UNIVERSITY OF EDUCATION, WINNEBA

AN EXPLORATION OF JHS 1 TEACHERS' KNOWLEDGE IN TEACHING FRACTION IN SOME SELECTED SCHOOLS IN ATWIMA NWABIAGYA NORTH DISTRICT

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DECLARATION

STUDENT'S DECLARATION

I, JOSEPHINE ADJEI, declare that this thesis, with the exception of quotations and references contained in published works which have all been identified and duly acknowledged, is entirely my own original work, and it has not been submitted, either in part or whole, for another degree elsewhere.

Signature:…………………………………. Date: …………………………………

SUPERVISOR'S DECLARATION

I hereby declare that the preparation and presentation of this work was supervised in accordance with the guidelines for supervision of thesis/dissertation/project as laid down by the University of Education, Winneba.

Signature:………………………………… Date: …………………………………

Prof. Ebenezer Bonyah

DEDICATION

I dedicate this project to my family, especially my husband(Frederick Asiedu), my mother(Alice Agyei Kuffour), and my children(Barffour Kyei Asiedu and Kofi Asare Asiedu), for their support, love and prayers throughout my academic career.

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ABSTRACT

The content area of fractions has been proven to be very complicated and troublesome for both learners and teachers. Teachers' knowledge in fraction has a very significant role in teaching it at the basic school level. This study aimed at exploring JHS 1 mathematics teachers' knowledge in teaching fraction and arithmetic. The study used a qualitative research approach that used a case study design. Twelve JHS 1 mathematics teachers were conveniently selected from a population of 80 mathematics teachers in the Atwima Nwabiagya North District in the Ashanti Region. Codes and themes were used to analysed the response obtained from the teachers. It was also revealed that half of the teachers could explain the addition procedure; however, they provided incorrect explanations for the division procedure. Teachers also were found to have used visuals and real-world examples in their explanations of addition procedures and explanations for fraction division. It was recommended that teachers have a deep understanding of the mathematics being taught in order to affect student understanding of fractions.

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CHAPTER ONE

INTRODUCTION

1.0 Introduction

This chapter is the introduction of the study and it comprises of the background to the study, statement of the problem, purpose of the study, research questions, significance of the study, delimitation, limitation, and organization of the study.

1.1 Background to the Study

The content area of fractions has proved itself to be very complicated and troublesome for learners to masters. Van de Walle, Karp, and Bay-Williams (2010. p.313) identified many possible factors contributing to poor understanding of fractions. They identified the reasons for learners' difficulties in fractions as follows: fractions include many meanings such as part-whole, measurement, division operator, the written format of fractions is unusual for learners, the conceptual understanding of fractions is ignored in instructions and whole-number knowledge is overgeneralized by learners. Pienaar (2014) aluded that one of the reasons teachers experience difficulties when teaching fractions may be how mathematics as a subject is viewed in the Ghanaian curriculum.

In support of the reasons above, the researcher believes that because the concept of fractions is one of the topics in the mathematics curriculum, which is challenging for learners, it is therefore important for teachers to teach learners the concept of fractions meaningfully and effectively. Teachers' mathematical knowledge for teaching fractions plays a significant role in this case, especially in primary schools or at the elementary level. Ball et al.(2008) framework of mathematical knowledge for teaching serves as a point of reference in this regard. Sowder and Wearner (2006) pointed out that learner consistently perform weakly, and as such, they have a weak understanding of fractions. Teachers' poor content knowledge of teaching fractions is one reason for learners' of some selected Basic Schools in Atwima Nwabiagya North District poor performance. Any incorrect teaching of fractions can affect learners' understanding of the topic and become a lifetime problem, influencing their schooling, tertiary education, and even at their performance at their workplace. Shulman (1986) stated that teachers must have a knowledge base specific to the subject matter. Pienaar (2014) concurs, saying that fractions play an important role in our ever-advancing technological society. Many careers today rely heavily on the ability to compute accurately, proficiently, and insightfully with fractions.

Many will agree with the idea that fractions are challenging concepts that most learners find it difficult to understand. Furthermore, Ma 1999; asserted that the understanding of fractions continues to be a challenging topic both for learning and for teaching. He also pointed out that teachers and researchers have typically defined the teaching of fractions as a thought-provoking area of the Mathematics curriculum. Moreover, it is true that fractions cannot be divorced from our daily life usage, and this is the reason enough for teachers to develop the fraction concept effectively to the learners. Steffe and Olive, (2010) alluded that this is especially tricky in light of the fact that learners have many everyday life experiences with fractions before they are introduced to formal teaching and learning about them.

