems. It was again found that representation used depends on teachers' understanding.

Monoyiou, Papageorgiou and Gagatsis (2007) conducted a study to examine the representations use in solving non-routine problems by school teachers in the classroom. A total of 20 teachers interviewed. The findings of the study indicated that the teachers mostly preferred algebraic representations. Consistent with this, Cai (2005) also found that teachers mostly preferred algebraic representations in problem-solving.

Herman (2007) also examined the representations of teachers in algebraic lessons and their beliefs about multiple representations. After ten weeks of the study, data were collected through interview, pre-test and post-test. The results of the study showed that the teachers preferred algebraic representations most even though they showed preference for graphic and tabular representations in the algebraic lessons. Similarly, Delice and Sevimli (2010) examined the representations teachers use in solving definite integral problems in the classroom and how they make transition between the representations. Forty-five (45) teachers were sampled for the study. The data were collected through interview and tests. At the end of the research, the findings revealed that the ability of the teachers to use multiple representations was not at the expected level. It was found that the teachers preferred algebraic representation for all the solutions of the problems with the reason that they were accustomed to this representation.

2.10 Representation Preferences of Students

Ozgun-Koca (2004) emphasized the need for creating learning environment that employs multiple representations. Because, individuals vary in their preferences for representations and such differences should be acknowledged (Ainsworth, 1999).

In this regard, many researchers have investigated representations students prefer in the classroom. Beyraneyard (2010) investigated how students recognize same linear equations with one unknown presented in multiple ways. The findings from the study revealed that low achieving students preferred pictorial representation while high achieving students preferred using verbal and symbolic (algebraic) representations when solving linear equations.

Hong, Thomas and kwon (1999) also investigated students understanding of linear algebraic equations via super-calculator. The students ($n = 35$) explored equations using symbolic (algebraic), tabular and graphical representations. The findings of the study revealed that students were more successful to solve equations in symbolic (algebraic) representation representing 86.7% and 69% in two skills questions that were given. However, it was reported that very few students could solve the equations using graphical or tabular representation representing 11.5% and 15.8% respectively.

Ozgun-Koca (1998) carried out a research study on students' representation preferences when they were asked to solve a mathematical problem. Data were collected from ($n = 16$) through interview, observation and questionnaire in both regular classroom and computer class settings. The results revealed that in the regular classroom setting, students had preference for equations to solve a mathematical problem and the least appealing representational mode among students was graph. However, in the computer class setting, most students showed preference for graph. Also, it was found that students' preferences were due to students' perception about the usage of a representation to generate an answer.

Knuth (2000) conducted another study to examine students' representation preferences and translation between algebraic and graphical representations. Data were collected in the five classes in small working groups where students' responses were categorized as algebraic and graphical solutions. The results of this study revealed that more than three fourths of all the students preferred the algebraic representation. It was also reported that many students did not even notice that the graphical representation was a solution.

Neria and Amit (2004) examined problem solving patterns and which representations were preferred by candidates. The representations were classified as algebraic, verbal, diagram and graphical. A total of 164 candidates were sampled for the study. Data were collected through open-ended and multiple-choice tests. The findings from the study revealed that the success of candidates using algebraic representation was higher than the other representational modes.

Herman (2002) conducted a study to investigate students' usage of multiple representations to solve algebra problems. The researcher examined students' usage of algebraic, graphical and tabular modes of representations in topics such as functions and equations. Also, representations students preferred to solve the algebra problems were examined. The results of the study revealed that most of the students preferred to use algebraic and graphical mode of representations to solve algebra. Tabular representation was least popular. The students explained that algebraic mode of representation was more mathematical and familiar than others. Boulton-Lewis (1998) also conducted a research study to investigate children's representation of symbols and operations. Students ($n = 29$) from first, second and third grades were interviewed. The classrooms teachers were also interviewed to know their representations and strategies employed in the classroom. The results from the study revealed that children from all the grade levels preferred to use manipulatives rather than carrying out the operations mentally.

Adding to the research, Keller and Hirsch (1998) investigated whether students have representation preferences and the extent to which representation preference was influenced. Pretest and posttest were administered to students $(n = 79)$.The results showed that students had preference for various representations. It was also revealed that students'experience with representations, perceptions of the usage of the representations, the level of the tasks among others were the factors that influenced students' representations preferences. Other factors such as frustration, anxiety, despair, satisfaction and dissatisfaction with the mathematical task were also investigated to influence the representations students used to solve problems (DeBelis Goldin, 2006; Goldin, 2003).

2.11 Summary

Teaching and learning linear equations require a very accommodating instruction. The review from most of the studies reported the relevant of using multiple representations in the classroom even though other studies acknowledged some hindrance of multiple representation-based instruction. The historical development and meaning of algebra and linear equations were also discussed. From the review, it was also found that basic concepts such as equal sign, variables, constants, coefficients, algebraic terms and expressions influenced greatly how students make sense of linear equations in one variable. Further representational modes such as algebraic, manipulatives and graphic representations in linear equations in one variable were examined.

In algebraic representation, it was that as one of the dominated modes of representation in linear equations. Manipulative alone was found not to guarantee understanding unless teachers' intervention through appropriate linkage with other representations while many of the studies reported students' limited understanding of graph and its construction.

The researcher again examined teachers' mode of representations and what accounts for teachers' choice of representations. It was found that most teachers prefer algebraic representation with the reason of being accustomed to it. Lastly, students' representation preferences were reviewed and was found that students' preferences for representations vary which depend on factors such as experience with the representation, perception of the usage of the representation and the level of the task.

CHAPTER THREE

RESEARCH METHODOLOGY

3.0 Overview

This chapter discusses the research design, population, sample and sampling techniques, research instruments, validity and reliability of the instruments, pilot study, data collection procedures and data analysis.

3.1 Research Design

The term research design has been defined in various ways by researchers and other authorities. According to Burns and Grove (2009), research design is a blueprint for conducting a study with maximum control over factors that interfere with the validity of the findings. Parahoo (2006) also defined research design as a plan that describes how, when and where data are to be collected and analyzed. In the view of Polik and Beck (2012), research design is the researcher's overall plan for answering research questions or testing the research hypothesis.

Therefore, in order to answer the research questions and formulate hypothesis, the researcher used a quasi-experimental design which draws quantitative and qualitative data for analysis. The researcher adopted quasi- experimental design because, the researcher wanted participants who were in their intact classes or naturally occurring groups without being randomly assigned to both experimental and control groups (Fraenkel & Wallen, 1996; Gall, Gall & Borg, 2007). This helped the researcher to assess the effects of the intervention on participants in detail as they occur in their intact classes. Also, this design was selected because, it has been recommended for educational evaluations and as a good alternative to randomized experiment (Schneider, Carnoy, Kilpatrick, Schmidt & Shavelon, 2007).

Quasi-experimental design exists in many forms; however, the non-equivalent control group was used. This is when both participants in the experimental and control groups take pre-test and post-test and only the experimental group receives the treatment. This was used by the researcher because, it has been found as one of the popular approaches of quasi-experimental design (Creswell, 1994).

Additionally, the researcher employed the design which draws quantitative and qualitatively data for analysis because, the researcher wanted a richer data and stronger evidence than using a single method (Johnson & Christensen, 2008; Gay, Mills & Airasian, 2006).

3.2 Population

There are so many definitions of research population. According to Burns and Grove (2005) a research population refers to all the elements that meet the criteria for inclusion in a study.

Parahoo (2006), also described research population as the total number of units such as individuals, artifacts, events and organizations from which data can be collected. In the view of De Vos, Strydom, Fouche and Delport (2002), research population is the entire group of persons or objects that is of interest to the researcher and meet the criteria which the researcher is interested in studying.

In this study the target population constituted all the Junior High School Two (2) Students in the forty-five (45) public Junior High Schools as well as the mathematics teachers in these schools in the Bimbilla Municipality in the Northern Region of Ghana. However, an accessible population of students from three different schools in three different circuits were used. These schools were named as School A, School B and School C. The population of teachers, Junior High School two (2) students in the Bimbilla Municipality as well as the Junior High School two (2) students in the selected schools are shown in Table 3.1.

Table 3.1: Summary of Teachers' and Students' Population

Participants	Total Population	Males	Females
Teachers	461	342	119
Students	2,958	1,671	1,287
		Population	J H S 2 Students
School A		310	124
School B		305	108
School C		245	102

Source: GES, Bimbilla Municipality, 2020

The data in Table 3.1 show that, the total J H S 2 students' population in the Bimbilla Municipality is 2,958 consisting of 1,671 boys and 1,287 girls. There were 461 J H S teachers comprising 342 males and 119 females for 2018/2019 academic year. School A had total population of 310 students and out of this, the J H S 2 constituted 124 students. School B had total population of 305 and the J H S 2 constituted 108 students while School C had total population of 245 students and the J H S 2 constituted 102 students.

3.3 Sample and Sampling Techniques

A sample is a fraction of the whole population selected for a study while sampling refers to a process of selecting a portion of the population to represent the entire population (Polik & Hunley, 1999). A sample is a subset of a population selected to participate in a study while sampling refers to the process of selecting a portion of a population that conforms to a designated set of specifications to be studied (Polik & Beck, 2004). Three Junior High Schools were purposively selected because their level of mathematics achievement was comparatively equal. Additionally, a convenient sample of 53 students was used in each school (School A, School B and School C). Thus, in all a total of 159 students were used in the three schools. The researcher adopted convenient sampling techniques because, it allowed the researcher to select students (participants) who were readily available at the time of the study and agreed to participate in the study (Frey, Carl & Gary, 2000; Henry, 1990; MacNearly, 1999; Fink, 1995). School A had 24 boys and 29 girls. School B had 16 boys and 37 girls and school C had 28 boys and 25 girls. The Junior High School students were targeted because, it is the stage where students are introduced to algebraic equations of the form $ax \pm b = c, ax \pm b = cx, ax \pm b = cx \pm d$. On the other hand, 86 classroom mathematics teachers were selected for the study because, they have been teaching mathematics and could provide the best information for the researcher to achieve the objectives of the study (Tashakkori & Teddie, 2003; Creswell, 2003).

However, to decide which school represent experiment group 1, experimental group 2 and control group, the researcher adopted simple random techniques using random numbers generated by computer. School A was represented by experimental group 1, School B as experimental group 2 and School C as control group 3. Experimental group 1 students experienced multiple representations-based instruction with three representations (algebraic, manipulative and graphic representations). Similarly, in experimental group 2 students experienced multiple representations-based instruction with only two representations (algebraic and manipulatives) while in control group, students experienced traditional instruction with only algebraic representation.

The sample size for the students ($n = 159$) is representative of the target population. This is because, Cohen, Manion and Morrison (2008) found that a sample size of 30 is accepted by many researchers to be the minimum number if a researcher wants to do a statistical analysis on any data in a research. Consistent with this, Roscoe (1975) found samples of 30 or more as recommended for any experimental research. Also, the sample size for the mathematics teachers ($n = 86$) is representative of the target population. This is because, out of the total 95 mathematics teachers, 86 of them were used. Consistent with Gay and Diehl (1992) view for a survey research (descriptive), 10% of respondents of the target population or 20% of respondents if the target population is small is accepted for a study. Therefore, the sample size used is valid for any statistical analysis and conclusions.

3.4 Research Instruments

According to Parahoo (2006), a research instrument is a tool used to collect data or one designed to measure knowledge, attitudes and skills. Because, the study employed quantitative and qualitative data analysis, the researcher used questionnaire, representation preference test (RPT), unstructured interview and linear equations achievement test (LEAT) as research instruments for data collection.

3.4.1 Questionnaire

According to Babbie (1990), a questionnaire is a document containing questions and other items designed to elicit appropriate information for analysis. Questionnaires contained systematically prepared documents of questions designed to elicit responses from respondents for the purpose of understanding the nature of the research problem under study. The questionnaire used in the study contained both closed and open-ended forms of questions. The researcher used questionnaire because, it allows wider coverage of data collection with minimum expenses both in the money and effort (Osula, 2001). The questionnaire was made of two parts (see Appendix A). The first section (Section A) sought information about the demographic background of teachers. This covered a wide range of characteristics such as teachers' gender, age in years, professional status, qualification and number of years served as mathematics teacher. The second section (section B) of the questionnaire investigated teachers' mode of representations in linear equations. In all, four questions were asked (from Q6 – Q9). Question 7 had five sub-questions. Each sub- question describes a particular mode of representation in teaching equations rated on Likert type of scale as: 1 = Never, 2 = Almost never, 3 = occasionally, 4 = Almost every time and 5 = Every time.

This was done to fine out mode of representations teachers use in teaching linear equations. Questions 8 and 9 had five sub-questions each investigating what accounts for teachers' choice of representations in teaching linear equations.

3.4.2 Representation Preference Test (RPT)

The representation preference test was developed by the researcher to address the third research question that sought to find students' representation preferences in linear equations. The test was made up of two parts (see Appendix B). The first part sought information about the demographic background of students. This covered students' gender and age in years. The second part investigated students' preferences. Question 3 had five sub-questions. Each sub-question describes a particular mode of representation in linear equations. The students were made to tick a particular representation they prefer in linear equations.

3.4.3 Interview guide

An interview is an interaction in which oral questions are posed by the interviewer to elicit oral responses from the interviewee (Annum, 2015). The researcher adopted semi-structured type of interview guide in this study to find out what contributed to students' representation preferences. This is because, this type of interview provides an atmosphere that is often casual and flexible. It ensures freedom in the interaction process. In all a total of 15 students were conveniently sampled for the interview.

3.4.4 Linear Equations' Achievement Test (LEAT)

Test according to Della et al (2001) is a set of questions, exercises or practical activities used to measure someone's ability, skills or knowledge. Test helps to measure students' weaknesses or strengths in a lesson so that remedial lessons can be administered from their weaknesses. The linear equations' achievement test consists of pre-test and post-test developed by the researcher to measure the level of students' achievement in linear equations in one variable. The test items were constructed based on the lesson taught and the learning objectives in the Junior High schools two (2) mathematics curriculums. These were administered to all the three groups before and after the introduction of multiple representations-based instruction.

Pre- test

The researcher used the pre-test (see Appendix C) to measure the amount of pre-existing knowledge and the level of students' understanding in linear equations in one variable in all the three groups. In all, ten theory questions on linear equations in one variable were given to the students and they were given 1 hour: 20 minutes to answer the questions.

Post-test

The researcher administered the post-test to all the three groups after multiple representations-based instruction in the experimental groups. The structure and the number of questions developed and administered were the same as the pre-test. However, it does not necessary mean that the same questions as in the pre-test were administered to students (see Appendix D). The values and figures in the pre-test were altered in the post-test. The students were then given 1 hour: 20 minutes to answer the questions.

3.5 Pilot Study of the Instruments

According to Bless and Higgon-Smith (2000), a pilot study is a small study conducted prior to a larger piece of research to determine whether the methodology, sampling, instruments and analysis are adequate and appropriate. Polit and Beck (2004) also indicated that the purpose of pilot study is to test the data collection instruments and other aspects of a study in preparation for a larger study. One of the advantages of conducting a pilot study is that it might give advance warning about main research project could fail, where research protocols may not be followed, or whether proposed methods or instruments are inappropriate or too complicated (van Teijlingen & Hundley, 2001). In this study, the researcher piloted all the research instruments in the Bimbilla Municipality in the Northern Region of Ghana. The questionnaire was piloted with 30 classroom mathematics teachers. Also, the linear equations' achievement test (pre-test and post-test) and the representation preference test were piloted with 30 Junior High School Two Students.

3.6 Validity and Reliability of the Instruments

Validity of an instrument is the extent to which an instrument measures what it supposed to measure (Durrheim, 1999). Thus, an instrument is said to be valid if it serves the intended function well. On the other hand, reliability deals with the consistency, dependability and explicability of the results obtained from a piece of research (Nunan, 1999). Reliability is when a measuring instrument yields the same results on repeated applications or It is the degree of consistency of the test scores (Thompson & Durheim, 1999). In order to have a valid instrument both content and face validity of the instruments were done. To ensure the content validity of the equation's achievement test, the mathematics course book for J H S 2, teaching syllabus and other dissertation work were considered. These reference materials helped the researcher to develop the content around what students are expected to learn. It was further given to experienced mathematics teachers, other experts as well as my supervisor for vetting. Also, other instruments were vetted by my supervisor. After the pilot study of the instruments, the internal consistency of the questionnaire and the linear equations' achievement tests were estimated by calculating liability coefficient using the Cronbach alpha and Split-half method. The SPSS was used to estimate reliability coefficient of all the instruments. In the Split-half method, students' scores in the pre-test were divided into two halves and scored. The same thing was done to the post-test. The scores of odd-numbered items and the scores for even numbered-items for each participant was determined. The reliability coefficients were then estimated using the SPSS. The value of the reliability coefficient for pre-test and post-test according to the Spearman- Brown coefficient were 0.71 and 0.72 respectively. Also, the reliability coefficient of the questionnaire had a Cronbach alpha (α) of 0.76. These ranges of reliability estimates were found to be reliable

(Gearge & Mallery, 2003). Therefore, the instruments can be considered as reliable for the study.

3.7 Data Collection Procedure

After seeking permission to carry out all the activities of the study in the selected sample schools and had the approval, the researcher then administered linear equations achievement test (pre-test, see Appendix C) to all the three groups to measure their pre-existing knowledge on linear equations in one variable and the work was collected for processing and analysis. The teachers were also given questionnaire to find out the mode of representations they use in teaching linear equations and what account for their choice of representations in teaching linear equations in one variable and the data was also collected for processing and analysis. After the administration of the pre-test for students and questionnaire for teachers, the researcher then carried out teaching activities with experimental group 1and 2 and whiles different teacher of the same experience taught control group class. Both of us were teaching the same topic based on the same lesson objectives but experimental group 1 were taught using manipulatives, graphic and algebraic representations and experimental group 2 were also taught using manipulative and algebraic representations whiles control group students were taught through traditional instruction or regular instruction with only algebraic representation.

In this study, teaching lesson plans formed the instructional design in which several activities of the various representations for teaching linear equations were carried out (see Appendix F).

The instructional design was implemented in the experimental group 1 and Experimental group 2 that were scheduled on Mondays and Tuesdays respectively from 2:05pm to 2:45pm for every week. Wednesdays were used for the control group. The instruction lasted for a period of 3 weeks. The researcher then carried out series of activities with students in the experimental group 1 using manipulative, graphic and algebraic representation in teaching linear equations in one variable.

The researcher then took students through additive identity (zero pair) using tiles. That any number or term and its opposite equals zero. For instance, -1 +1 = 0; x + (-x) = 0, etc. The researcher guided the students to know that, one (1) red bar and one (1) blue bar equals zero and one (1) red square and one (1) blue square equals zero as well. Also, the rules of subtraction were discussed. For instance, 4 + (-2), represent 4 blue squares and 2 red squares. The equality property of whatever is done to one side must be done to the other side to keep the sides equal was explained using tiles. For instance, using tiles to model -3x + 2 = -4, we first add 2 red squares to both sides. Finally, the researcher guided the students to know that, the positive variables (x) represented by blue, green and yellow bars and positive constants represented by blue, green and yellow squares while negative variables (-x) represented by red bars and negative constants represented by red squares.