Taylor and Vinjevold (1999), Carnoy, Chisholm and Chilisa (2012) alluded that over the past years, the ongoing low learner performance in mathematics has led to increasing interest in understanding how teacher pedagogical practices and content knowledge may contribute to patterns of poor academic performance. Research and evaluation of mathematics intervention point to the lack of foundational mathematical knowledge as one of the key factors for poor performance.

In addition, Fleisch (2008) maintained that poor performance crises start early in the primary school where learners acquire basic skills that they need as they further their studies. This is where primary school teachers should equip learners with the relevant mathematical knowledge, skills, and attitudes. Basic school mathematics teachers of Atwima Nwabiagya North District should have extensive mastery of the fractional mathematical knowledge for teaching. According to my observation, it is surprising to find learners who cannot tell what a fraction is. The researcher's concern is that if learners were taught or mastered fractional concepts from lower classes, it could not be difficult for them to recall what they learned in lower class. Fleisch (2008) argues that the foundation phase is the level where learners should acquire the basic and foundational mathematics skills. If they fail to acquire these fundamental mathematical skills, they will continue performing poorly as they progress to higher classes. Learners, who are inadequately prepared in lower classes, pose lots of challenges to the J.H.S 1 teacher; this causes the J.H.S 1teacher to deviate from the original pacesetter and struggle to close the gap caused by primary school teachers. This may result in the incomplete coverage of the curriculum at the end of the term or year.

All these concerns will create a significant problem for the entire economy which may lead to high failure rate and high unemployment rates of young people as they would have performed poorly with no attainment of a complete qualification. In the light of the reasons given above, one is likely to think and believe that Atwima Nwabiagya

North District will continue to perform poorly as long as the teachers' mathematical knowledge is lacking. Fleisch et al. (2008) indicated that mathematics learning problems appear at a very early stage in children, but mostly in elementary school, and then that problem continues up to high school. Any incorrect teaching of fractions can affect learners' understanding of the topic and become a lifetime problem. Ultimately this will influence their schooling, tertiary education and working situation.

1.2 Problem Statement

The teaching of fractions is vital since it connects other topics such as decimals, percentages, ratios and proportions. Kong (2008) alluded that the topic of fractions is important in the basic school mathematics curriculum. ln her teaching experience, the researcher observed that year by year when the Basic 6s are promoted to JHS1, they bring along with them shallow and insufficient mathematical knowledge on fractions, referring to the mathematical knowledge on fractions expected to have been mastered in upper primary.

It appeared that teachers at Atwima Nwabiagya North District struggle with content of teaching of fractions. It was further alluded that teachers' poor content knowledge of teaching fractions and the incorrect way of teaching fractions could be one reason for learners' of Atwima Nwabiagya North District poor performance. Pienaar (2014) support that the teaching of fractions is difficult, and Ma (1999) stated that teachers have insufficient knowledge of fractions necessary for classroom teaching.

1.3 Purpose of the Study

This study explored JHS1 teachers' knowledge in teaching fractions in three selected schools in the Atwima Nwabiagya North District.

1.3.1 Objectives of the Study

Based on the statement of the problem and the purpose of the study mentioned above, the following objectives were set out:

1. To explore the accuracy of teachers' explanations of algorithms.
2. To explore the relationship between teachers' explanations for fraction addition and multiplication.
3. To explore the characteristics of teachers' explanations with concepts and representation of fraction addition and multiplication.

1.4 Research Questions

Specifically, this research study aimed to answer the following research questions:

1. What is the difference in the mathematical accuracy of teachers' explanations for the addition and multiplication algorithms?
2. What is the relationship between teachers' explanations for addition and multiplication?
3. What are the characteristics of teachers' explanations with concepts and representations of addition and multiplication?

1.5 Significance of the Study

This study aimed at addressing the gap in literature by collecting and analyzing data on teachers in a district from a sample of JHS1 mathematics teachers on the concept of fractions and arithmetic. It is conceived that understanding teachers' in-depth understanding of the subject they teach allows for the addressing of teachers' needs in teacher education and professional development programmes in a better manner. It will also help for a better understanding of what learning opportunities teachers are providing to students to make sense of school subjects.

1.6 Delimitation of the Study

The Ghanaian JHS 1mathematics syllabus as guided by the national curriculum encapsulate several content domain. However, this study focused on one aspect, which is the concept fraction and arithmetic. This study also used only one research approach (qualitative) to explore teachers knowledge on the concept of fraction and arithmetic. The study was conducted in the Atwima Nwabiagya North District, specifically JHS 1 mathematics teachers from three schools in the district. For this reason, the findings, is not for generalization purpose except mathematics teachers elsewhere with similar characteristics.

1.7 Limitation of the Study

The sample for this study was conveniently selected this makes it not possible to generalize the findings to all JHS 1 mathematics teachers in the country. Another factor that could be a limitation to this study is that, the study includes schools that the researcher is not working, and considering the fact that the researcher is inexperience in conducting such a qualitative study, there could be issues of biasness in some of the procedures in the study.

1.8 Organization of the Study

This study is organized into five chapters, the chapter one consist of the background to the study, statement of the problem, the purpose of the study, research questions, significance of the study, delimitation, limitation, and this section. In chapter two, relevant literature on the study is reviewed under theoretical, empirical and conceptual. The chapter three present the study area, research design, population, sample and sampling techniques, instrument used in the data collection process, as well as its procedures, and data analysis procedure. The chapter four is where the result on the analyses of the data obtained fro the participant is presented and discussed. And the final chapter, which is chapter five, The summary of the findings, conclusions, and recommendations are also presented.

CHAPTER TWO

LITERATURE REVIEW

2.1 Introduction

The study aimed to explore mathematical knowledge for teaching JHS 1 teachers in the teaching of fractions at the various schools in Atwima Nwabiagya North District. The chapter review studies and concepts on mathematical knowledge for teaching JHS 1 teachers in the teaching of fractions.

2.2 The meanings of Mathematics

According to the royal society Thai dictionary (2013), Mathematics is the science of numbers that is in primary curriculum B.E. 1978. Mathematics of primary level is aimed at enabling all children and youths to create activity in relations and relate to daily life. Mathematics is a foundation of many subjects and is the key leads to core subjects, whether in the arts, even science. Mathematics is a study of reasoning, relations and logic that all steps are logical and cannot be separated from each other (Ministry of Education Thailand, 2010). Mathematics is a communication of information, synchronizes the clarity of information between the senders and receivers.

The clarity of the information is given by the number, this number is valuable in various fields. Mathematics is groups of arithmetic, geometry, algebra, calculus that calculate by using quantity, size, shape and symbol. Mathematics is the science of placements and numbers. According to the meanings, Mathematics is an important subject. It doesn’t mean only numbers and symbols. Mathematics has a board meaning. Mathematics is about the foundation of prosperities in various disciplines. In addition mathematics has mathematical language as well as accurate and appropriate communication. Mathematics is the structure of reasoning, planning, exact methods and principles. Mathematics is both of science and arts to develop teaching and learning. To enhance Mathematics is to create activity in relations and relate to daily life.

2.3 Primary 6 Graduates

Primary 6 graduates have numerical knowledge, understanding, and sense of cardinal numbers and zero, fractions, decimals of not more than three places, percentages, operation of numbers and properties of numbers; can solve problems involving addition, subtraction multiplication and division of cardinal numbers, fractions, decimals of not more than three places and percentages; are aware of validity of the answers reached; and can find estimates of cardinal numbers and decimals of not more than three places. Have knowledge and understanding of length, distance, weight, area, volume, capacity, time, money, direction, diagrams and size of angles; can measure correctly and appropriately; and can apply knowledge of measurement for solving problems faced in various situations. Have knowledge and understanding of characteristics and properties of triangles, squares, circles, cuboids, cylinders, cones, prisms, pyramids angles and parallel lines. Have knowledge and understanding of patterns and can explain their relationships and solve problems involving patterns; can analyze situations or problems as well as write linear equations with an unknown that can be solved. Can collect data and information and discuss various issues from pictograms, bar charts, comparative bar charts, pie charts, line graphs and tables that are availed of for presentation; and can apply knowledge of basic probability in projecting various possible situations. Can apply diverse methods for problem-solving, availing of mathematical and technological knowledge, skills, and processes appropriately to solve problems faced in various situations; can suitably provide reasoning for decision-making and appropriately present the conclusions reached; can use mathematical language and symbols for communication as well as accurate and appropriate communication and presentation of mathematical concepts; can link various bodies of mathematical knowledge and can link mathematical knowledge with other disciplines; and have attained ability for creative thinking