The researcher then modeled linear equations such as $3x + 2 = 8 + x, -2x - 4x = 2x - 8,\ 4x + 6 = 10,\ 3(x - 2) = -4x + 1,\ 3x - 6 = x + 8,\ \frac{1}{2}x + 5 = 2x + 3$ etc using manipulatives and guided the students to solve similar examples using manipulative representation. The students were further guided to solve linear equations using manipulatives as shown in Figure 3.1.

Figure 3.1: Students solving linear equations with manipulatives

In the figure, the students' modeled the equation; $3x + 2 = 8 + x$ using concretemanipulatives. They were asked to use these tiles to solve the equations such as $4 - x = 2 - 3x$, $4x + 3 = x + 9$, $2(x + 3) = -3x - 4$ etc on their own. The researcher further supervised the students to discuss among themselves to solve more equations using manipulative.

The students were guided to solve such equations using graphs as shown in Figure 3.2

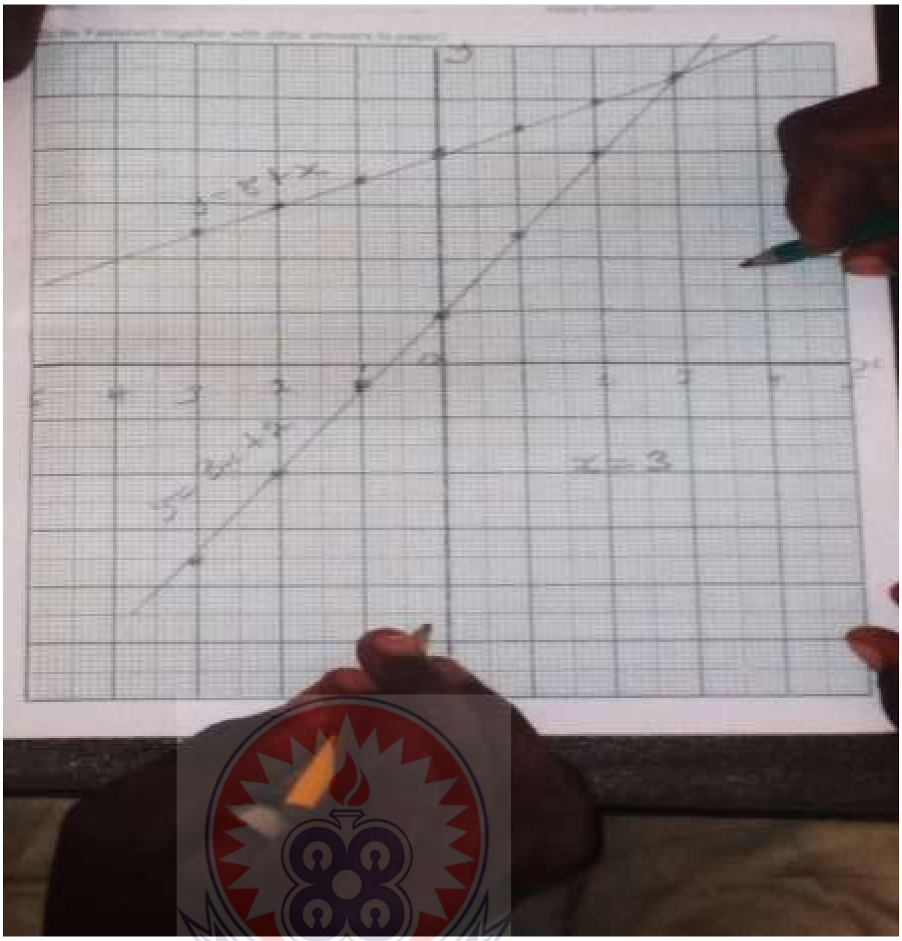

Figure 3.2: Students plotting linear equations in one variable on graphs

The graph in Figure 3.2 shows students plotting simple linear equations in one variable. The researcher explained to the students to know that, the point of intersection of the two lines drawn on the graph satisfied the truth set of the equations. The equation; $3x + 2 = 8 + x$ was plotted on the graph using the form y $=3x + 2$ $and\ y = 8 + x$. The point of intersection of the two lines was obtained as 3 on the graph which satisfies the truth set of the equation. Similarly, the students were guided again to plot the equation $4 - x = 2 - 3x$ on the graph. The equation was plotted on the graph using the form $y = 4 - x\ and\ y = 2 - 3x$. The point of intersection of the two lines was obtained as -1 on the graph which satisfies the truth set of the equation.

The researcher finally guided the students to present the equations algebraically without using manipulatives and graphs as shown in Figure 3.3.

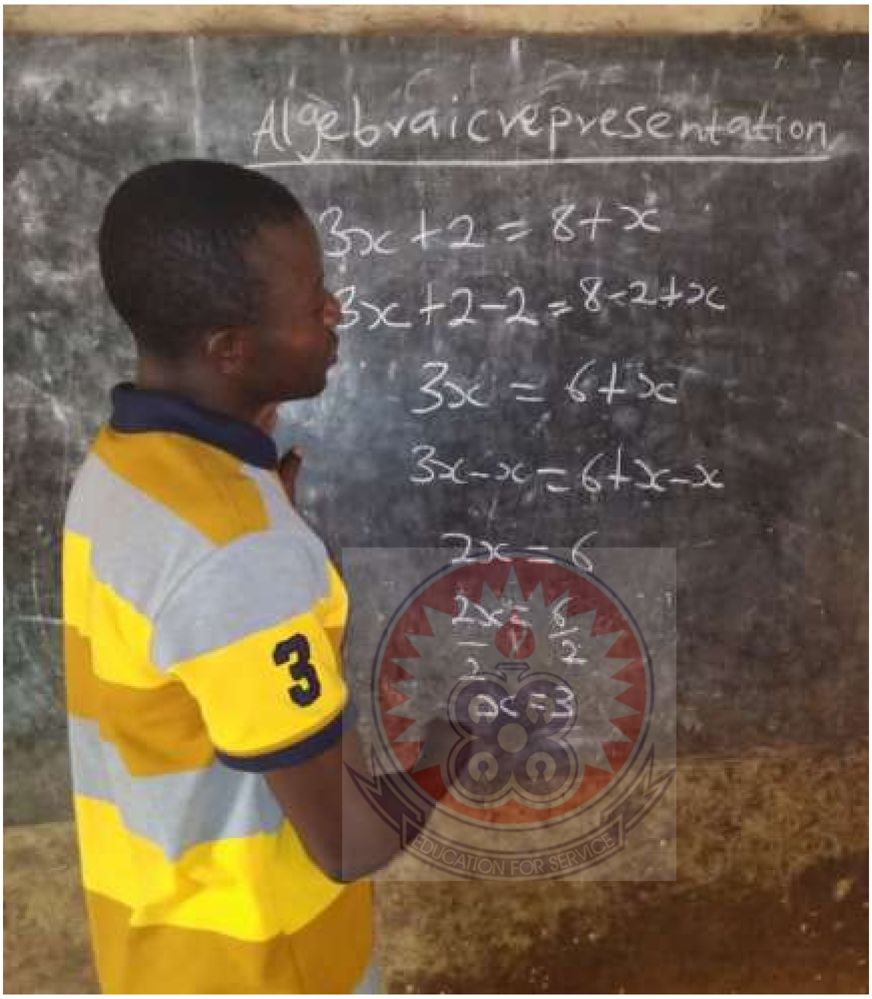

Figure 3.3:Algebraic representation of linear equations in one variable.

The picture in the Figure 3.3 shows the researcher guiding students to solve simple linear equations; $3x + 2 = 8 + x$ using algebraic representation. The students were guided to use the idea of whatever is done to one side of an equation must be down to the other side to keep the sides equal to solve the equations such as$4 - x = 2 - 3x,\ 4x + 3 = x + 9,\ 2(x + 3) = -3x - 4$ etc.

The experimental group 2 students were also taught linear equations in one variable with only two representations such as manipulatives and algebraic representations by the researcher.

Using manipulatives to solve linear equations, the researcher again took students in experimental group 2 through additive identity (zero pair) using tiles. That any number or term and its opposite equals zero. For instance, -1 +1 = 0; x + (-x) = 0, etc. The researcher guided the students to know that, one (1) red bar and one (1) blue bar equals zero and one (1) red square and one (1) blue square equals zero as well. Also, the rules of subtraction were discussed. For instance, 4 + (-2), represent 4 blue squares and 2 red squares. The equality property of whatever is done to one side must be done to the other side to keep the sides equal was explained using tiles. For instance, using tiles to model -3x + 2 = -4, we first add 2 red squares to both sides. Finally, the researcher guided the students to know that, the positive variables (x) represented by blue, green and yellow bars and positive constants represented by blue, green and yellow squares while negative variables (-x) represented by red bars and negative constants represented by red squares.

The researcher then carried out series of activities with students in the experimental group 2 using manipulatives in teaching linear equations in one variable. The researcher guided students to also solve the following linear equations using manipulatives$3x + 2 = 8 + x, -2x - 4x = 2x - 8,\ 4x + 6 = 10,\ 3(x - 2) = -4x + 1,\ 3x - 6 = x + 8,\ \frac{1}{2}x + 5 = 2x$ +3 etc

The researcher further supervised the students to discuss among themselves to solve more equations using manipulative.

Finally, students in experimental group 2 were guided again to solve linear equations using algebraic representations. The students were guided to solve simple linear equations; $3x + 2 = 8 + x, 4 - x = 3 - 3x$ etc using algebraic representation. The students were guided to use the idea of whatever is done to one side of an equation must be done to the other side to keep the sides equal to solve the equations such as $3x + 2 = 8 + x, -2x - 4x = 2x - 8,$ $4x + 6 = 10,$ $3(x - 2) = -4x + 1,$ $3x - 6 = x + 8,$ $\frac{1}{2}x + 5 = 2x$ +3 etc

Lastly, control group students were taught linear equations in one variable through traditional instruction or regular instruction using only algebraic representation. The students were guided to solve the following linear equations through traditional instruction with only algebraic representation 3x+$2 = 8 + x, 3x + 7 = 2x, -2x - 4x = 2x - 8,$ $4x + 6 = 10,$ $3(x - 2) = -4x + 1,$ $3x - 6 = x + 8,$ $\frac{1}{2}x + 5 = 2x$ +3 etc All instruction was lecture-based and examples were provided to guide the students using algebraic representation.

The researcher then administered Linear Equations Achievement Test (Post-test, see Appendix D) to all the three groups after the introduction of the multiple representations-based instruction and students used various representations for solving the questions (see Appendix E). Post-test was used to measured students' achievement in linear equations in one variable after the treatment.

The researcher then administered Representation Preference Test (see Appendix B) to all the three groups students to find out the mode of representations they prefer to be taught linear equations in one variable and also a fellow-up semi-structured interview guide was also conducted to find out what account for their representation's preferences in linear equations in one variable and the data was collected for processing and analyzing.

3.8 Data Analysis

Data collected were analysed both qualitatively and quantitatively. Qualitative data were obtained from interview guide. The qualitative data were grouped into different categories/themes consistent with the research objectives and deduction and generalizations made using patterns and trend of responses. The quantitative data were also obtained from Questionnaire, Representation Preference Test (RPT) and Linear Equations Achievement Test and the data were entered in the computer using SPSS program version 16. Specifically, the data were analysed using simple descriptive statistics: percentages, means and frequencies. The data were presented with the aid of tables, bar graphs, box plots and scatter plots and further analysed using inferential statistics by testing the research hypothesis using one-way Analysis of Covariance (ANCOVA).

3.9 Ethical considerations

David and Resnik (2009) defined ethics in research as the discipline that studies standards of conduct, such as philosophy, theology, law, psychology or sociology. In other words, it is a method, procedure or perspective for deciding how to act and for analysing complex problems and issues. Protection of participants and their responses were assured by obtaining informed consent, protecting privacy and ensuring confidentiality. In doing this, the description of the study, the purpose and the possible benefits were mentioned to participants. The researcher permitted participants' to freely withdraw or leave at any time if they deemed it fit. As a way of preventing plagiarism, all ideas, writings, drawings and other documents or intellectual property of other people were referenced indicating the authors, title of publications, year and publishers.

CHAPTER FOUR

RESULTS AND DISCUSSION

4.0 Overview

The purpose of this study is to investigate the effects of multiple representations-based instruction on Junior High School students' achievement in linear equations in one variable in the Bimbilla Municipality. The findings of the study and discussion of results are presented according to the research questions and hypothesis. Also, the following hypothesis was tested:

H_0: There is no statistically significant difference between students' scores in linear equations achievement test using multiple representations-based instruction and traditional instruction after controlling for students' age, gender, and pre-test scores.

4.1 Demographic Information about Teachers

Information about the demographic background of teachers for this study covered a wide range of characteristics such as gender status, age, professional status, qualification and number of years served as mathematics teacher. All these were done to solicit in-depth information of teachers who were involved in the study. Data gathered on teachers' demographic characteristics were presented in Table 4.1.

Table 4.1: Summary of Demographic Characteristics of Teachers

Demographic factors	Category	Frequency	Percentage (%)
Gender	Male	68	79.1
	Female	18	20.9
Total		86	100.0
Age	20 – 25 years	7	8.1
	26 – 30 years	36	41.9
	31 – 35 years	28	32.6
	36 – 40 years	11	12.8
	41 years and above	4	4.7
Total		86	100.0
Professional Status	Pupil-teacher	1	1.2
	Non-professional	8	9.3
	Professional	77	89.5
Total		86	100.0
Academic Qualification	SSCE/WASSCE	1	1.2
	Certificate 'A'	2	2.3
	Diploma	33	38.4
	HND	6	7.0
	Degree (B.SC. /Bed etc	38	44.2
	Masters	6	7.0
	Total	86	100.0
Years of Teaching Mathematics	1 – 5 years	45	52.3
	6 – 10 years	26	30.2
	11 – 15 years	10	11.6
	16-20years	3	3.5
	21 years and above	2	2.3
Total		86	100.0

Source: GES, Bimbilla Municipality.

The statistics in Table 4.1 on gender status of teachers show that 68 (79.1%) of mathematics teachers that participated in the study were males and 18 (20.9%) represented female mathematics teachers. It is evident from the data on gender that more male teachers teach mathematics at the Junior High School level than their female counterpart in Bimbilla Municipality at the time of the study. On teachers' age, the statistics show that most of the teachers were within their youthful ages of 26 and 30 years and 31 to 35 years, representing 36 (41.9%) and 28 (32.6%) respectively. The number of teachers within 20 to 25 years and 41 years and above was found to be small representing 7 (8.1%) and 4 (4.7%) teachers respectively. Also, on teachers' professional status the results show that the majority of teachers, 77 (89.5%) were professional teachers and only one teacher was a pupil-teacher representing 1.2%. This indicates that many of the teachers sampled for the study were trained teachers.

Therefore, statistics gathered on teachers' academic qualification show that 33 (38.4%) of the teachers were Diploma holders whilst 38 (44.2%) were Bachelor degree holders. Both qualifications represent the majority of the teachers' academic attainment. Only 1 (1.2%) teacher was SSCE/WASSCE holder and 6 (7.0%) teachers holding masters' degrees and also 6 (7.0%) holding HND certificates.

Moreover, the results on number of years taught by mathematics teachers reveal that 45 (52.3%) teachers taught mathematics for 1 to 5 years and 26 (30.2%) teachers for 6 to 10 years. This implies that majority of the teachers have taught mathematics between 1 to 5 years and 6 to 10 years. Only 2 (2.3%) teachers have taught mathematics for 21 years and above.

4.2 Demographic Information about Students

The students' background information covered their gender status and age. Data gathered on students' demographic characteristics were presented in Table 4.2.

Table 4.2: Summary of Demographic Characteristics of Students

Demographic factors	**Category**	**Frequency**	**Percentage (%)**
Gender	Male	68	42.8
	Female	91	57.2
Total		159	100.0
Age	13 – 15 years	109	68.6
	16 – 18 years	49	30.8
	19 years and above	1	0.6
Total		**159**	**100.0**

Source: GES, Bimbilla Municipality.

The statistics in Table 4.2 on gender status of students show that 68 (42.8%) of students that participated in the study were males and 91 (57.2%) represented female students. It is evident from the data on students' gender status that more females participated in the study than males. Also, the results on students' age reveal that 109 (68.6%) students' age ranged from 13 -15 years only 1 (0.6%) student was 19 years and above. This implies that majority of the students sampled for the study were within 13 to 15 years.

4.3 Presentation of Research Questions

4.3.1 Results of Research Question1: *What mode of representations do teachers use to teach linear equations in one variable in the Bimbilla Municipality?*

Research Question 1 was intended to find out the mode of representations teachers' use to teach linear equations in one variable. Data gathered on teachers' mode of representations were presented in table 4.3. The response scale was shortened as: Never (N), Almost Never (AN), Occasionally (O), Almost every time (AE) and Every time (ET) in Table 4.3.

Table 4.3: Teachers' mode of representations on linear equations in one variable

Frequency of Use							
Description of mode of Representations	**N** **N (%)**	**AN** **N (%)**	**O** **N (%)**	**AE** **N (%)**	**ET** **N (%)**	Mean	SD
Algebraic	0 (0%)	0 (0%)	0(0%)	29(33.7%)	57(66.3)	4.66	0.48
Manipulatives	32(37.2%)	29(33.7%)	4(4.7%)	15(17.4%)	6(7.0%)	2.23	1.31
Graphic	43(50.0%)	19(22.1%)	14(16.3%)	8(9.3%)	2(2.3%)	1.92	1.12
Multiple representations	19(22.1%)	13(15.1%)	26(30.2%)	16(18.6%)	12(14.0)	2.87	1.34
Single representation	0(0%)	2(2.3%)	11(12.8%)	34(39.5%)	39(45.3)	4.28	0.78

Source: Author's Construct with field data, 2020.

The descriptions of modes of representations in Table 4.3 were classified as use of transposition and other related techniques, use of tiles and other accessories, teaching linear equations by plotting on graphs, use of a blend of modes and teaching linear equations using only one representation respectively by the researcher. The results in Table 4.3 indicate that 57(66.3%) teachers sampled for the study used algebraic representation every time. No teacher responded never or almost never in terms of using algebraic and single representation in linear equations. Only few teachers stated that they used manipulatives, graphic and multiple representations every time representing 6(7.0%), 2(2.3%) and 12(14.0%) teachers respectively.

4.3.2 Discussion of Results of Research Question 1

The results revealed that most teachers (66.3%) used algebraic representation every time in teaching linear equation. Thus, most of the teachers sampled for the study teach linear equations by using transposition or other related techniques. This shows consistency with the findings of other studies reviewed in the literature (Bal, 2014; Monoyiou, Gagatsis & Papageorgion, 2007; Herman, 2007; Cai, 2005) that most teachers use algebraic representation in solving mathematics problems. The possible reason for teachers' uses of algebraic representation is that, most described algebraic representation as understandable and others believed that it is faster as found from data in Table 4.4. On the other hand, only few teachers used manipulatives, graphic and multiple representations in teaching linear equations. The finding is consistent with the finding by Delice and Sevimli (2010) when they found that multiple representations usage among teachers was not at the expected level. Similarly, Gagatsis and Shiakalli (2004) reported in their study that graphic representation was least used by teachers. The possible reason is that teachers have challenges integrating multiple representations in their teaching environment (Even, 1998; Celik & Baki, 2007).