2.4 Meanings of Fractions

Fraction means a fraction represents a numerical value, which has to be divided into four parts, and then it is represented as 1/4. Fraction means a number that compares part of an object or a set with the whole, especially the quotient of two whole numbers, for example there are 2 boys of 6 boys, written in the form 2/6 = 1/3 of 6 boys. Fraction represents a numerical value, which defines the parts of a whole. Suppose a number has to be divided into equal parts, then it is represented the quotient as the numerators is a dividend and the denominator is a divider. Example, there are 6 children, group them into 3 group in equal. How many children are there in each group? So the fraction represents 6/3 = 2 children.

2.4.1 Addition of fraction

To make sure the denominators are the same, add the numerators, put that answer over the denominator.

2.4.2 Subtraction of fraction

To Make sure the denominators) are the same, subtract the numerators. Put the answer over the same denominator.

2.4.3 A fraction multiplied by fraction

To multiply the numerator by the denominator and the denominator by the numerator. If there are common factors of the numerator and the Denominator. To take the common factors divide both the numerator and the Denominator.

2.4.4 A fraction divided by a fraction

To reverse the numerator and denominator) of the second fraction, multiply the two numerators. Then, multiply the two denominators.

2.5 Theoretical Framework

Teachers' knowledge can be perceived from different perspectives. Grounded in Shulman's (1986) work, some new conceptualizations on mathematics teachers' knowledge have emerged (Rowland, Huckstep & Thwaites, 2005; Davis & Simmt, 2006). In our focus on teachers' knowledge, we focus on the mathematical knowledge for teaching (MKT) conceptualization with its various sub-domains (Ball et al., 2008). One reason for favoring this conceptualization of knowledge is that we perceive the sub-domains of MKT (Ball et al., 2008) as a relevant starting point for designing tasks for the mathematical preparation of teachers, and for doing research on what inputs to teachers training shows effects on students and practices. Interestingly, the Michigan group has found a connection between teachers' MKT, as measured by their MKT items, and students' achievement in mathematics (e.g., Hill et al., 2005).

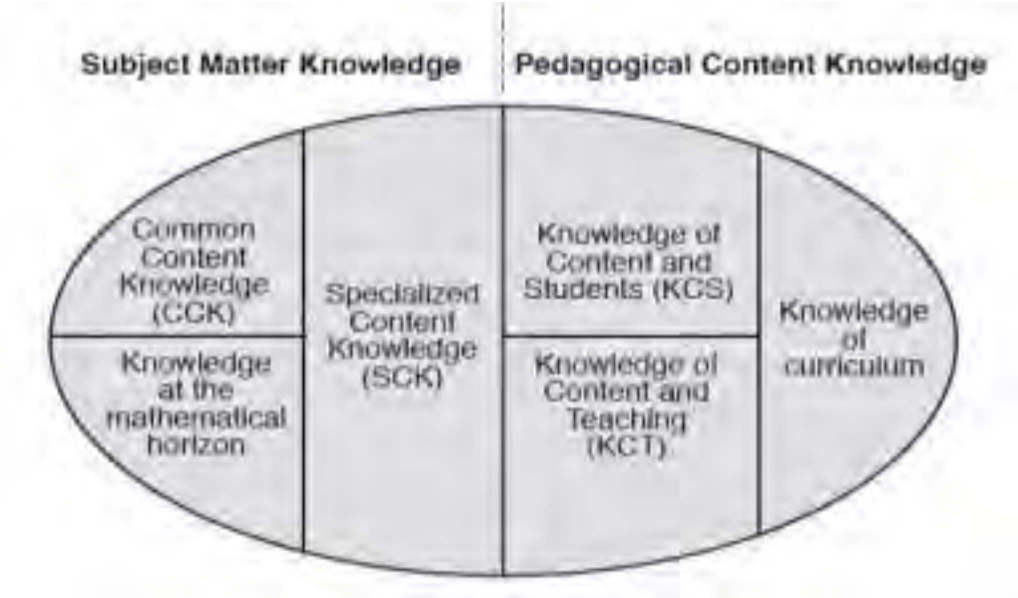

Figure 1: Domains of MKT (Ball et al., 2008, p. 403)

The MKT conceptualization of teacher knowledge comprises Shulman's domains (subject matter knowledge (SMK) and pedagogical content knowledge (PCK)) and considers each one of them as being composed of three sub-domains. We will here approach only the sub-domains concerning SMK. SMK comprises what is termed "common content knowledge" (CCK), "specialized content knowledge" (SCK), and "horizon content knowledge' (HCK). CCK is knowledge that is used in the work of teaching, but also commonly used in other professions that use mathematics. It can be seen as an individual's knowing the topic for themselves – e.g. knowing how to obtain the correct answer when multiplying fractions. Teachers (obviously) need to know how to do this, but it is also common knowledge within a variety of other professions.

However, in order to give students' opportunities to achieve a deeper understanding of the topics (here fractions), besides knowing how to perform the calculations (find the correct result or identify incorrect answers), teachers' need to know the mathematical hows and whys behind such calculations. Such knowledge on the hows and whys related with fractions is a core knowledge in order to allow teachers' to (amongst others) being able to explain it to students', listen to their explanations, understand their work, and choose useful representations of fractions that can support students' learning.

This is knowledge that requires additional mathematical insight and understanding (Ball, Hill & Bass, 2005), and is considered SCK. The last sub-domain is termed HCK, which is described as "an awareness of how mathematical topics are related over the span of mathematics included in the curriculum" (Ball et al. 2008, p. 403), and is important for developing students' connectedness in mathematical understanding along the schooling. Teachers' knowledge and what concerns the specificity of the topic being approached (mathematics) is inter-related, it influence and is influenced by a large span of dimensions and aspects. Examples of these dimensions and aspects are teachers' role, actions and goals (Ribeiro, Carrillo & Monteiro, 2009). Teacher's participation in professional development programs can contribute to an important part on their awareness of practice (Muñoz-Catalan, Carrillo & Climent, 2006). It also contributes to the development of their MKT and on their awareness of the role of teachers' professional knowledge dimensions in practice (Ribeiro et al., 2009). We assume that teachers' professional development starts, explicitly and in a formal way, in pre-service teachers' education, and thus, this is (should be) the starting point for discussing, promoting and elaborating teachers' knowledge allowing them to teach with and for understanding.

Within the new Portuguese National Curriculum (Ponte et al., 2007), the understanding, representation and interpretation of fractions is transversal to all the first nine years of schooling. In this new curriculum, it is mentioned that the approach to rational numbers should start on the first two years of schooling, in an intuitive manner. Thereafter, one should progressively introduce the representation of fractions, using simple examples. In years three and four, the different interpretations of fractions should be deepen, starting from situations involving equitable sharing or measuring, refining the unit of measure – using discrete and continuous quantities. Discussing the importance of the role of the whole is a core element in allowing for understanding of all the different interpretations and representations of fractions (Kieren, 1976), and is perceived as a "prerequisite" for such understanding (Ribeiro, in preparation).

Fractions are among the most complex mathematical concepts that children encounter in their years in primary education (Newstead & Murray, 1998). These difficulties can be originated from the fact that fractions comprise a multifaceted construct (Kieren, 1995) or they can be conceived as being grounded in the instructional approaches employed to teach fractions (Behr et al., 1993). These identified difficulties illustrate the importance of improving teachers' initial training. A consequence of such an improvement will be increase students' CCK concerning fractions, contributing to a new and better direction at all educational levels.

2.5.1 Transformative Learning Theory

Transformative learning theory (TLT) is a model of andragogy that attempts to reveal and clarify a learner's prior assumptions and then transform these assumptions into new understandings (Mezirow, 2012). TLT was developed by Jack Mezirow in the mid to late 1980s and early 1990s. He based his initial theory on a study of 83 women returning to college in 12 different reentry programs in 1975. Mezirow's initial transformation process included ten phases:

Experiencing a disorienting dilemma;

Undergoing self-examination;

Conducting critical assessment of internal assumptions and feeling alienation from traditional social expectations;

Relating discontent to the similar experiences of others. In other words recognizing the that problem is shared;

Exploring options for new ways of acting;

Building competence and self-confidence in new roles;

Planning a course of action;

Acquiring the knowledge and skills for implementing the new action;

Trying new roles and assessing them;

Reintegrating into society with the new perspective (Cranton, 2006).