4.4.1 Results of Research Question 2: *What accounts for teachers' choice of representations in linear equations in one variable in the Bimbilla Municipality?*

The researcher used this research question to find reasons why teachers use or did not use certain representations in teaching linear equations in one variable. The data obtained (from questionnaire, open-ended part) were summarized in Table 4.4.

Table 4.4: Reasons for Teachers' mode of Representations in Linear Equations

Representations	Reasons	Frequency	Percentage (%)
Algebraic	Easy, simple and understandable	54	62.8
	Pupils have knowledge on it	8	9.3
	Well-known representation	12	14.0
	Faster	4	4.6
	Others	8	9.3
	Total	**86**	**100**.0
Manipulatives	Better understanding	13	52.0
	Makes lesson real	10	40.0
	Others	2	8.0
	Total	**25**	**100**.0
Graphic	Better understanding	10	41.6
	Makes lesson practical	7	29.2
	Others	7	29.2
	Total	**24**	**100.0**
Multiple representations	Better understanding	17	31.5
	Motivates students	14	25.9
	Address different learning styles	12	22.2
	Others	11	20.4
	Total	**54**	**100.0**

Source: Author's Constructs with field data, 2020.

The statistics in Table 4.4 revealed that, for teachers who used algebraic representation, 54 (62.8%) of them indicated that algebraic representation is easy, simple and understandable while 12(14.0%) reported that it is well-known and most widely used representation. For teachers using manipulatives and algebraic representations in teaching linear equations, majority gave their reasons that two representations promote better understanding, representing 13(52.0%) and 10 (41.6%) teachers respectively. For teachers using multiple representations, 17(31.5%) stated that it promotes understanding and 14 (25.9%) indicated that it motivates students. However, for teachers using single representations, most provided similar reasons as that of algebraic. Data on why teachers do not use certain representations in teaching linear equations was also found and reported in Table 4.5.

Table 4.5: Reasons for teachers not using certain mode of representations in linear equations

Representations	**Reasons**	**Frequency**	**Percentage (%)**
Manipulatives	Time consuming	9	14.8
	Difficult to understand	10	16.4
	Lack of materials	19	31.1
	Have no idea	16	26.2
	Others	7	11.5
	Total	**61**	**100.0**
Graphic	Time consuming	13	21.0
	Difficult to understanding	24	38.7
	Not found in syllabus	13	21.0
	Have no idea	8	12.9
	Others	4	6.5
	Total	**62**	**100.0**
Multiple representations	Time consuming	13	40.6
	Lack of materials	8	25.0
	Students get confuse	3	9.4
	Have no idea	6	18.8
	Others	2	6.3
	Total	**32**	**100.0**

Source: Author's Constructs with field data, 2020.

The research results in Table 4.5 revealed that, out of the 61 teachers who do not use manipulatives in teaching linear equations, 19(31.1%) of them gave their reasons as lack of materials while 16(26.2%) indicated that they are lack of ideas about how to use manipulatives in teaching linear equations. Other reasons such as cost, not recommended in the syllabus, not relevant to the pupils, to mention such a few recorded 11.5%. Out of 62 teachers who do not use graphic representation, 24 (38.7%) of them indicated that it is difficult for their students to understand and 13(21.0%) of them gave reasons as time consuming and not found in syllabus.

Other reasons such as cost, lack of materials, examinations questions not set on it, to just mention a few recorded 6.5%. Out of teachers who do not use multiple representations, 13(40.6%) indicated that it is time consuming and 8(25.0%) of them gave reasons as lack of materials. However, for algebraic representation, almost all the teachers indicated that they used it hence did not give any reason. This was similar to single representation.

4.4.2 Discussion of Results of Research Question 2

The results revealed that most teachers used algebraic representation in teaching linear equations due to reasons such as easy, fast, simple, widely used, understandable, to just mention a few. Thus, the teachers' choice of algebraic representation was influenced by these factors. The finding is consistent with the finding by Bal (2014) when teachers described their reasons for using algebraic representation as understandable. The possible reason is that most of the teachers believed their students easily understand linear equations when being taught through algebraic representation. Similarly, it was revealed that teachers do not use manipulatives, graphic and multiple representations in teaching linear equations due to reasons such as time, lack of materials, lack of ideas, difficulty for students, to just mention a few. Again, this finding is consistent with the finding by Bal (2014) found that teachers use of representations depend on their understanding. Once a teacher lacks ideas of a representation, the possibility that such representation will not be used is high. It was revealed that, only few teachers used manipulatives, graphic and multiple representations in teaching linear equations.

4.5.1 Results of Research Question 3: *What forms of representations do students prefer to be taught linear equations in one variable in the Bimbilla Municipality?*

The researcher investigated this research question with the intention of finding out which representation students prefer to be taught linear equations in one variable. Data gathered were reported in Table 4.6

Table 4.6: Students' representation preferences in linear equations

Description of mode of representations	Frequency	Percentage (%)
Algebraic	59	37.1
Manipulatives	28	17.6
Graphic	13	8.2
Multiple representations	27	17.0
Single representations	32	20.1
Total	159	100.0

Source: Author's Constructs with field data, 2020.

The descriptions of mode of representations in Table 4.6 were classified as transposition and other related techniques, use of tiles and other accessories, teaching linear equations by plotting on graphs, use of a blend of modes and teaching linear equations using only one representation respectively by the researcher. The statistics obtained show that majority, 59 (37.1%) students preferred algebraic representation.

This was closely followed by single representation with 32 (20.1%) students. The least preferred was graphic representation representing 13 (8.2%) students. The statistics in Table 4.6 were further represented diagrammatically in the form of a bar chart in Figure 4.1.

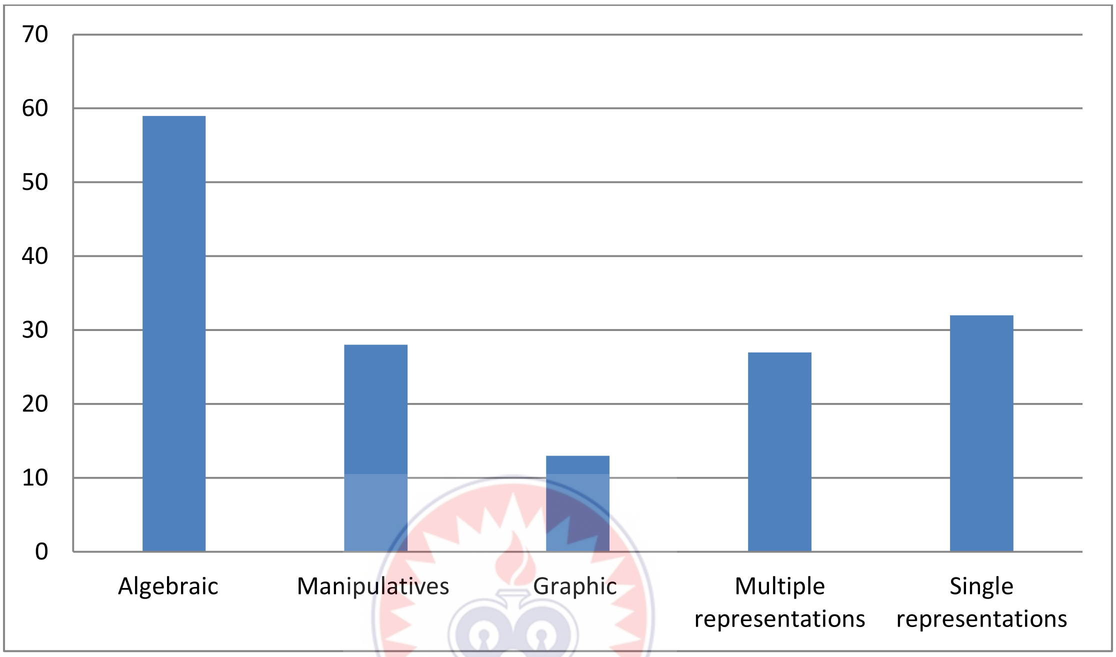

Figure 4.1: Bar chart Showing Students' Representation Preferences in Linear Equations

The bar chart shows that algebraic representation had the longest bar indicating the most preferred representation in linear equations by students. This was followed by single representation. The shortest bar representing graphic representation indicated the least preferred representation by students in linear equations in one variable. In a follow-up interview to find out what contributed to students' representation preferences, some views expressed by some students for algebraic representation preferences were given as follows (labelled A to F).

A: *I am used to algebraic representation in linear equations and is faster to work with.*

B: *At times, I become confused working with manipulatives in linear equations, so I like algebraic representation.*

C: *when I read my answers on graph, I find it difficult because of this, I like algebraic representation.*

D: *I easily understand algebraic representation but for manipulatives, understanding takes time.*

E: *Sir, I only know algebraic representation.*

F: *The algebraic representation is like you are doing mathematics.*

Looking at the views expressed by students for algebraic representation preferences, respondent A indicated that algebraic representation is familiar and faster. Also, respondent B indicated that algebraic representation is not confusing as compared to other representations. Respondent C indicated that algebraic is not difficult. Respondent D indicated that algebraic representation is more understandable. Respondent E indicated that algebraic representation is the only known representation and in the view of the respondent F, algebraic representation is more mathematical than other representations. The views from the respondents on algebraic representation show that it is the most preferred representation in linear equations by the students due to its familiarity among other reasons.

For students showing preferences for manipulatives, some views expressed by them were as follows (labelled G to I):

G: *I enjoy working with manipulatives in linear equations.*

H: Sir, when I add say $-4x + 3x$*, and other terms like this, at times negative and positive confuse me. But when I use manipulatives to do, I get the answers.*

I: *Sir, manipulatives in linear equations do not make me bore. Even if you teach with manipulatives the whole day, I will not get tired.*

From students' views, respondent G indicated that manipulatives instruction is enjoyable. Respondent H indicated that manipulatives make operations on negative and positive variables and constants easier. Also, respondent I said that manipulatives never make learning bored as compared to other representations. The respondents' views on manipulatives show that manipulatives can be used to take away the abstract nature of linear equations. Further, for students showing preferences for graphic representation, some views expressed by some students were given as follows (labelled J to K).

J: *I see my answers clearly on the graph even though, I struggle to trace them on x-axis*.

K: Sir, using graph is difficult but at times is interesting to use.

Looking at the views from respondents on graphic representation, respondent J indicated that answers on the graph are clearly seen and more practical, even though admitted that it is difficult to trace answers on graph. Also, respondent K indicated that using graph is difficult, however, the respondents said that at times using graph is interesting. The respondents' views on graphic representation show that even though students appreciate the use of graph, they still have fear in its usage. For students showing preferences for single representation, some views expressed by some students were given as follows (L to M):

L: *Sir, using only one representation at a time is easy to follow than combining more representations. Because, when you use them like that I cannot follow.*

M: *Sir, when you used single representation is simple to follow.*

For students showing preferences for single representation, respondent L and respondent M admitted that using single representation is simple and easy to follow. The respondents' views on single representation show that it is one of the comfortable representations in linear equations.

Additionally, for students showing preferences for multiple representations, some views expressed by some students were given as follows (labelled N to O):

N: *Sir, when you use more representations, at least I will get one that I understand.*

O: *I want to know more ways of solving linear equations.*

Looking at the views expressed by students on multiple representations, respondent N indicated that when using more representations in linear equations, it gives opportunity to get at least one that is understandable. Also, respondent O indicated that using more representations gives opportunity to know different ways of solving linear equations. The respondents' views on multiple representations show that students do not want to be restricted by one method or approach of solving linear equations. They prefer to have more approaches when solving linear equations.

4.5.1 Discussion of Results of Research Question 3

The students revealed that most students 59 (37.1%) prefer algebraic representation compared to other representations in linear equations. The students' preference for algebraic representation showed consistency with the findings of other studies in the literature (Naria & Amit, 2004; Herman, 2002; Girard, 2002; Knuth, 2000). The possible reasons are that most of the students believed algebraic representation is familiar, more mathematical, easy, faster, understandable, to mention just a few. These views expressed by students were similar with what accounted for majority of teachers' choice of algebraic representation. Therefore, it can be concluded that teachers' choice of representations can influence students' representation preferences.

4.6.1 Results of Research Hypothesis

H_0: There is no statistically significant difference between students' scores in linear equations achievement test using multiple representations-based instruction and traditional instruction after controlling for students' age, gender, and pre-test scores.

Before testing the research hypothesis, the descriptive statistics of students' pre-test and post-test scores were found and presented in Table 4.7.

Table 4.7: Mean Scores of Students' pre-test and post-test by group

Variable	Group	Mean	SD	Min	Max
Pre-test	Exp group 1	3.13	1.13	1	6
	Exp Group 2	3.02	1.84	0	9
	Cont. group	2.91	1.04	1	6
Post-test	Exp group 1	4.11	1.71	1	9
	Exp group 2	3.23	2.30	0	9
	Cont. group	2.51	1.42	0	6

Source: Author's Constructs with field data, 2020.

The descriptive statistics in Table 4.7 show that the pre-test mean scores of students in experimental group 1 (*M = 3.13, SD= 1.13*) increased in post-test mean scores (*M=4.11, SD = 1.71*). Similarly, the pre-test mean scores of experimental group 2 (*M =3.02, SD =1.84*) increased in post-test mean scores (*M =3.23, SD =2.30*). However, the pre-test mean scores of Control group (*M =2.91, SD = 1.04*) did not increased in post-test mean scores (*M= 2.51, SD =1.42*). Both Experimental group 1 and Control group had the same minimum and maximum values (1 and 6) for the pre-test. Also, for the post-test, the maximum values of experimental group 1 and experimental group 2 was 9. This indicates that in the initial pre-test, the lowest and highest scores obtained by the students in both experimental group 1 and control group were the same, however, in the post test after the intervention, students in the experimental groups obtained the highest scores indicating improvement in students' scores.

In addition to the descriptive statistics, simple boxplots were formed for students' pre-test and post –test scores among the groups. The boxplots were presented in Appendix G and H. For students' pre-test scores, the boxplots (see Appendix G) revealed that experimental group 1 and the control group had the same median value.

However, for students' post-test scores, the boxplot (see Appendix H) reveals that, the experimental group 1 had the largest median value among the groups. This indicates that students' achievement was much improved in the experimental group 1 after the introduction of the multiple representations-based instruction.

The inferential statistics was done by testing the research hypothesis using one-way Analysis of Covariance (ANCOVA). The following assumptions were checked.

1. Independency of observation
2. Normality
3. Measurement of the covariates
4. Reliability of the covariates
5. Correlation between the covariates and the dependent variable
6. Linearity
7. Homogeneity of variance
8. Homogeneity of regression (slopes)

The researcher administered and supervised the pre-test and post-test personally and made sure that each student answered the questions independently. Therefore, independency was observed. The assumptions of normality were checked for students' pre-test and post-test scores using histogram with normality curve (see Appendix L and Appendix M). From the histogram, since the bars are clustered at the Centre, then it means the scores are reasonably normally distributed. Also, the covariates were measured before the intervention was introduced. The pre-test had reliability coefficient of 0.71. The results of the correlation between the covariates and the dependent variable (students' post-test scores) were checked and presented in Table 4.8.

Table 4.8: Correlations between the covariates and the dependent variable

Correlations		
Students' pre-test scores		**Students' Post-test scores**
	Pearson Correlation	0.512**
	Sig. (2-tailed)	0.000
	N	159
Students' gender	Pearson correlation	-.131
	Sig. (2-tailed)	0.100
	N	159
Students' age in years	Pearson correlation	0.170*
	Sig. (2-tailed)	0.032
	N	159

***Correlation is significant at the 0.01 level (2-tailed)*
**Correlation is significant at the 0.05 level (2- tailed)*
Source: Author's Constructs with field data, 2020.

The results of the correlation between the covariates and the dependent variable in Table 4.8 reveal positive correlation between students' pre-test and post-test scores, $r(159) = 0.51, P < 0.01$. This correlation range (r= 0.51) is considered by Cohen (1988) as medium or reasonable correlation. Between students' age and post-test scores, correlation was small, $r\ (159) = 0.17,\ P < 0.05$. However, correlation between students' gender and post-test scores did not reach significant level, $r(159) = -.13$, $P = 0.10$. Therefore, students' age and gender were not included as covariates. As part of the test of assumptions, the linearity between students' pre-test and post – test scores, students' age and post-test scores and students' gender and post-test scores across the groups were ascertained to contribute to the evidence gathered in Table 4.8. Scatter plots of these relationships were summarized in Appendices I, J and K respectively

The scatter plot of students' pre-test and post-test scores (see Appendix I) indicates linear relationship across the groups since the lines are straight showing no violation of assumption of linearity. The scatter plot of students' age and post-test scores (see Appendix J) indicates no violation of assumption of linearity. However, the relationship was weak. On the other hand, the scatter plot of students' gender and post-test scores (see Appendix K) did not show linear relationship at all since the lines are crossover indicating violation of assumption of linearity. Therefore, students' age and gender were not included as covariates. Only students' pre-test scores were used as covariate. It was further required to examined assumption of homogeneity of variance to ensure that the variance of post-test scores across the groups is equal. The results were presented in Table 4.9.

Table 4.9: Levene's Test of Equality of Error Variance

Dependable variable: Post-test scores of students

F	df1	df2	Sig.
2.784	2	156	0.065

Design: Intercept + Pretest + Group

Source: Author's Constructs with field data, 2020.

The results in Table 4.9 show no violation of assumption of homogeneity of variance as evidenced by, $F(2, 156) = 2.784., P > 0.05$. This means the variance of post-test scores across the groups is equal and hence the assumption met. Moreover, the assumption of homogeneity of regression was examined. This was done to ensure that there is no interaction between pre-test scores and the groups. The results were presented in Table 4.10.

Table 4.10: Interaction between the pre-test scores and the group

Dependent Variable: Post-test scores of students.

Source	Type III Sum of Squares	Df	Mean Square	F	Sig.
Group	5.909	2	2.955	1.011	0.366
Pretest	61.731	1	61.731	21.120	0.000
Group*Pretest	0.267	2	0.134	0.046	0.955
Error	447.202	153	2.923		
Corrected Total	515.109	158			

Source: Author's Constructs with field data, 2020.

The results in Table 4.10 show no violation of assumption of homogeneity of regression as evidenced by, $F(2, 153) = 0.046\ P > 0.05$. This means, there is no interaction between pre-test scores and the groups and hence the assumption met. The researcher then conducted the One-way Analysis of Covariance (ANCOVA) after satisfying the necessary assumptions to test the research hypothesis. The independent variable (group) included three levels: Experimental group 1, Experimental group 2 and Control group. The students' pre-test and post-test scores were used as covariate and dependent variable respectively. The ANCOVA results were reported in Table 4.11.