Mezirow wrote about and amended his theory of adult learning and development in articles beginning in the mid-1980s and continuing for over 30 years. While the initial theory included the ten phases above, later versions used by Mezirow (1991) and others (Dirkx & Smith, 2009) included a subset of these phases. Key ideas in TLT include the notions that “we transform our frame of reference through critical reflection on assumptions” (Mezirow, 1991), and that “rational discourse through communicative learning” is a key concept in understanding (Mezirow, 1991). Mezirow says that this reflection and discourse often take place within the context of problem solving (Mezirow, 2012). His theory claims that only after learners are aware of their assumptions can they develop strategies to transform these assumptions (Mezirow, 2012). This suggests that in order for PTs to make shifts in their understanding and become proficient at the mathematics content, they need to have their reasoning challenged in ways that encourage reflection on their prior assumptions about mathematics. Kasworm and Bowles (2012) reviewed 250 published reports on TLT in higher education settings including faculty development, mentoring settings, and experiential learning. They note that success in higher education is a natural site for transformative learning theory to occur because “ideally, higher education offers an invitation to think, to be, and to act in new and enhanced ways. These learning environments sometimes challenge individuals to move beyond their comfort zone of the known, of self and others” (Kasworm & Bowles, 2012).

2.5.2 Cognitive Structure Learning

Theory Bruner believes that the essence of learning is that one connects the similar things and organizes them into meaningful structures, and learning is the organization and reorganization of cognitive structures (Kohlberg, 1968). Knowledge learning is to form the knowledge structure of all subjects in the minds of students. Bruner holds that cognitive structure is a general way for people to perceive and generalize the external physical world, and it is a psychological structure formed in the process of human activities to recognize the external things (Wen, 2018). Cognitive structure is progressive and multi-level, developing from low level to advanced level. And it is formed on the basis of past experience and is constantly changing in the process of learning. In addition, the formation of cognitive structure is an important internal factor and foundation for further learning and understanding of new knowledge. Wen (2018) further presented that Bruner calls cognitive structure "representation" and holds that representation can be divided into three types: action representation, image representation and symbolic representation.

The so-called behavioral characteristics mainly refer to relying on action to perceive the world, for example, a two-year-old infant often relies on action to perceive the world (Wen, 2018). As children grow older, they begin to use visual and auditory representations or images in their minds to represent external things and try to solve problems through images (Flavell, 1988). We call this representation as image representation. From the age of about six or seven, individuals can use symbols such as language and numbers to represent experience, while using these symbols to learn and gain experience (Wen, 2018). We call this representation as symbolic representation. The three representations do not exist in isolation. As the individual develops to a certain stage, the three representations coexist in individual cognitive structure, complement each other and work together on cognitive activities. Bruner thought that knowledge learning is to form certain knowledge structure in the minds of students. This knowledge structure is made up of the basic concepts, basic ideas or principles of subject knowledge. The structural form of knowledge structure is made up of human coding system.

2.5.3 Discovery of Learning Theory

In terms of teaching method, Bruner put forward the "discovery learning method". "Discovery is not limited to the search for unknown things, but rather it includes all the means of obtaining knowledge through one's own mind," Bruner said. Bruner's "discovery" is not a scientist's invention, but "an activity in which students organize what they know in their own way rather than books" (Bruner, 2006). What he calls discovery learning is a process in which students acquire new knowledge for them by reading books and literature independently and thinking independently. Bruner attaches great importance to discovery and believes that students are not passive or passive recipients, but active explorers (Wen, 2018). Cognitive process is a process in which people mainly choose, transform, store and apply the things that come into their senses, in which people actively study, adapt to and transform the environment. He suggested that teachers should provide more materials for students to analyze and synthesize the deserved conclusion rules and become - discoverers. In this way, we can better explore the potential of wisdom, arouse the enthusiasm of students' thinking, stimulate students' excitement, self-confidence and interest in learning, and help to maintain memory.

2.6 Designing the Knowledge for Teaching Fractions Test

Measuring teacher knowledge is not simple due to its complex and multi-faceted nature (Gülmez-Dağ & Yıldırım, 2016). It is internal and it cannot be entirely measured by observing teachers' mathematical instruction or behaviors (Kazemi & Rafiepour, 2018). Therefore, researchers used a variety of methods such as written tests (Lin, Becker, Byun, Yang, & Huang, 2013), interviews (Behr, Khoury, Harel, Post, & Lesh, 1997), classroom teaching experiments (Tobias, 2013), and a combination of two or more of the aforementioned methods (Lo & Luo, 2012) to measure pre-service teachers' knowledge for teaching fractions. The current study attempts to contribute to the body of literature on pre-service teachers' knowledge for teaching fractions by proposing the Knowledge for Teaching Fractions Test that was designed based on the limitations of large- and small-scale studies conducted in the past. Large-scale studies such as the Mathematical Knowledge for Teaching in the United States (Ball, Thames, & Phelps, 2008), the Cognitive Activation in the Classroom Study in Germany (Baumert et al., 2010; Kunter et al., 2013), and Teacher Education and Development Study in Mathematics in seventeen countries (Senk et al., 2012) used single surveys with items covering many different topics to measure participants' mathematical knowledge.