Table 4.11: Results of ANCOVA for post-test scores of students

Dependent variable: Post-test scores of students

Source	**Type III Sum of Squares**	**Df**	**Mean Square**	**F**	**Sig.**	**Partial Eta Square**
Pretest	82.339	1	82.339	28.521	0.000	0.155
Group	58.897	2	29.449	10.201	0.000	0.116
Error	447.470	155	2.887			
Corrected Total	588.706	158				

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Source: Author's Constructs with field data, 2020.

The statistics in Table 4.11 show that after satisfying the assumption of homogeneity of regression to ensure no interaction between pre-test scores and groups, $F(2, 153) = 0.046, P > 0.05$, relationship between pre-test scores and post-test scores, $F(1, 155) = 28.52. P < 0.05$, $n^2 = 0.16$ and other assumption in Table 4.9, the ANCOVA results indicated a statistically significant group difference in students' achievement scores, $F(2, 155) = 10.20, P < 0.05, n^2 = 0.12$. Thus, 12% of the variance in post-test scores was explained by the intervention and this according to Cohen's (1988) criterion, indicates a large effect size. The follow-up test based on

LSD pair wise comparisons among the adjusted means was ascertained to find the group difference. The results obtained were reported in Table 4.12.

Table 4.12: ANCOVA pair wise comparisons of the adjusted means among the groups

Dependent Variable: Post-test scores of students

Groups	Adjusted Mean	Comparisons	Mean Difference	Std. Error	Sig.
Exp. group 1	4.11	Exp. group 1 Vs. Exp. group 2	0.88*	0.330	0.014
Exp. group 2	3.23	Exp. Group 1 Vs. Cont. group	1.60*	0.329	0.000
Control group	2.51	Exp. Group 2 Vs. Cont. group	0.72*	0.332	0.048

**. The mean difference is significant at the 0.05 level.*

Source: Author's Constructs with field data, 2020.

The results in Table 4.12 showed that students in Experimental group 1 had the largest adjusted mean (*M =4.11),* followed by Experimental group 2 ($M = 3.23$). and the least was the control group ($M = 2.51$). Besides, the LSD pairwise comparisons of the adjusted means among the groups showed that the mean difference is statistically significant in students' achievement scores between Experimental group 1 and Experimental group 2 ($M = 0.88$ $P = 0.014$), Experimental group 1 and Control group ($M = 1.60, P = 0.000$) and Experimental group 2 and Control group ($M = 0.72,\ P = 0.048$) after controlling for the effects of students' pre-test scores.

Therefore, based on the results of ANCOVA, $(2, 155) = 10.20,\ P < 0.05,\ \eta^2 = 0.12$, the null hypothesis was rejected in favor of alternative hypothesis. This suggest that statistically significant difference exists between students' scores in linear equations achievement test using multiple representations-based instruction and the traditional instruction after controlling for the effects of students' pre-test scores. This improvement in students' achievement in linear equations in one variable was as a result of the introduction of multiple representations-based instruction.

4.6.2 Discussion of Research Hypothesis Results

The analysis of students' scores in linear equations' achievement test based on the ANCOVA revealed a statistically significant difference among the groups in favor of multiple representations-based instruction, $F(2, 155) = 10.20,\ P < 0.05,\ \eta^2 = 0.12$. Also, the mean difference of the adjusted means among the groups was significant. The students' in the Experimental group 1 had the largest adjusted mean, indicating the highest performance. The possible reason for this result is that the Experimental group 1 students had the chance to experience linear equations through three different representations namely: manipulatives, graphic and algebraic representations. The students in the Experimental group 2 who experienced linear equations through two different representations namely: manipulatives and algebraic representation also did well with the second highest adjusted mean.

However, the students in the Control group had the least adjusted mean, indicating the lowest performance. This finding is consistent with the finding by Doktoroglu's (2013) who investigated the effects of teaching linear equations with a dynamic mathematics software on seventh grade students' achievement. The results indicated a significant effect on students' achievement in linear equations in favor of the Experimental group that utilize multiple representations of the dynamic mathematics software meanwhile no significant effects were found on activities that could not apply the multiple representations of the dynamic mathematics software. Similarly, these findings were consistent with other studies conducted by (Cikla, 2004; Thompson & Senk, 2001; Hong, Thomas & Kwon, 1999). This possibly indicates that teaching mathematics concept by utilizing more representations may increase students' achievement.

CHAPTER FIVE

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

5.0 Introduction

This chapter discusses the major findings of the study, educational implications, conclusions, limitations of the study, recommendations from the findings and suggestions for further studies.

5.1 Summary

The study investigated teachers' mode of representations in teaching linear equations in one variable, what accounts for teachers' choice of representations and students' representations preferences, and effects of multiple representations-based instruction on students' achievement in linear equations in one variable. In all 159 students (Junior High School 2) and 86 mathematics teachers were selected for the study through purposive and convenient sampling techniques respectively. Questionnaire, representation preference test, interview and linear equations achievement test were used as research instruments through quasi-experimental design. The students in experimental group 1 experienced multiple representations-based instruction with three different representations, experimental group 2 experienced multiple representations-based instruction with two different representations while control group students experienced traditional instruction with only algebraic representation. Both quantitative and qualitative forms the data analysis.

5.1.1 Summary of Major Findings

The major findings of the study were summarized as follows:

1. Majority of the teachers used algebraic representations every time when teaching linear equations.
2. Few of the teachers used manipulatives, graphic and multiple representations-based instruction when teaching linear equations.
3. Majority of the teachers used algebraic representations when teaching linear equations in one variable due to the following reasons such as easy, faster, simple, well-known, understandable, and widely used and many more.
4. Some teachers do not use manipulative, graphic and multiple representations when teaching linear equations due to the following reasons such as difficulty for students, time, lack of ideas, lack of materials, no relevance and absence from syllabus and many more.
5. Most of the students prefer to be taught linear equations using algebraic representations to the other representations.
6. The views expressed by students for algebraic representations preference were similar with what accounted for majority of teachers' choice of representations.
7. There was much improvements in students' achievement in linear equations achievement test when multiple representations-based instruction was used in teaching linear equations.

5.2 Educational Implications of the Study

As indicated by Pape, Bell and Yetkin (2003), instruction in the classroom is altered nowadays in order to help students appreciate what is taught. When multiple representations-based instruction and traditional instruction were used in teaching linear equations, students' achievement in experimental group 1 and experimental group 2 were higher than the control group students because of the introduction of multiple representations-based instruction in the experimental groups. Multiple representations serve as a strong instrument that eases understanding of mathematical concepts for students (Tripathi, 2008). Findings of (Cooper & Warren, 2011; Rose &Willson, 2012) revealed that multiple representations support abstraction of mathematical concepts and enhance students' learning. In this regard, teaching linear equations should not be limited to one representation such as algebraic. Other representations such as manipulatives and graphic should be used alongside with traditional use of algebraic representations.

The study also found that some teachers do not used manipulatives and other representations because of lack of ideas. If teacher does not have enough ideas about how to use a particular representation, the possibility of him or her using it to teach is high. In this regard, teachers' knowledge should be improved on how to use various representations in teaching linear equations in one variable. This will make it possible to integrate unfamiliar representations easily in the course of teaching and learning of linear equations in one variable. Moreover, reasons such as absence from syllabus, no relevant and many more limited teachers use certain representations in teaching linear equations. Therefore, this should be addressed by designing mathematics teaching text books to include all representations necessary in teaching linear equations in one variable to make them useful and relevant to teachers and students.

5.3 Conclusions

The study investigated teachers' mode of representations in teaching linear equations in one variable, what accounts for teachers' choice of representations and students' representations preferences, and effects of multiple representations-based instruction on students' achievement in linear equations in one variable. The following research questions were formulated:

1. What mode of representations do teachers use to teach linear equations in one variable in the Bimbilla Municipality?
2. What accounts for teachers' choice of representations in teaching linear equations in one variable in the Bimbilla Municipality?
3. What forms of representations do students prefer to be taught linear equations in one variable in the Bimbilla Municipality?

Also, the following hypothesis was tested:

H_0: There is no statistically significant difference between students' scores in linear equations achievement test using multiple representations-based instruction and traditional instruction after controlling for students' age, gender, and pre-test scores.

With the help of questionnaire, representations preference test, interview and linear equations achievement test as research instruments, it was found that majority of teachers and students used algebraic representation due to reasons such as easy, simple, fast, well-known, and understandable and many more. Finally, there was much improvement in students' achievement in linear equations achievement test when multiple representations-based instruction was introduced. Therefore, teaching linear equations by utilizing more representations increase students' achievement.

5.4 Recommendations

Based on the findings of the study, it was recommended that:

1. Mathematics teachers should adopt multiple representations-based instruction in teaching linear equations in one variable. This will enhance students' understanding of linear equations which serve as a hallmark for students' algebraic proficiency in mathematics (Huntley &Terrel, 2014).
2. Mathematics teachers' knowledge on how certain representations in teaching linear equations should be improved by providing professional courses such as INSET and other related capacity building seminars to help them integrate unfamiliar representations easily.
3. Students should be exposed to different forms of representations to increase their representation preferences without being influenced by limited choice of representation at their disposal.
4. Representational materials such as computers, graphs, algebra tiles, and other related accessories should be made available to mathematics for them to use in teaching linear equations in one variable.
5. The teaching syllabus and pupils' textbooks should be designed to include all forms of representations in teaching linear equations in one variable.

5.5 Suggestions for Further Studies

Based on the findings of the study, it is recommended that further research should be conducted to investigate:

1. Mathematics teachers' knowledge of representations of linear equations in one variable.
2. How mathematics teachers use various representations in teaching linear equations in one variable in the course of classroom instruction.
3. Long term effects of multiple representations-based instruction on students' achievement in linear equations and other algebra content areas.

REFERENCES

Aburime, F. E. (2007). How Manipulatives Affect the Mathematics Achievement of Students in Nigerian Schools. *Educational Research Quarterly*, *31*(1), 3-15.

Ainsworth S. (2006). A conceptual framework for considering learning with multiple representation. *Learning and Instruction*, 16, 183-198.

Ainsworth S. (2008). The educational value of multiple-representations when learning complex scientific concepts. In J. K. Gilbert, M. Reiner, & M. Nakhleh, *Visualization*: *Theory and Practice* in Science *Education*. *Modelsand* Modelling *in Science Education* (3,196).

Ainsworth S. E., Bibby, P. A., & Wood, D. J. (1997). Information Technology and multiple representations: New opportunities-new problems. *Journal of information Technology for Teacher Education*, 6 (1), 93-104.

Ainsworth S. E., Bibby, P. A., & Wood, D. J. (1998). Analysing the coasts and benefits of multi-representational learning environments. In M. W. Someren, P. Reimann, H. P. Boshuizen, & T. d. Jong, *Learning Multiple Representation*. Amsterdam: Pergamon.

Ainsworth S. E., Bibby, P. A., & Wood, D. J. (2002). Examining the effects of different representational system in learning primary mathematics. Journal *of the Learning Science*s 11(1), 25-62.

Ainsworth, S. (1999). The functions of multiple representations. *Computers& Education*, 33(2-3), 131-152.

Akpalu, R., Adaboh, S., & Boateng, S. S. (2018). Towards improving senior high school PSTs' conceptual understanding of system of two linear equations. *International Journal of Educational Research Review, 3(1), 28-40.*

Akyüz, D. & Hanggul (2013). Algebraic thinking in middle school students at different grades: Conceptions about generalization of patterns. *Necatibey Faculty of Education Electronic Journal of Science and Mathematics Education*, *10*(2), 243-272.

Alagic, M. (2003). Technology in the mathematics classroom: Conceptual orientation. *The Journal of Computers in Mathematics and Science Teaching*, 22(4), 381-360.

Alio, B., & Harbor-peters, V. F. (2000). The effects of Polya's problem solving technique on secondary school students' achievement in mathematics Abacus. *Journal of Mathematical Association of Nigeria*, 25 (1), 27-37.

Amissah, S. E. (1998). The role of mathematics in the FCUBE Program. *Journal of Mathematics Connection*, *Mathematics Association of Ghana (MAG).*

Anamuah-Mensah, J., & Mereku, D. (2005). Ghanaian JSS 2 students' Abysmal Mathematics Achievement in Timss-2003: A consequence of the Basic school mathematics curriculum. *Paper presented at the* West *Africa Examination Council (WAEC)Monthly Seminar at WAEC Conference Hall*, *Accra*

Annum, G. (2015). *Research Instrument for Data Collection*. Kumasi: KNUST Gh.

Arnold, S. (1992). Images of functions in Australian Schools and Universities. *Australian Senior Mathematics Journal*, 6(2), 108-126.

Austin, J.D., &voltrath, H. J (1989). Representing Solving and Using Algebraic Equations, *Mathematics Teachers* 82, 608-612.

Avant, M.J., & Heller, K.W. (2011). Examining the effectiveness of Touch Math with students with physical disabilities. *Remedial and Special Education*, 32(4), 309-321.

Babbie, E. (1990). *Survey Research Methods* (2nd Ed). California: Wadsworth Publishing Company.

Bal, A.P. (2014). The examination of representation used by classroom teacher candidates in solving mathematical problems. *Educational Sciences*: *Theory & Practice*, 14(6), 1-17.

Bansila, S., James, A., & Naidoo, M (2010), *whose voice matters*? Learners, South Africa.

Bardini, C., Pierce, R. U., & Stacey, K. (2004). Teaching linear equations in context with graphic calculators: Students' responses and the impact of the approach on their use of algebraic symbols. *International Journal of Science and Mathematics Education,* 2(3), 353-376.

Baroody, A. J. (1989). Manipulatives don't come with guarantees. *Arithmetic Teacher*, 37(2), 4-5.

Bashmakova, I. G., &Smirnova, G. S. (2000). In A. Shenitzer, & J. Stillwell, *Mathematical evolutions*. Washington, DC: The Mathematical Association of America.

Battista, M. T. (1999). *The mathematical miseducation of America's youth*. Bloomington: Phi Delta Kappan.

Beatty, R.A. (2010). Pattern rules, patterns and graphs: Analysing grade 6 students' learning of linear functions through the process of webbing, situated abstraction, and convergent conceptual change (Ph. D. Dissertation*).* Canada*,* University of Toronto*.*

Bednarz, N., Kieran, C., and Lee, L. (1996) *Approaches to Algebra*: *Perspective for Research and Teaching*. Dordrecht: Kluwer Academic Publishers.

Berlinghoff , W. P., & Gouvea, F. Q. (2002). *Math through the ages. A gentle historyrom teachers and others*. Farmington, ME: Oxton House Publishers.

Berman, B., &Friederwitzer, F. (1989). Algebra can be elementary when it's concrete. Arithmetic Teacher, 36, 21-24.

Best, J. W., & Kham, J. V. (1993). *Research in Education* (7th Ed). Boston: Allyn and Bacon.

Beyranevand*,* M*. (*2010*). Investigating mathematics students' use of multiple representations when solving linear equations with one unknown (Unpublished doctoral dissertation).* University of Massachusetts*,* Lowell*,* USA*.*

Blanton, M., & Kaput, J. (2003). Developing elementary teachers' "algebra eyes and ears". *Teaching Children Mathematics*, 10(2), 70-77.

Bless, C., & Higson-Smith, C. (2000). Fundamentals of Social Research Methods. Cape Town: Juta*.*

Blume, G. W., & Heckman, D. S (2000). Algebra and functions. In E. A. Silver, & P. A. Kenny, *Results from the Seventh Mathematics Assessment of the National Assessment of Educational Progress*. Reston, VA: NCTM.

Boakye, K. (2018). The use of Manipulatives in teaching Mathematics in upper primary schools in Antoa Community, Ghana. Theses, Presbyterian University College, Ghana.

Boggan, M., Harper, S., &Whitmire, A. (2010). Using manipulatives to teach elementary mathematics. *Journal of Instructional Pedagogies*, 3, 1-6.

Borenson, H. (1994). *The hands-on equations learning system*: *Instruction and level* I. Allentown, PA: Borenson and Associates.

Borenson, H., & Barber, L. (2008). The effect *of hands-on equations on the learning of Algebra*. Allen own, PA: Borenson and Associates.

Boulton-Lewis, G. M. (1998). Children's strategy, use and interpretations of mathematical representations. *Journal of Mathematical Behaviour*, 17(2), 219-237.

Boulton-Lewis, G., Cooper, T., Atweh, B., Pillay.,Wilss, L., &Mutch, S. (1997).Processing load and the use of concrete representations and strategies for solving linear equations. *Journal of Mathematical Behaviour*, 16(4), 379-397.

Brizuela, B. M., & Schliemann, A. D. (2004). Ten-year old students solving linear equations. *For the Learning of Mathematics, 24(2), 33-40.*

Brookie, E. B., (2014). *The effectiveness of manipulatives materials in a High School Algebra II class.* Bachelor of Science Project Work, Ball State University, Muncie, Indiana.

Bruner, J. (1966). *Toward a theory of instruction. Cambridge:* MAL Belknap Press.

Bruner, J. S. (1964). Learning and Thinking. In R. C. Anderson & D.P. Ausubel (Eds.), *Reading in the Psv ychology of Recognition*, 12(2). Berkeley: University of California Press.

Burns, N., & Grove, S. K. (2005). *The practice of nursing research*: *conduct*, *critique and utilisation* (*5th Ed*). Philadlphia: Elsevier.

Burns, N., &Grove, S.K. (2009). *The practice of nursing research: Appraisal, synthesis, and generation of evidence (6th Ed).* St. Louis, Missouri: Elsevier.

Butler, F. M., Miller, S.P., Crehan, K., Babbitt, B., & Pierce, T. (2003). Fraction instruction for students with mathematics disabilities: Comparing two teaching sequences. *Learning Disabilities Research and Practice*, 18(2), 99-111.

Caglayan, G., & Olive, J. (2010). Eighth grade students' representations of linear equations on a cups and tiles model. *Educational Studies in Mathematics*, 74,143-162.

Cai, J. (2005). U.S and Chinese teacher's constructing, Knowing, and evaluating representation to teach mathematics. *Mathematical Thinking learning*, 7(2), 135-169.

Calvin. (1990). *Conquering Mathematics Phobia*. Canada: Wiley & Sons Inc.

Canterbury, S.A (2007). *An investigation of conceptual knowledge: Urban Africa American Middle School students' use of fraction representations and computations in performance-based tasks (Doctoral dissertation).* University of Georgia.

Capraro, M.M., & Joffrion, H. (2006). Can middle-School students meaningfully translate from words to mathematical symbols? *Reading Psychology*, 27(2&3, 147-164).

Carry, L. R., Lewis, C., & Bernard, J. (1980). *Psychology of Education Solving*: *An Information Processing Study*. Austin: University of Texas at Austin, Department of Curriculum and Instruction.

Celik,*D.*, &Baki, A. (2007). *A paper presented at the 7th International Educational Technology Conference.* Near East University*:* North Cyprus.

Chappell, M. F (2001). Creating connections: promoting algebraic thinking with concreate models. *Mathematics teaching in the Middle School*, 7, 20-25.

Charles, D.M. (1994). *Mathematical Ideas*. New York: Harper Collins College Publishers.

Charles-Ogan, G. I. & Otikor, M. S. (2016). Practical utility of mathematics concepts among senior secondary school students in river state. *Europeans Journal of Mathematics and Computer Science,* 3(1), 2059-9951.

Cheng, P.C. (2000). Unlocking conceptual learning in mathematics and Science with effective representational systems. *Computers & Education*, 33, 109-130.

Cikla*,* O. A. (2004). *The effects of multiple representations-based instructions on seventh grade students' algebraic performance, attitude toward mathematics, and representation preference (Ph. D. dissertation). Turkey:* Middle East Technical University.

Clement, D.H., &McMillen, S. (1996). Rethinking concretes manipulatives. *Teaching Children Mathematics* 2(5), 270-279.

Cobb, P., Yackel, E., & Wood, T. (1992). A constructivist alternative to the representational view of mind in mathematics education, *Journal for Research in Mathematics Education*, 23(1), 2-33.

Cockcroft, W. (1982). Mathematics Counts: Report of the Committee of Enquiry into the Teaching of Mathematics in Schools. London: HMSO.

Cohen, L., Maniom, L., & Morrison, K. (2008). *Research methods in Education* (6th Ed). USA: Routledge.

Cole, J.E., & Washburn-Moses, L. H. (2010). Going beyond "math Wars". *Teaching Exceptional Children*, 42(4), 11-20.

Cooper, T.J., & Warren, E. (2011), Years 2 to 6 Students' ability to generalize: Models representations and theory for teaching and learning. In J. Cai, & E. Knuth, *EarlyAlgebraization*. Berlin: Springer Berlin Heidelberg.

Cortes, A., & Pfaff, N. (2000). *Solving equations and inequations: Operational invariants and methods constructed by students. Proceedings of the 24 the Conference of the International Group for the Psychology of Mathematics Education.* Hiroshima*:* Japan.

Cramer, K., &Bezuk, N. (1991). Multiplication of fractions: teaching for understanding. *Arithmetic Teacher*, 39(3), 34-37.

Creswell, J. W. (1994). *Research design*: *Qualitative quantitative approaches*. Thousand Oaks, CA: Sage Publications.

Creswell, J. W. (2003). *Research design*: *Qualitative*, *quantitative*, *and mixed methods approach*. Thousands of Oaks, CA: Sage publications.

Creswell, J.W., & Garrett, A. L. (2008). The "movement" of mixed methods research and the role of educators. South Africa. *Journal of Education*, 28(3), 321-333.

Davis, J.D. (2007). Real-world contexts, multiple representations, students invented terminology, and y-intercept. *Mathematical Thinking and learning*: *An international Journal*. 9(4), 387-418.

DeBellis, A., &Goldin, G.A. (2006). Affect and meta-affect in mathematical problem-solving: A representational perspective. *Educational Studies in Mathematics*, 63(2), 131-147.

Delice, V. A., &Sevimli, E (2010). Educational Sciences. *Theory & practice*, 10, 111-149.

Demana, F. D., Schoen, H. L., & Waits, B. K. (1993). Graphing in the K-12 curriculum: the impact of the Graphing calculator. In T. A. Rombrg, E. Fennema, & T. P. Carpenter, *Integrating research on the graphical representation off unction's*. Mahwah, NJ: Lawrence Erlbaum.

Denscombe, J. (2007). The good research guide for small scale social research projects. Maiden head, England*:* Open University Press.

Derbyshire, J. (2006). Unknown quality: *Area land imaginary history of algebra*. Washington DC: Joseph Henry press.

Dienes, Z. P. (1960). *Building up Mathematics*. London: Hutchinson Education.

Dogbe, B., Mereku, D. K., & Quarcore, E. B. (2004). *Junior Secondary Mathematics* 2. Unimax Macmillan Ltd: Accra, Ghana.

Doktoroglu, R. (2013). *The effects of teaching linear equations with dynamic Mathematics Software on seventh grade students' achievement (Master's Thesis).* Turkey: Middle East Technical University.

Dressler, I., & keenan, E. P. (1998). *Integrated Mathematics-Course* 1(3rd Ed) New York, NY: AMSCO School publication inc.

Dugopolski, M. (2002). *Elementary and Intermediate Algebra* (1st Ed). USA: McGraw-Hill Companies.

Dupour-Janvier, B., Bednarz, N., &belanger, M. (1987). Pedagogical considerations concerning the problem of representation. In C. Janvier, *Problems of Representation in the learning and Teaching of Mathematics*. New Jersey: Yayinevi.

Durrheim, K. (1999). Quantitative measurement. In M. T. Blanche, & K. Durrheim, *Research in practice: Applied methods for the social sciences*. Cape Town: University of Cape Town press.

Duval, R. (2006). A cognitive analysis of problems of comprehension in a learning of mathematics. *Educational Studies in Mathematics*, 61, 103-131.

Egodawatte, G. (2011). *Secondary school students' misconceptions in algebra* (Ph.D dissertation).University of Toronto: Canada.

Elia, I., panaoura, A., Eracleous, A., &Gagatsis, A. (2007). Relations between secondary pupils' conceptions about functions and problem solving in different representations. *International Journal of Science and Mathematics Education*, 5, 533-556.

Eraslam, A. (2008). The notion of reducing abstraction in quadratic functions. *International Journal of Mathematical Education in science and Technology*, 39, 1051-1060.

Essien, A., & Setati, M. (2006). Revisiting the equal sign: Some grade 8 and 9 learners' interpretations. *African Journal of Research in Mathematics, Science and Technology Education*, 10(1), 47-58.

Even. R. (1998). Factors involved in linking representations of functions. *Journal of Mathematics Behaviour*, 17(1),105-121.

Fadel, C. (2008). Multimodal learning through media: what the research says. SanJose, CA: Cisco systems.

Filloy, E., & Rojano, T. (1989). Solving equations: The transition from arithmetic to algebra. *For the Learning of Mathematics*, 9(2), 19-25.

Filloy, E., & Sutherland, R. (1996). Designing curricula for teaching and learning algebra. In A Bishop, K Clements, C. Keitel, J. Kilpatrick, & C. Laborde, *International handbook of mathematics education*. Dordrecht: Kluwer.

Filloy, E., Rojano, T., &Puig, L. (2007). *Educational algebra*: *Atheoretical and empirical approach*. Berlin Heidelberg, New York: Springer.

Fink, A. (1995). *How to Sample in Surveys*. London: Sage publications.

Flores, M. M (2009). Teaching subtraction with regrouping to Students experiencing difficulty in mathematics. *Preventing School Failure*: *Alternative Education for Children and Youth*, 53(3), 145-152.

Fraenkel, J. R., &Wallen, N. E. (1996). *How to design and evaluate research in education(3rd Ed).* USA: McGraw Hill Inc.

Freitas, M. A. (2002). Equacao do primeirograu: Methods de resolucoaeanalise de error noensi no medio. In R. N. Lima, *Procedural Embodiment and Quadratic Equations*.

Frey, L. R., Carl, H. B., &Gary, L. K. (2000). *Investigating Communication*: *An introduction to Research Methods(2nd Ed).* Boston: Allyn Bacon.

Gagatsis, A., Shiakalli, M. (2004). Ability to translate from one representation of the concept of function to another and mathematical. Problem solving. *Educational Psychology*, 24(5), 645-657.

Gall, M., Gall, J., & Borg, W. (2007). *Educational Research*: *An introduction(8th Ed).* Boston: Pearson.

Gardner, H. (1993). *Multiple intelligences*: *The theory in practice*. New Yok: Basic Books.

Gay, L. R., &Deihl, P. L. (1992). *Research Methods for Business and Management*. New York: Macmillan.

Gay, L. R., Mills, G. E., &Airasian, P. (2006). *Educational research*: *Competencies for analysis and applications(8th Ed).* Nj: Pearson Education Inc.

George, D., &Mallery, P. (2003) SPSS for *Windows step by step*: *A simple guide and reference(4th Ed).* Allyn & Bacon.

Ginsburg, H. (1989), *children's arithmetic.* Austin, TX: Pro-Ed.

Girard, N. R. (2002) *students' representational approaches to solving calculus: Examining the role of graphing calculators (Unpublished)*Pittsburgh: University of Pittsburgh.

Goldin, G. A. (1990). Epistemology, constructivism, and discovery learning mathematics. In R. B. Davies, C. A. Maher, & N. Nodding, *Constructivist views on the Teaching and Learning of Mathematics*. Reston, VA: NCTM.

Goldin, G. A. (1998). Representational systems, learning, and problem solving in mathematics. *Journal of Mathematics Behaviour*, 17(2),137-165.

Goldin, G. A. (2000). Affective pathways and representational in mathematical problem solving. *Mathematical thinking and learning*, 2(3), 209-219.

Goldin, G. A. (2003). Representation in school mathematics: A unifying research perspective. In J. Kilpatrick, W.G. Martin, & D. Schifter, *A research companion to principles and standard for school mathematics*. Reston, NJ: NCTM

Goldin, G. A., &Janvier, C. (1998). Representations and the psychology of mathematics education. *Journal of mathematics Behaviour*, 17(1),1-4.

Goldin, G. A., Shteingold, N. (2001). System of mathematical representations and development of mathematical concepts. In F. R. Curcio, *The roles of representation in school mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Goldin, G.A., & Kaput, J.J. (1996). A joint perspective on the idea of representations in learning and doing mathematics. In S. P. Leslie, & N. Pearla, *Theories of mathematical learning*. Mahwah, NJ: Lawrence Erlbaum Associates.

Hall, R. (1996). Representation as shared activity: Situated and cognition and Dewey's cartography of experience. *The Journal of the Learning Science*, 5(3) 209-238.

Hammer, A. S., D., & Sherin, B. (1991). Inventing graphing: Meta representation expertise in children. *Journal of Mathematical Behaviour*, 10, 117-160.

Hazan, O., & Zaskis, R. (2005). Reducing abstraction: the case of school mathematics. *Educational and Studies in Mathematics*, 58,101-119.

Henry, G. T. (1990). *Practical Sampling*. London: Sage publications.

Herman, M. F. (2002). *Relationship of college students' visual preference to use of representations: Conceptual understanding of functions in algebra (Unpunished Ph.D. dissertation)*. Columbus: Ohio State University.

Herman, M. F. (2007). What students choose to do and have to say about use of multiple representations in college algebra. *The Journal of Computers in Mathematics and Science Teaching*, 26(1), 27-54.

Herscoviecs, N., & Kieran, C (1980). Constructing meaning for the concept of equation. *Mathematics Teacher*, 73,572-581.

Hiebert, J., & Carpenter, T. P. (1992). Learning and Teaching with understanding. In D. Grouws, *Handbook of research on mathematics teaching and learning.* New York: Macmillan Library Reference.

Hitt**,** F**.** (2001). Working group on representations and mathematics visualization. *Representation and mathematics Visualization*, *2*, *3*.

Hong, Y. Y., Thomas, M., & Kwon, O-N. (1999). *Understanding linearalge braicequationsviasuper-calculatorrepresentations*. *Seoul*: *Korea*

Hudson, P., & Miller, S.P. (2000). *Designing and implementing instruction for students with diverse learning needs*. Boston: Pearson Education.

Huntley, M. A., Terrel, M. S. (2014). One-step and multi-step linear equations: a content analysis of five textbook series. *ZDM Mathematics Education*, *46*, *751-766*.

Huntley. M. A., Marcus, R., Kahan J., & Miller J. L. (2007). Investigating high school students reasoning strategies when they solve linear equations. *Journal of Mathematics Behavior*, 26, 115-139.

In F. Seeger, I. Voight, & V. Werschescio, *The culture of the mathematics classroom*. Cambridge: Cambridge University Press.

Janvier, C. (1987). Representation and understanding: the notion of function as an example. In C. Janvier, *Problems of Representations in the Learning and Teaching of Mathematics*. New Jersey: Lawrence Erlbaum Associates.

Janvier, C. (1998). The notion of chronicle as an epistemological obstacle to the concept of function. *Journal of Mathematics Behavior*, 17(1), 79-103.

Janvier, C., Girardon, C., & Morand, J. (1993). Mathematical Symbols and Representations. In P. S. Wilson, *Research Ideas for the Classroom*: High School Mathematics. Reston, VA: NCTM.

Johnson, B., & Christensen, L. (2008). *Educational research: Quantitative, qualitative, and mixed approaches (3rd Ed).* LA: Sage.

Kaput J. J. (1994). The representational roles of technology in connecting mathematics with authentic experience. In R. Biehler, R. W. Scholz, R. Strasser, & B. Winkelman, *Didactics in Mathematics as a Scientific Discipline*. Netherlands: Kluwer Academic Publishers.

Kaput, J. J. (1989). Linking representations in the symbol system of algebra. In S. Wagner, & C. Kieran, *Research Issues in the Learning and Teaching of Algebra*. Reston, VA: National Council of Teachers of Mathematics.

Kaput, J.J. (1991). Notations and representations as mediators of constructive processes. In E.V. Glasersfeld *Radical Constructivism in Mathematics Education*. Netherlands: Kluwer Academic Publishers.

Karakırık, E., & Aydın, E. (2011). Matematik Nesneleri. Retrieved on 12/5/2018 from https://www.academia.edu/9634384/MATEMAT%C4%B0K_NESNEL ER%C4%B0 performance in algebra at the junior high school (*Unpublished master's thesis*). University of Cape Coast, Ghana.

Keller, B. A., & Hirsch, C. R. (1998). Student preferences for representation of functions. *International Journal in Mathematics Education*, *Science*, *and Technology*, 29(1), 1-17.

Kieran C. Booker, G., Filloy, E., Vergnaud, G., & Wheeler, D. (1990). Cognitive processes involved in learning school algebra. In P. Nesher, & J. Kilpatrick, *Mathematics and Cognition*. Cambridge University Press.

Kieran C., & Drijvers, P. (2006). The co-emergence of machine techniques, paper-and pencil techniques, and theoretical reflection: A study of CAS use in secondary school algebra. *International Journal of Computers for Mathematical Learning,* 11(2), 205-263.

Kieran, C. (1992). The learning and teaching of algebra. In D. A. Grouws, *Handbook of Research on Mathematics Teaching and Learning*. NY: Macmillan Publishing Company.

Kieran, C. (2004). The core of algebra. Reflection on its main activities. In K. Stacey, H. Chick, & M. Kendal, *the future of the teaching and learning of algebra: The 12th ICMI study*. Boston, MA: Kluwer.

Kilpatrick, J., &Izsak, A. (2008). A history of algebra in the school curriculum. In C. E. Greenes, *Algebra and algebraic thinking in school mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding it up: helping children learn mathematics.* Washington, DC: National Academy Press.

Klein, P.D. (2003). Rethinking and multiplicity of cognitive resources and curricular representations: alternatives to 'learning styles' and 'multiple intelligences. *Journal of Curriculum Studies*, 35(1),45-81.

Knuth, E. J. (2000). Students understanding of the Cartesian connection: an exploratory study. *Journal for Research in Mathematics Education*, 31(4), 500-508.

Knuth, E.J., A. C., McNeil, N. M., &Alibali, M. W. (2006). Does understanding the equal sign matter? Evidence from solving equations. *Journal of Research in Mathematics Education*, 37, 297-312.

Krabbendam, H. ((1982). The non-qualitative way of describing relations and the role of graphs: some experiments. In G. V. Barnveld, & H. krabbendam, *Conference on Functions*. Enschede, Netherlands: Foundation for Curriculum Development.

Kramarski, B., &Mevarech, Z. R. (1997). Cognitive-metacognitive training within a problem-solving Based logo environment. *British Journal of Educational Psychology*, 67(4), 425-445.

Kuchemann, D. (1981). Algebra. In K. M. (Eds.), *Children's' understanding of mathematics*. London: Murray.

Kurumeh, M. S., Chiawa, M. A., & Ibrahim, M. O. (Eds.), (2010). Dienes multi-base blocks approach: An effective strategy for improving interest in number bases among secondary school students in mathematics. *Research Journal of Mathematics and statistics, 2(3),* 101-104.

Larbi, E. (2011). *The effects of algebra tiles on students' performance in algebra at the junior high school (Unpublished Master's Thesis).* University of Cape Coast: Cape Coast.

Larbi, E., &Okyere, M. (2014). Algebra tiles manipulative and gender differences in learning and achievement in mathematics: A Case of Sunyani West. *Journal of Education and Practice*, 5(38), 1-8.

Laud, C. (1995). *Canadian Business Mathematics Book One*. Montreal Canadian Pacific Inc.

Legner, P. (2013, January 12). *The value of teaching mathematics*. Retrieved from http://mathigon.org/resources/value-of-mathematics.pdf

Leinhardt, G., Zaslevsky, & Stein, M. K. (1990). Functions, graphs, and graphing: tasks, Learning and teaching. *Review of Educational Research*, 60(1), 1-64.

Lesh, F.K., & Behr, M. (1987b). Representations and translations among representations in mathematics learning and problem solving. In C. Janvier, *Problems of Representations in the Teaching and Learning of Mathematics*. New Jersey: Lawrence Erlbaum Associates.

Lesh, R., Post, T., & Behr, M. (1987a). Dienes revisited: multiple embodiments in computer environment. In I. Wirsup, & R. Streit, *Development in School Mathematics Education around the World*. Reston, VA: National Council of Teachers of Mathematics.

Leung, F.K. Clarke, D., Holten, D., & Park, K. (2014). *Algebra teaching* around *the world*. Rotterdam, The Netherlands: Sense Publishers.

Li, K. (2007). *An investigation of secondary school algebra teachers' mathematical knowledge for teaching algebraic equations solving (Ph. D dissertation).* University of Taxas: Austin.

Lima, R. N., & Tall, D. (2008). Procedural embodiment and magic in linear equations. *Educational Studies in Mathematics*, 67, 3-18.

Linsell, C. (2008). *Solving equations: Students algebraic thinking. In finding from the New Zealand Secondary Numeracy Project 2008.* Wellington, New Zealand: learning media.

Linsell, C. (2009) Students' knowledge and strategies for solving equations. In finding from the New Zealand Secondary Numeracy Project 2008. Wellington, New Zealand: Learning Media.

Lowrie, T. (2001). The influence of visual representation on mathematical problem solving and numeracy performance. In B. Perry, *Numeracy and beyond*. Sydney: Mathematics Education Research Group of Australia.

Lubinski, C. A., & Otto, A. D. (2002). Meaningful mathematical representation and early algebraic reasoning. *Teaching Children Mathematics*, 9(2), 76-91.

Maccini, P., &Ruhl, K. L. (2000). Effects of graduated instructional sequence on the algebraic subtraction of integers by secondary students with learning disabilities. *Education and Treatment of Children*, *23*(4), 465-489.

Macnealy, M. S. (1999). *Strategies for Empirical Research in Writing*. New York: Longman.

Martin, J. L. *(1994). Mathematics for Teacher Training in Ghana. Students' Activities.* Accra-North: Unimax.

Mason, J. (1996). *Approaches to Algebra*: *Perspective for Research and Teaching*. Dordrecht: Kluwer Academic publisher.

Matz, M. (1981). Building Metaphoric Theory of Mathematics Thought. *Journal of Mathematical Behavior*, *3*(1), 93-166.

Matz, M. (1982). Towards a process model for school algebra error. In D. Sleeman, & J. S. Brown, *Intelligent tutoring systems*. New York: academic press.

McIntosh, G. V. (2012). *Testing instrumentation validity for measuring teachers' attitudes towards manipulative use in the elementary mathematics classroom.* Retrieved from http://search.proquest.com/docview/1312423428?accountid=14771.

Mcneil, N. M., & Uttal, D. H. (2009). Rethinking the use of concrete materials in learning: perspective from development and education. *Child Development Perspectives*, 3, 137-139.

Mercier, E. M., & Higgins, S. E. (2013). Collaborative learning with multitouch technology: Developing adaptive expertise. *Learning and Instruction*, *25*, 13-23.

Mereku, D. K. (2001). An investigation into factors that influence teachers' content coverage in primary mathematics. *African* Journal *of Educational Studies in Mathematics and Science*, 1, UEW, SACOST.

Ministry of Education (2012). Teaching Syllabus for Mathematics (Junior High School). CRDD, Accra.

Ministry of National Education (MoNE) (2013). *İlköğretim matematik dersi (5-8, Sınıflar) öğretim program.* Ankara: MEB.

Monk, S. (2003). Representation in school mathematics: learning to graph and graphing to learn. In J. Kilpatrick, W. G. Martin, & D. Schifter, *A research companion to principles and standards for school for mathematics*. Reston, VA: NCTM.

Monk, S. (2004). Representation of in school mathematics: learning to graph and graphing to learn. In J. Kilpatrick, *A research companion to principles and standards for school mathematics*. Reston, VA : NCTM, Inc.

Monoyiou, A., Papageorgiou, P., &Gagatis, A. (2007). Students' and teacher's representation in problem solving. Working Group. *The roles of images and metaphors in the learning and understanding mathematics*. CERMES, 5, 141-150.

Moritz, J. (2000). Graphical representations of statistical associations by upper primary students. In A. Chapman, *Mathematics education beyond.* Fremantle: Mathematics Education Research Group of Australia.

Moseley, B. & Brenner, M.E. (1997). *Using multiples representation for conceptual change in pre-algebra: A comparison of variable usage with graphics and text-based problems.* Washington, DC: Officer of Educational Research and Improvement.

Moseley, B. (2005). Students' early mathematics representation knowledge: the effects of emphasizing single or multiple perspectives of the rational number domain in problem solving. *Educational Studies in Mathematics*, 60, 37-69.

Mulryan, C. M. (1994). Perceptions of intermediate students' cooperative small-group work in mathematics. *The Journal of Educational Research*, *87*(5), 280-291.

Nabie, M. J. (2009). *Fundamentals of the Psychology of Learning Mathematics. Revised edition.* Accra: Akwa Impression Ltd.

National Council of Teachers of Mathematics (2000). *Principles and Standards for School Mathematics.* Reston, VA: NCTM Publications.

NCTM. (2000). *Principles and Standards for School Mathematics.* Reston VA: NCTM Publications.

Neria, D., &Amit, M. (2004). Students' preference of non-algebraic representations in mathematics communication. *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education*, 3, 409-426.

Nickson, M. (2000). *Teaching and learning mathematics: a teachers' Guide to recent research and its application.* London: Cassell.

Numan, D. (1999). *Research methods in language learning*. Cambridge: CUP.

Nunes, T. (1996). Children Doing Mathematics. Blackwell publisher's ltd: Oxford, United Kingdom.

Odili, G. A. (1990). *Teaching Mathematics in the Secondary School*. Obosi: Anachuna Educational Books.

Okpube, N. M. (2016). Card games and algebra mathematics on achievement of junior secondary ii students in algebraic expressions. *International Journal of Evaluation and Research in Education*, *5*(2), 93-100.

Osula, E. (2001). *Introduction to research methodology(3rd Ed).* Nigeria: African-Feb Publishers Ltd.

Outhred, L., &Saradelich, S. (1997). Problem solving in kindergarten: the development of children's representations of numeral situations. In K. Carr, *People in Mathematics Education*. Retorura: Mathematics Research Group of Australasia.

Owens, K. D., & Clements, M. A. (1997). Representations in spatial problem solving in thE classroom. *Journal of mathematics Behavior*, 17(2), 197-218.

Ozgun-koca, S. A. (1998). *Students' use of representations in mathematics education Psychology of Mathematics Education* NC: Raleigh.

Ozugun-Koca, S. A. (2004). The effects of multiple link representations on students' learning of linear relationships. *Psychology of Mathematics Education*, 26, 82-90.

Padmore, E. A. (2017). Examining the use of manipulative materials in teaching Mathematics among junior high school teachers in the Wa municipality. MPhil. Thesis. Department of Basic Education of the Faculty of Education. University of Education, Winneba.

Panasuk, R. (2010). Three-phase ranking framework for assessing conceptual understanding in algebra using multiple representations. *Education*, 131(4), 235-257.

Panasuk, R. (2011). Taxonomy for assessing conceptual understanding in algebra using multiple representations. *College Student Journal*.

Pape, S. J. &Tchoshvanov, M. A. (2001). The role of representations(s) in developing mathematical understanding. *Theory into practice*, 40(2), 118-127.

Pape, S. J. Bell, J. &Yetkin, I. E. (2003). Developing mathematical thinking and self-regulated learning: A teaching experiment in a seventh-grade mathematical classroom. *Educational Studies in Mathematics*, 53,179-202.

Parahoo, K. (2006). *Nursing research*: principles, process, and issues. New York: Palgrave Macmillan.

Perso, T. (1996). Teaching equation-Again? *The Australian Mathematics Teacher,* 52, (1) 19-21.

Polit, D. F. & Beck, C. T. (2004). Nursing research: *appraising evidence for nursing practice(7th Ed).* Philadelphia: wolters Kluwer/Lippincott Williams & Wilkins.

Polit, D. F. & Beck, C. T. (2012). *Nursing research: generating and assessing evidence for nursing practice*. Philadelphia: Wolters Kluwer Health/Lippincott William & Wilkins.

Polit, D. F. &Hungler, B. P. (1999). *Nursing research: Principles and methods(6th Ed).* Philadelphia: Lippincott.

Poon, K., & Leung, C. (2010). Pilot study on algebra learning among junior secondary students. *International Journal of Mathematics Education in Science and Technology*, 41, 49-62.

Post, T., Behr, M. J., &Lesh, R. (1988). Proportionality and the development of pre-algebra understanding. In A. F. Oxford, *The ideas of algebra.* Reston VA: NCTM.

Prosser, M., &Trigwell, K. (1999). *Understanding Learning and Teaching: The Experience In High Education*. SRHE & Open University Press.

Proulx, J. Beisiegel, M., Miranda, H., &Simmt, E. (2009). Rethinking the teaching of systems of equations. *Mathematics teacher*, 102(7), 526-533.

Puchner, L., Taylor, A., O'Domell, B., & Fick, K. (2008). Teacher learning and mathematics manipulatives: A collective case study about teacher use of manipulatives in elementary and middle school mathematics lessons. *School Science and Mathematics*, 108(7), 313-325.

Rajano, T. (2002). Mathematics learning in the junior secondary school: students' access to significant mathematical ideas. In English, *Handbook of International Research in Mathematics Education*. Mahwah, NJ: Lawrence Erlbaum Associates.

Resnick, L. B., & Ford, W. W. (1981). *The Psychology of Mathematics for Instruction*. Lawrence Erlbaum Associates.

Richard, D. G. H. (2002) An Analysis of Errors Made in the Solution of Simple Linear Equations; Philosophy of Mathematics Education Journal 15.

Rider, R. L. (2004). *The effects of multi-representational methods on students' knowledge of function concepts in developmental college mathematics.* North Carolina State University.

Roscone, J. K. (1975). *Fundamental Research Statistics for the Behavioral Sciences* (2nd Ed). New York: Holt Rinehart & Winston.

Rosli, R., Goldsby, D. & Capraro, M. (2015). Using Manipulative materials in Solving and Posting Mathematical Problems. *Creative Education, 6,* 1718-1725.

Ross, A., &Willson, V. (2012). The effects of representations, constructivist approaches, and engagement on middle school students' algebraic procedure and conceptual, understanding. *School Science and Mathematics*, 112, (2), 117-128.

Schneider, B., Carnoy, M., Kilpatrick, J., Schmidt, W. H., &Shavelson, R. J. (2007). *Estimating causal effects using experimental and observational designs*. Washington, DC: American Educational Research Association.

Seeger, F. Voight, I. & Werschescio, V. (1998). Representations in the mathematics classroom: reflections and constructions.

Sfard, A. (2000). Symbolizing mathematical reality into being: How mathematical discourse and mathematical objects create each other. In P. Cobb, E. Yackel, & K. McClain, *Symbolizing and communicating*: *Perspective on mathematical discourse*, *tools*, *and instructional design*. Mahwah, NJ: Erlbaum.

Sharp, J. M. & Adams, B. (2002). Children's construction of knowledge for fractions division after solving realistic problems. *The Journal of Educational Psychology*, 95(6), 333-347.

Sharp, J. M. (1995). *Results of using algebra tiles as meaningful representations of algebra concepts. Paper Presented at the Annual Meeting of the Mid-Western Education Research Association.* Chicago, IL.

Sherman, J., &Bisanz, J. (2009). Equivalence in symbols and non-symbolic contexts: Benefits of solving problems with manipulatives. *Journal of Educational Psychology*, 101, 88-100.

Sherrod, S. E., Dwyer, J., & Narayan, R. (2009). Developing science and mathematics integrated activities for middle school students. *International Journal of Mathematics in Science and Teaching*.

Sidney, L. R. (1993). Algebra I: *A Process Approach*. Hawaii: University of Hawaii Press.

Silver, H. E., & Strong, R. W. (2003). *Learning style inventory*. Ho-Ho-Kus: thoughtful education Press.

Singletary, J. (1995). *Central Mathematics for GCES and Standard Grade*. London: Heineman Educational Bsooks Ltd.

Slavin, R. E. (2000). Educational Psychology: *Theory and Practice(6th Ed).* Needham Heights, MA: Allyn& Bacon.

Smith, S. P. (2004). Representation in school mathematics: Children's representations of problems. In J. Kilpatrick, *A Research Companion to Principles and Standards for School Mathematics*. Reston, VA: NTCM, Inc.

Sowell, E. J. (1989). Effects of manipulatives materials in mathematics instruction. *Journal for Research in Mathematics Education*, 20(5), 498-505.

Spiro, R. J. (1990). Cognitive flexibility and hypertext: Theory and technology for the nonlinear and multidimensional traversal of complex subject matter. In D. Nix., & R. Spiro, *Cognition*, *education and multimedia*: *Exploring ideas in high technology*. Hillsdale, NJ: Lawrence Erlbaum Associates.

Stacey, K. & Chick, H. (2004). Solving the problem with algebra. In K. Stacey, H. Chick, & M. Kendal, *the future of the teaching and learning of algebra*, *the12th ICMIStudy*. Boston, MA: Kluwer Academic Publishers.

Star, J. R. (2005). Conceptualizing procedural knowledge. *Journal of Research of Mathematics Education*, 36, 404-411.

Star, J. R., & Seifert, C. (2006). The development of flexibility in equation solving. *Contemporary Educational Psychology*, 31, 280-300.

Stein, M. K., Baxter, J. A., &Leinhardt, G. (1990). Subject-Matter and Elementary Instruction: A case from functions and graphing. *Educational Research Journal*, 27(4), 639-663.

Streter, J., Hutchison, D., &Hoelzle, L. (2000). *Intermediate Algebra* (*3rd Ed*). McGraw-Hill Co, Inc., USA.

Suh, J., &Moyer, P.S. (2007). Developing students' representational fluency using virtual and physical algebra balances. *The Journal of computers in Mathematics and Science Teaching*, 26(2), 155-173.

Swafford, J. O., &Brown, C. A. (1989). Variables and relations. In M. M. Linguist, *Results from the fourth mathematics assessment of the National Assessment of Educational progress.* Reston, VA: NCTM.

Swafford, J.O., & Brown, C.W. (2000). Grade 6 students' pre-instruction use of equation to describe and represent problem situations. *Journal for Research in Mathematics Education*, 31(1), 89-112.

Swatton, P., & Taylor, R. M. (1994). Pupil performance in graphical tasks and relationship to the ability to handle Variables. *British Education Research Journal*, 20, 227-243.

Tashakori, A., &Teddie, C. (2003.). *Handbook of mixed methods in social and* behavioural research. Thousand Oaks, CA: sage.

The West African Examination, (2011). Chief Examiners" Report on the Basic Council Education Certificate Examination, Accra, WAEC-Ghana.

Thompson, B. (1998). *Understanding coefficient alpha*. Colleges Station, Texas: USA.

Thompson, D. R., &Senk, S.L. (2001). The effects of curriculum on achievement in second-year algebra: The example of the University of Chicago School of Mathematics project. *Journal of Research in Mathematics Education*, 32(1), 58-85.

Thompson, P. W. (1999). Notation, Convention and Quality in Elementary Mathematics. In J. Sower, & B. Schapelle, *A Foundation for Teaching Middle School Mathematics*. Albany, NY: SUNY press.

Tomlinson, C. (2000). Reconcilable differences? Standards-based teaching and differentiation. *Education Leadership*, 58(1), 1-7.

Tossavainen, T. (2009). Who can solve 2x = 1? An Analysis of cognitive load Related to Learning Linear Equations Solving. The Montana *Mathematics Enthusiast*, 6(3), 435-448.

Tripathi, P.N. (2008). Developing mathematical understanding through multiple representations. *Mathematics Teaching in Middle School*, 13(89), 438-445.

Tsamir, P., & Almog, N. (2001) Students' strategies and difficulties: The case of algebraic inequalities. *International Journal of Mathematics Educationin Scienceand Technology*, *32*(4), 513_524.

Uribe-Flòrez, L. J. & Wilkins, J. L., (2010). Elementary School Teachers' manipulatives use. *School science and mathematics, 110*(7), 363-371. doi:101111/j.1949-8594.2010. 00046.

Usiskin, Z., Pereesini, A., Marchisoto E. A., & Stanley, D. (2003) *Mathematics for High School Teachers*: An Advanced Perspective. New Jersey: Prentice Hall, Pearson Education, Inc.

Vlassis, J. (2002). The balance model: Hindrance or support for the solving of linear equations with one unknown. *Educational Studies in Mathematics*, 49(3), 341-359

WAEC. (2001). *Chief Examiners' Report on the Basic Education Certificate Examination.* Accra.

WAEC. (2002). *Chief Examiners' Report on the Basic Education Certificate Examination.* Accra.

WAEC. (2006). *Chief Examiners' Report on the Basic Education Certificate Examination,* Accra.

Wagner, S., & Kieran, C. (1989). *Research Issues in the Learning and Teaching of Algebra*. Reston, Virginia: NTCM.

Wainer, H. (1992). Understanding graphs and tables. *Educational Research*, 21,14-23.

Wang, J., & Goldschmidt, P. (2003). Importance of middle school mathematics on high school students' mathematics achievement. *Journal of Educational Research*, *97*, 3-19.

Wang, J., & Goldschmidt, P. (2010). Importance of middle school mathematics on high school students' mathematics achievement. *Journal of Educational Research*, *97*, 3-19.

Wavering, M.J. (1989). Logical reasoning necessary to make line graphs. *Journal of Research in Science Teaching*, 26,373-379.

Welder, R. M. (2012). Improving Algebra Preparation: Implications from Research on Student Misconceptions and Difficulties. *School Science and Mathematics*, 112, 255-264

Wheeler, D. (1996). Backwards and Forward: Reflection on Different Approaches to Algebra. In N. Bednarz, C. Kieran, & L. Lee, *Approaches to Algebra*: *Perspective for Research and Teaching*. Dordrecht: Kluwer Academic Press.

Witzel, B. (2005). Using CRA to teach Algebra to students with math difficulties in inclusive settings: Learning Disabilities. *A Contemporary Journal*, 3(2), 49-60

Yerushalmy, (1991). Students' perceptions of aspects of algebraic function using multiple representation software. *Journal of Computer Assisted Learning*, 7, 42-57.

Yıldız, B. (2012). *A case study of the use of manipulatives in upper elementary mathematics classes in a private school: Teachers' and students' views*. (Unpublished doctoral dissertation). Middle East Technical University, Ankara.

Zhang, J. (1997) the nature of external representations in problem solving. *Cognitive Science*, 21(2), 179-211.

APPENDICES

APPENDIX A

Questionnaire for teachers

CODE……………………

Dear teacher,

I am a graduate (MPhil Mathematics Education) student of the University of Education, Winneba. I am researching into effects of multiple representations-based instruction on Junior High School students' achievement in linear equations in the Bimbilla Municipality. Hence, your responses are for academic purposes only and not meant to assess you. Therefore, I would like you to answer all questions as honestly and carefully as you can. All information provided would be treated with the strictest confidence. Your cooperation is highly needed to help me gather the appropriate data. Thank you. Please **tick [√]** one option.

SECTION A

TEACHER'S' BACKGROUND INFORMATION

1. Gender

   Male [ ] Female [ ]

2. Age in years

   20 – 25 [ ] 26 -30 [ ] 31-35 [ ] 36 – 40 [ ] 41 and above [ ]

3. Professional status

   Pupil-teacher [ ] Non-professional [ ] Professional [ ]

4. Qualification

SSCE/WASSCE [ ] Certificate 'A' [ ] Diploma [ ] HND [ ] B.SC/Bed [ ]

Masters [ ]

5. Number of years as mathematics teacher

1 – 5 [ ] 6-10 [ ] 11-15 [ ] 16-20 [ ] 21 and above [ ]

SECTION B

TEACHERS' MODE OF REPRESENTATION IN TEACHING LINEAR EQUATIONS

6. Have you taught linear equations in one variable before?

   Yes [ ] No [ ]

7. If **yes** please indicate how often you use the following in linear equations in one variable in the table below (circle one option in each). Rating are:

   1 = Never, 2 = Almost never 3 = Occasionally, 4 = Almost every time, 5 = Every time **Frequency of use**

Description of mode of representations	1=Never	2=Almost never	3=Occasionally	4=Almost every time	5=Every time
A. Algebraic (use of transposition and other related techniques)	1	2	3	4	5
B. Manipulatives (use of tiles and other accessories)	1	2	3	4	5
C. Graphic (teaching linear equations by plotting on graphs)	1	2	3	4	5
D. Multiple representations (use of a blend of modes)	1	2	3	4	5
E. Single representation (teaching linear equations using only one representation)	1	2	3	4	5

8. In your own view, state reason(s) for **using** your choice of representation(s).

Answer only representation (s) you use

A. Algebraic (use of transposition and other related techniques)

………………………………………………………………………………………

………………………………………………………………………………………

B. Manipulatives (use of tiles and other accessories)

………………………………………………………………………………………

………………………………………………………………………………………

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C. Graphic (teaching linear equations by plotting on graphs)

………………………………………………………………………………………

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D. Multiple representations (use of a blend of modes)

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E. Single representation (teaching linear equations using only one representation)

………………………………………………………………………………………

………………………………………………………………………………………

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9. In your own view, state reason(s) for not using the representations below:

Answer only representation (s) you do not use.

A. Algebraic (use of transposition and other related techniques)

…………………………………………………………………………………

…………………………………………………………………………………

………………………………………………………………………………...

B. Manipulatives (use of tiles and other accessories)

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C. Graphic (teaching linear equations by plotting on graphs)

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D. Multiple representations (use of a blend of modes)

…………………………………………………………………………………

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E. Single representation (teaching linear equations using only one representation)

…………………………………………………………………………………

…………………………………………………………………………………

THANK YOU FOR YOUR TIME

APPENDIX B

Students' Representation Preference Test

CODE……………………..

INSTRUCTIONS

1. Please answer all questions
2. Please indicate your answer by **ticking** the appropriate response.

STUDENTS' BACKGROUD INFORMATION

1. Gender

   Male [ ]   Female [ ]

   Age in years

2. 10 -12 [ ]   13 -15 [ ]   16-18 [ ]   19 and above [ ]

SECTION B

1. Please tick the column labelled '**preference**' to indicate which forms of representations you prefer to be taught linear equations in one variable.

Description of mode of representations	Preference
A. Algebraic (use of transposition and other related techniques)	
B. Manipulatives (use of tiles and other accessories)	
C. Graphic (teaching linear equations by plotting on graphs)	
D. Multiple representations (use of a blend of modes)	
E. Single representation (teaching linear equations using only one representation)	

APPENDIX C

Linear Equations Achievement Test (Pre-test)

CODE……………………

INSTRUCTIONS

PLEASE, ANSWER ALL THE QUESTIONS

1. Find the value of x that satisfy the equation $3x - 8 = -x - 12$.

2. Solve for x in the equation $x - 3 = 4x - 5$

3. If$5x - 3 = -3(x + 4)$, what is the value of x?

4. What value of x makes the statement $-4x + 6 = 10$ true?

5. Find the value of x that satisfied the statement $2(x - 1) = 3(x - 6)$.

6. Determine the value of x in $3(x - 2) = -4x + 1$

7. Solve for x in $4\ (x + 3) = 3(x - 2)$.

8. Find the value of x in $2(3x + 4) = -(x + 6)$.

9. Solve for x in $4x - 8 = 4$

10. Find the value of x in $\frac{1}{4}x - 4 = \frac{-1}{2}x - 1$.

THANK YOU FOR YOUR TIME

APPENDIX D

Linear Equations Achievement Test (Post-test)

CODE…………………..

INSTRUCTIONS

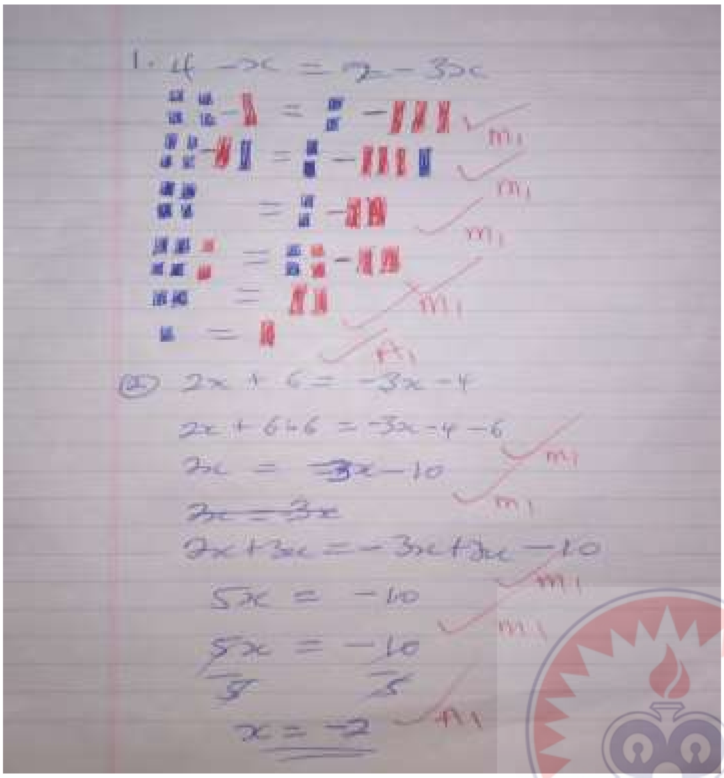
PLEASE, ANSWER ALL THE QUESTIONS

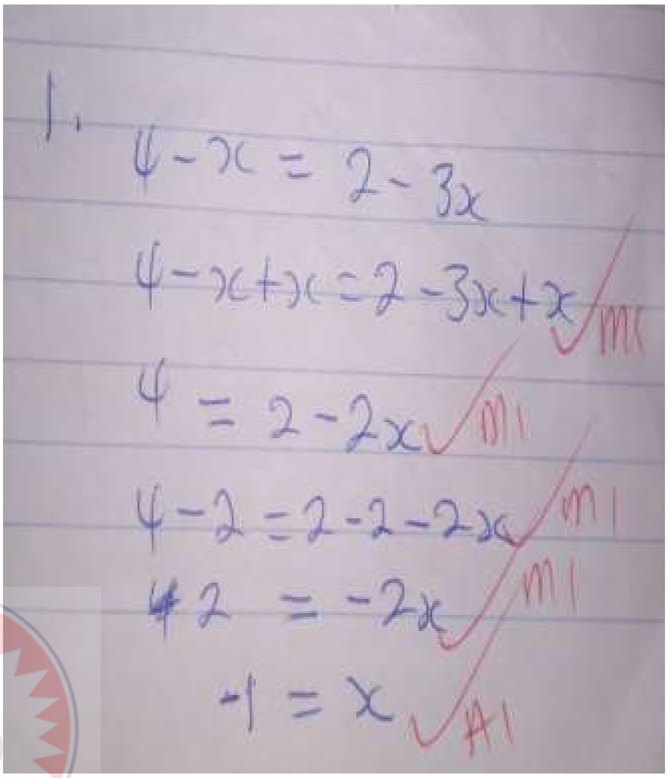
1. Solve for x in $4 - x = 2 - 3x$
2. If $2x + 6 = -3x - 4$, what is the value of x?
3. Solve for x in $5x - 20 = -6 + 3x$
4. Determine the value of x in $2(3x - 4) = -4(x + 6)$.
5. If $-2(x + 1) = 3(2x - 1)$, what is the value of x?
6. Given $2(-2x + 3) = 3(x - 2)$,determine the value of x
7. Solve for x in $3(2x - 1) = 4(x - 1)$
8. Solve for x in $2(3x + 5) - 3(x - 4) = 1$
9. Find the value of x in $3x - 5 = -(x - 3)$.
10. If $\frac{1}{3}x - 2 = \frac{-1}{2}x + 3$, find the value of x

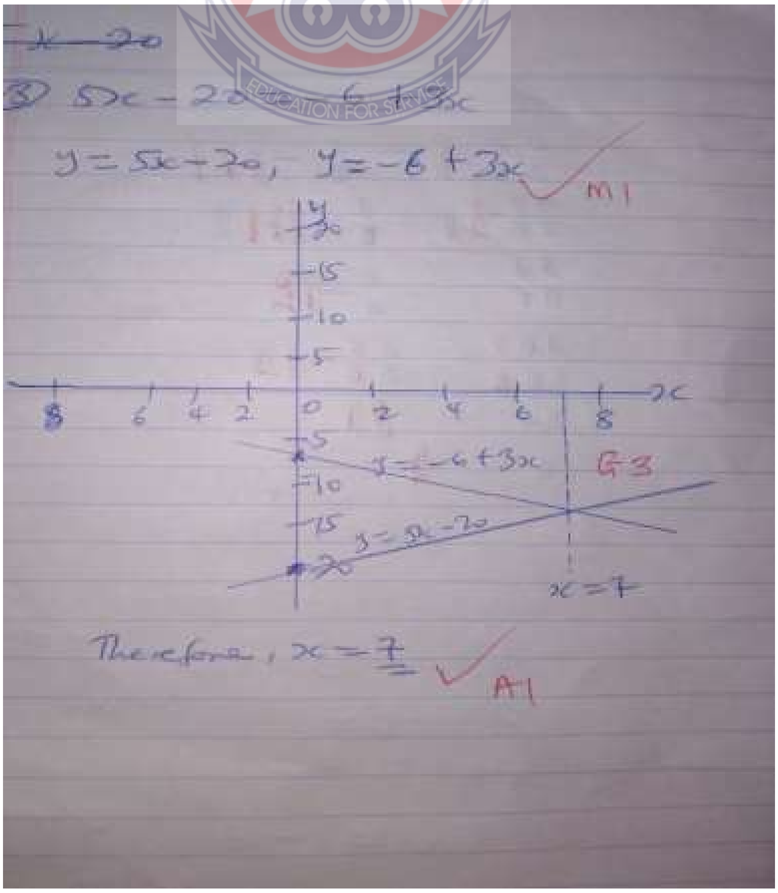
THANK YOU FOR YOUR TIME

APPENDIX E

Students solving linear equations in one variable using various representations.







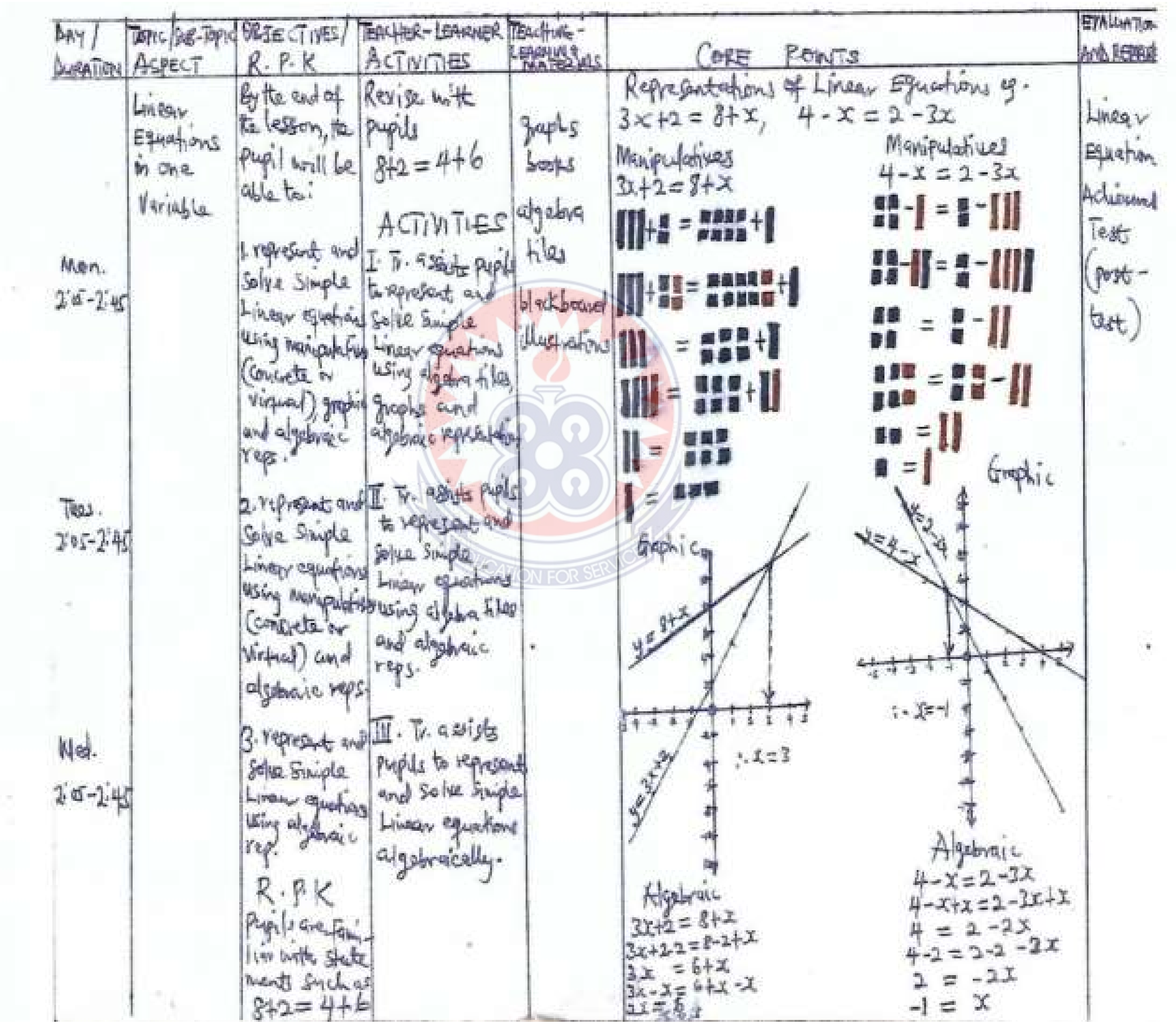
APPENDIX F

Teaching Lesson Plan

WEEK ENDINGS: 07/02/2020 and 28/02/2020

SUBJECT: MATHEMATICS

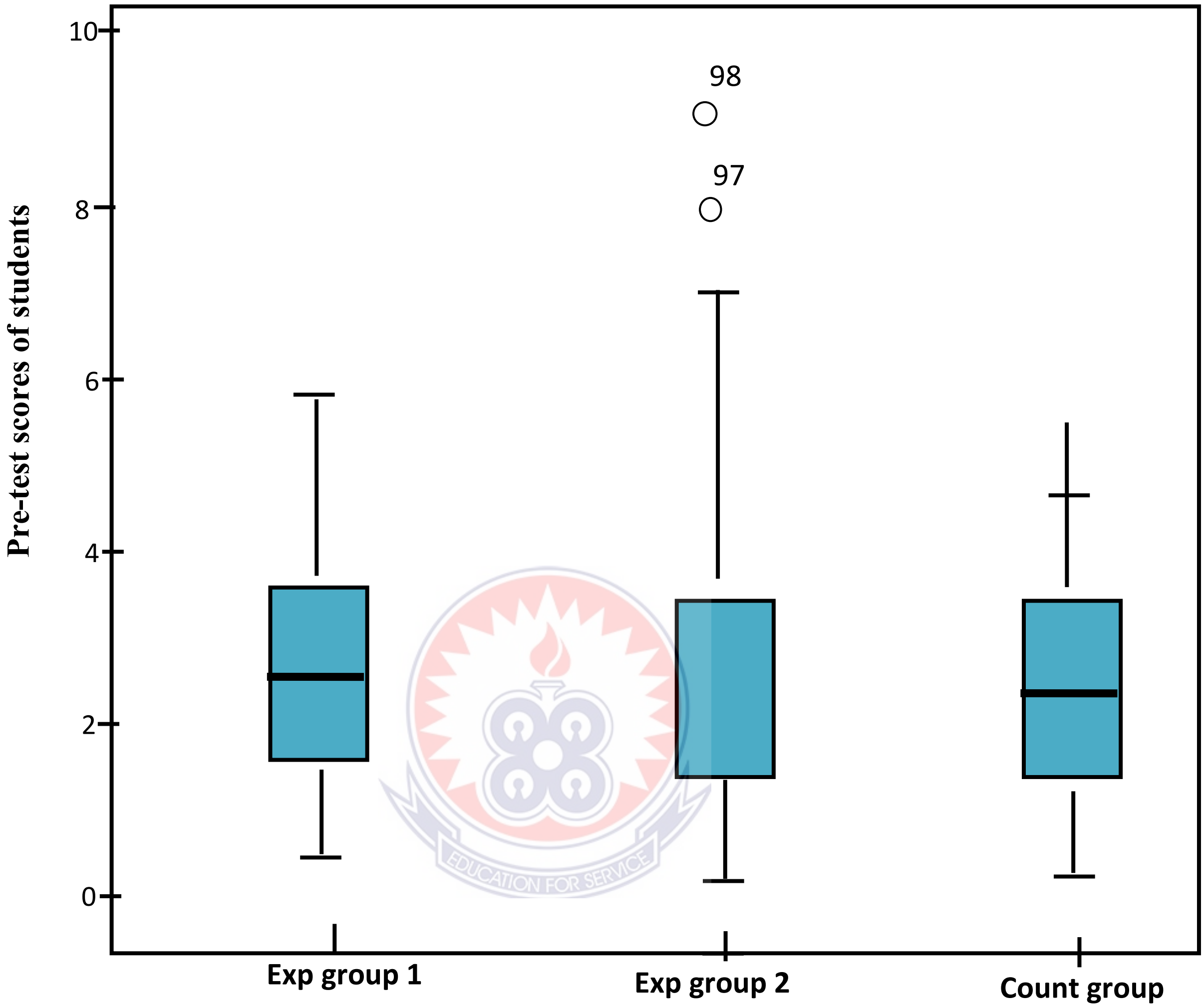
REFERENCE: Mathematics syllabus, Pupils text book2, Teachers guide, Aki-Ola Series



DAY / DURATION	TOPIC/SUB-TOPIC ASPECT	OBJECTIVES/ R.P.K	TEACHER-LEARNER ACTIVITIES	TEACHING-LEARNING MATERIALS	CORE POINTS	EVALUATION AND REMARKS
	Linear Equations in one Variable	By the end of the lesson, the pupil will be able to:	Revise with pupils $8+2=4+6$	graphs books	Representations of Linear Equations eg. $3x+2=8+x$, $4-x=2-3x$	Linear Equation Achievement Test (post-test)
Mon. 2:05-2:45		1. represent and solve simple Linear equations using manipulatives (Concrete or virtual), graphic and algebraic reps.	ACTIVITIES I. Tr. assists pupils to represent and solve simple Linear equations using algebra tiles, graphs and algebraic representations	algebra tiles blackboard illustrations	Manipulatives $3x+2=8+x$ Manipulatives $4-x=2-3x$	
Tues. 2:05-2:45		2. represent and solve simple Linear equations using manipulatives (concrete or virtual) and algebraic reps.	II. Tr. assists pupils to represent and solve simple Linear equations using algebra tiles and algebraic reps.		Graphic: $y=8+x$, $y=3x+2$, $\therefore x=3$ Graphic: $y=4-x$, $y=2-3x$, $\therefore x=-1$	
Wed. 2:05-2:45		3. represent and solve simple Linear equations using algebraic rep. R.P.K Pupils are familiar with statements such as $8+2=4+6$	III. Tr. assists pupils to represent and solve simple Linear equations algebraically.		Algebraic $3x+2=8+x$ $3x+2-2=8-2+x$ $3x=6+x$ $3x-x=6+x-x$ $2x=6$ ... Algebraic $4-x=2-3x$ $4-x+x=2-3x+x$ $4=2-2x$ $4-2=2-2-2x$ $2=-2x$ $-1=x$	

APPENDIX G

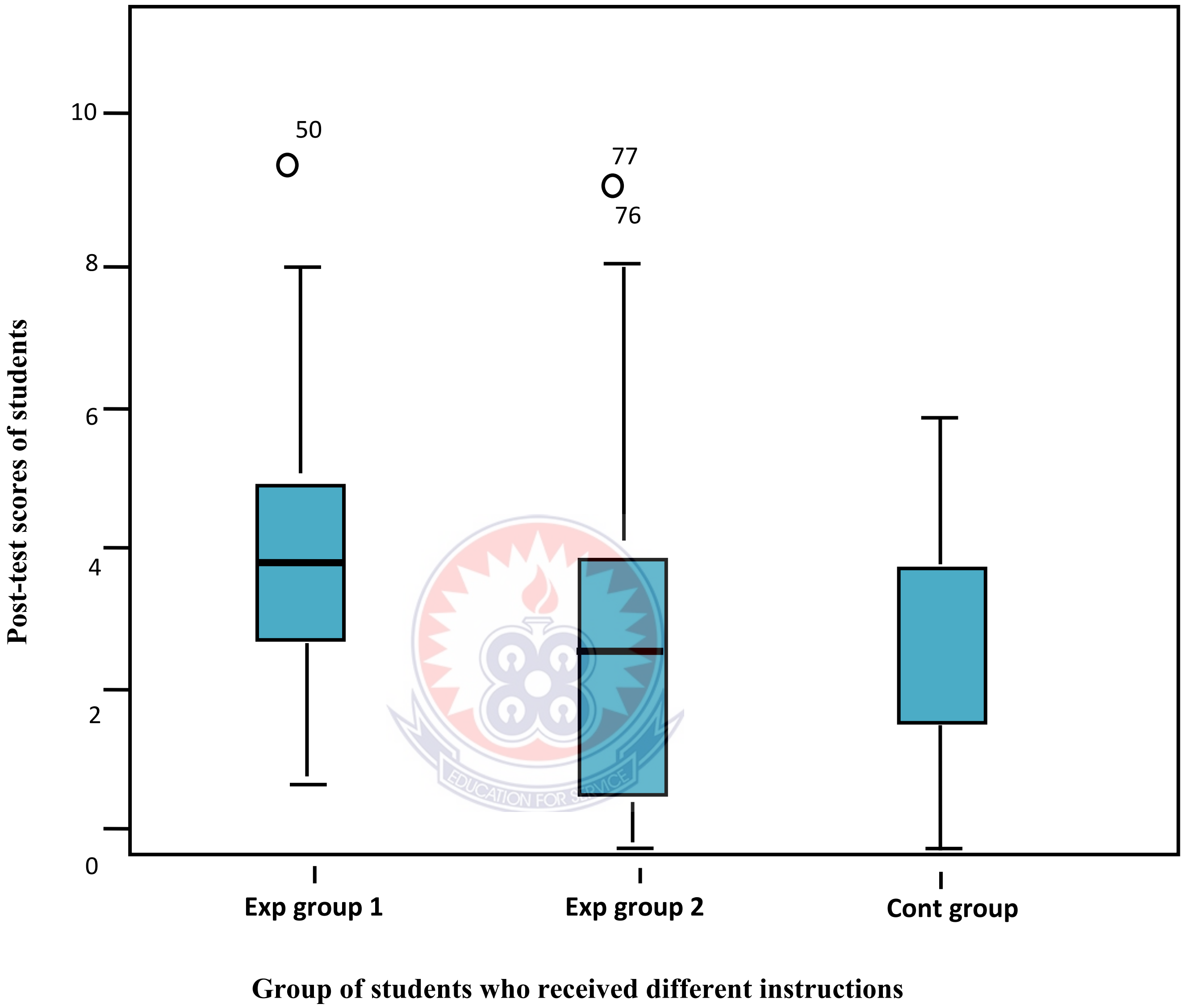
A Simple Boxplot for Students' Pre-test Scores



Group of students who received different instructions

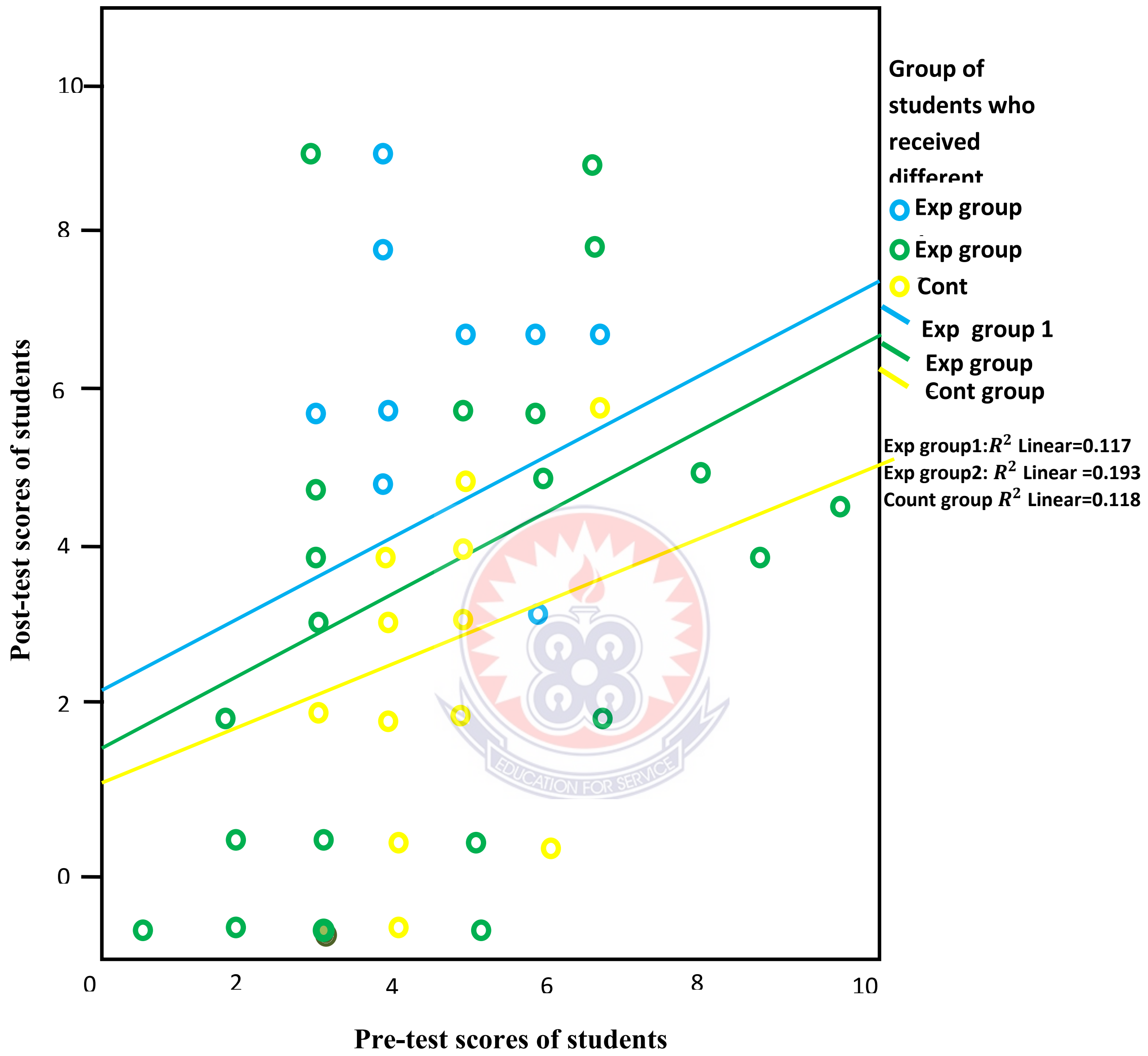
APPENDIX H

A Simple Box plot for Students' Post-test Scores



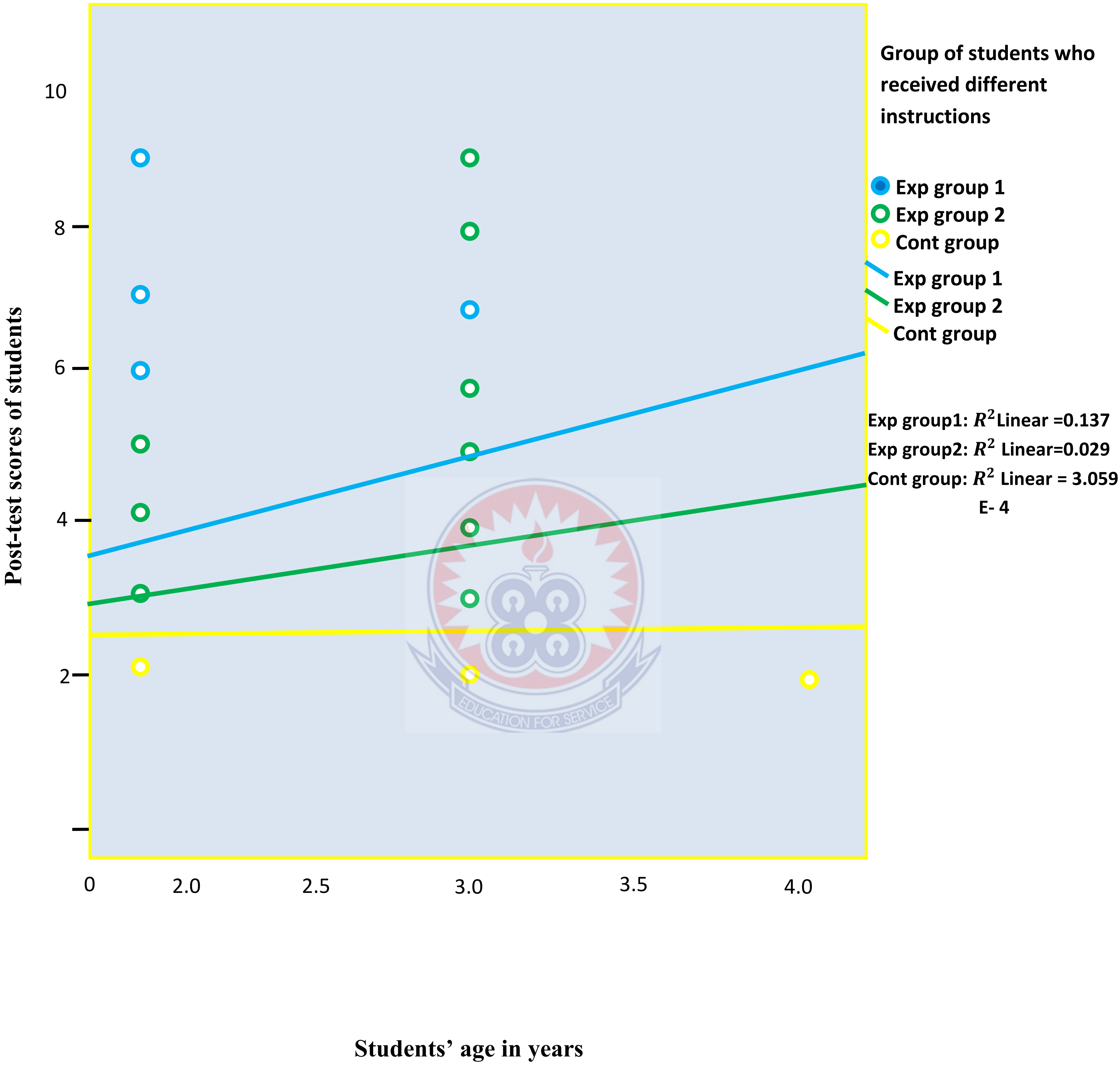
APPENDIX I

Test of Linearity between Students' Pre-test and Post-test Scores



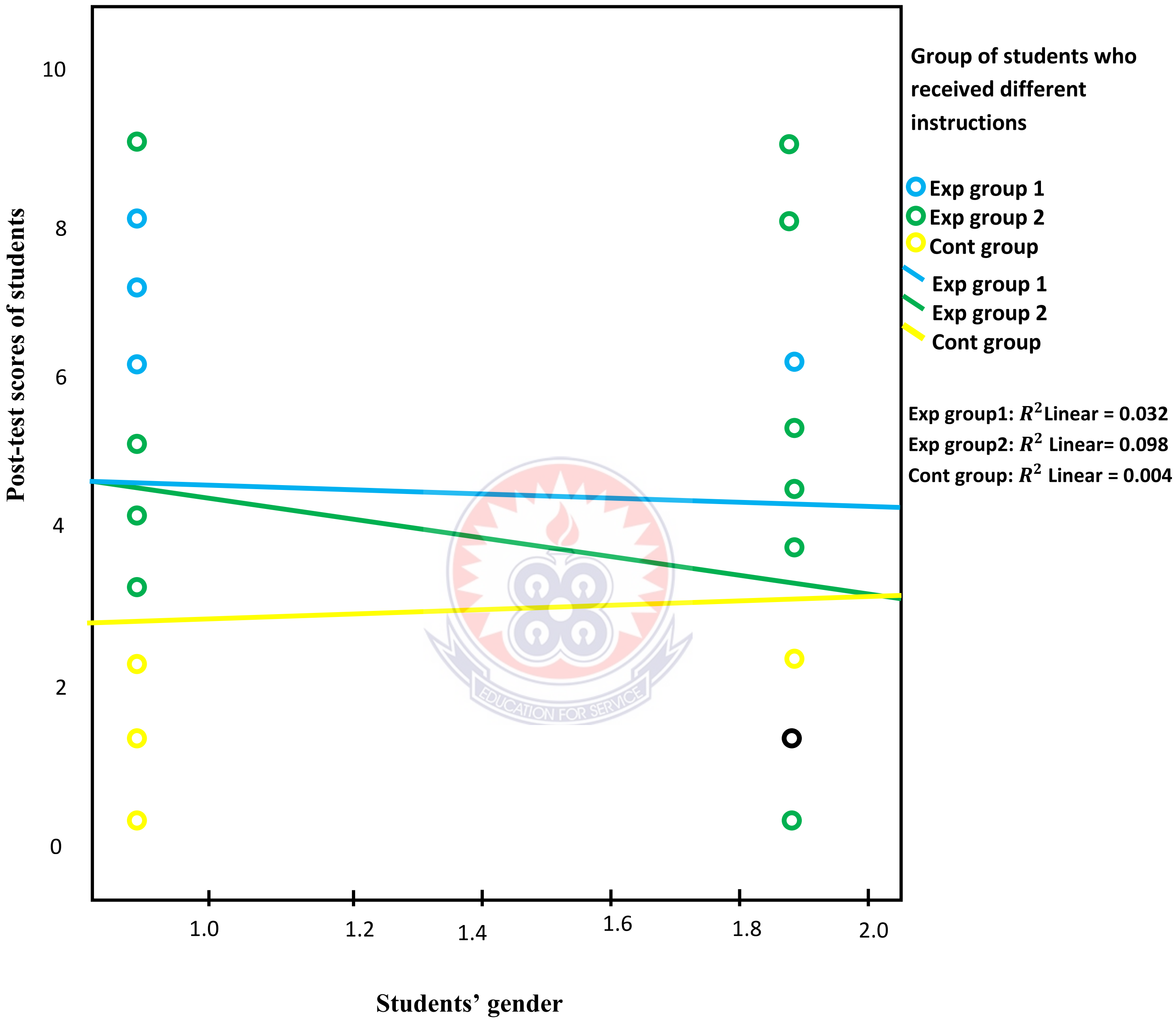
APPENDIX J

Test of Linearity between Students' Age and Post-test Scores



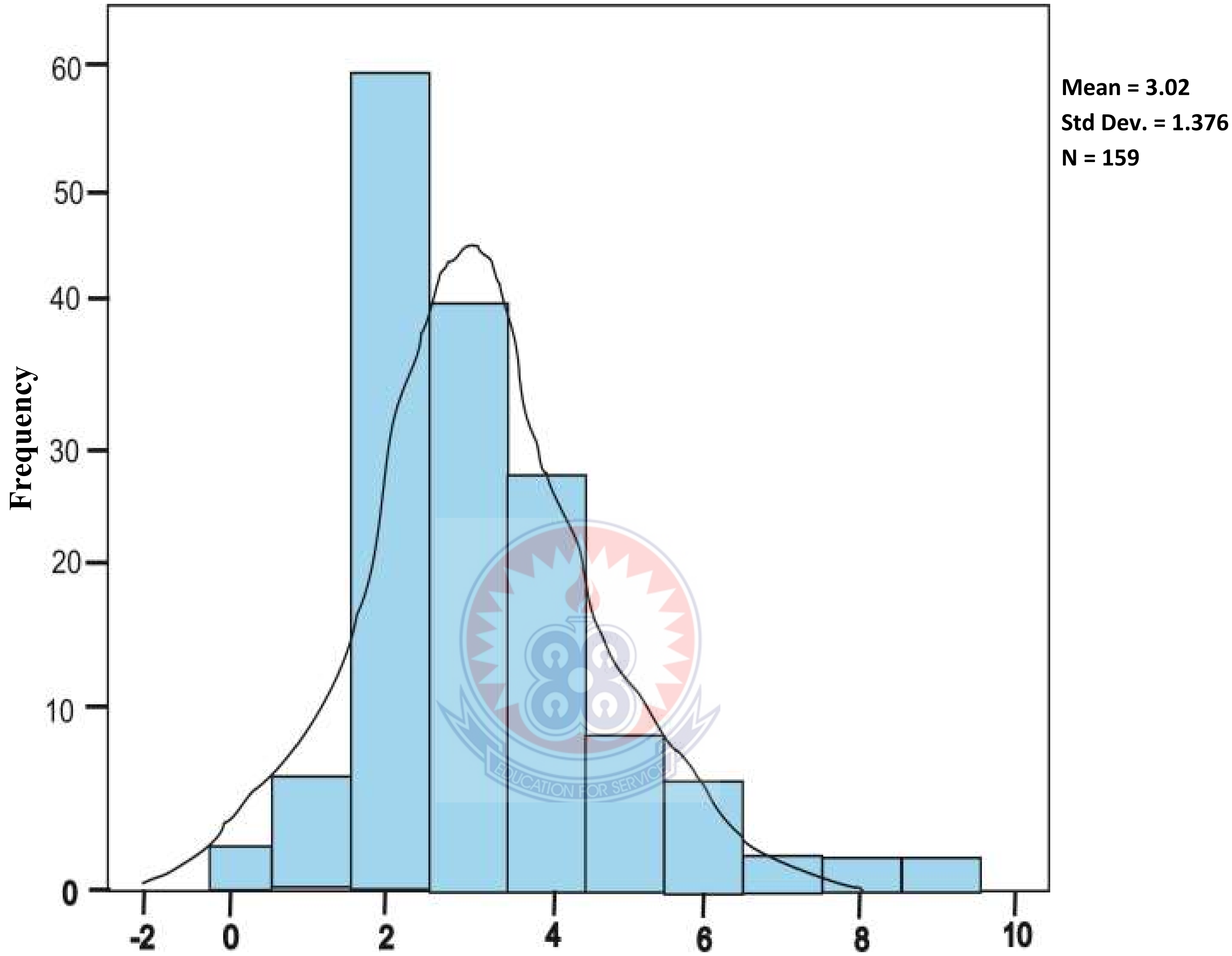
APPENDIX K

Test of Linearity between Students' Gender and Post-test Scores



APPENDIX L

Test of Normality for Students' Pre-test Scores



Pre-test scores of students

APPENDIX M

Test of Normality for Students' Post-test Scores.