However, such surveys may not help to uncover participants' knowledge of mathematics in a deep and comprehensive way and consequently may not provide a holistic and rich picture about their knowledge (Kazemi & Rafiepour, 2018). Moreover, there are relatively few small-scale studies that focus on pre-service teachers' knowledge for teaching fractions in the Turkish context (Işıksal & Çakıroğlu, 2011). On the other hand, there are many similar studies exploring pre-service teachers' knowledge for teaching fractions in an international context (Lo & Luo, 2012; Tobias, 2013). However, these studies display a rather uneven distribution among components of knowledge for teaching fractions. More clearly, pre-service teachers' common content knowledge and specialized content knowledge for teaching fractions received more attention from researchers, but teachers' knowledge of content and students, knowledge of content and teaching, and knowledge of mathematics curriculum for teaching fractions has not been the focus of much of the research (Olanoff, Lo, & Tobias, 2014). Thus, the author of the current study paid particular attention to including several tasks for each component of teacher knowledge when designing the fractions test.

2.7 Measuring Pre-service Teachers' Knowledge for Teaching Fractions

Fractions are widely used in mathematics education and have great importance in other disciplines as well (Ben-Chaim, Keret, & Ilany, 2012). For instance, fractions form the basis of introductory mathematics and other mathematical topics such as algebra and probability (Clarke & Roche, 2007). However, fractions are notorious for the difficulty encountered not only by students (Vamvakoussi & Vosniadou, 2010) but also by teachers (Izsak, 2008). As Lamon (2007) expressed, fractions "arguably hold the distinction of being the most protracted in terms of development, the most difficult to teach, the most mathematically complex, the most cognitively challenging, the most essential to success in higher mathematics and science, and one of the most compelling research sites" (p. 629).

The difficulty for teachers in teaching fractions to their students may stem in part from the conceptual richness of fractions (Li & Kulm, 2008) because teaching fractions conceptually requires teachers to make connections with other sources of mathematical knowledge and employ different representations and real-life contexts (Li, 2008). Moreover, teaching fractions in an inappropriate way or procedurally may lead to student misconceptions and consequently may inhibit their understanding of future concepts related to fractions (Kazemi & Rafiepour, 2018). Thus, mathematics teachers need to have a robust understanding of fractions in order to teach them to their students conceptually. Accordingly, the topic of fractions may provide a rich context for exploring the extent of pre-service teachers' mathematical knowledge for teaching and this may, in turn, prove useful in shedding light on their strengths and weaknesses related to the teaching of fractions in a deep and comprehensive way.

2.8 The conceptual understanding of Teachers

Teachers' knowledge of the subject matter, along with their content-specific pedagogical skills, has been the focus of numerous studies (e.g., Baumert et al., 2010; Blomeke, et al, 2016; Campbell et al., 2014; Copur-Gencturk, 2015; Copur-Gencturk, Tolar, Jacobson, & Fan, 2019; Hill, Rowan, & Ball, 2005; Kersting, et al., 2012; Ma, 2010; Sadler, Sonnert, Coyle, Cook-Smith, & Miller, 2013). In fact, in the last three decades, much more progress has been made toward identifying the components of the knowledge teachers need to teach school subjects effectively (Ball, et al., 2008; Grossman, 1990). One of the primary knowledge domains for effective teaching as agreed by almost everyone studying teacher knowledge is subject matter knowledge (Ball, 1991; Ball et al., 2008; Grossman, 1990; Leinhardt & Smith, 1985; Shulman, 1986). Logically, teachers cannot teach things they themselves do not know (Ball, 1991). Thus, the most easily recognized component of subject matter knowledge is the knowledge of facts, rules, and concepts (Ball, 1991; Shulman, 1986).

However, subject matter knowledge involves more than knowing the key facts, rules, or procedures (e.g., Ball, 1991; Grossman, 1990; Shulman, 1986), and it also involves having an explicit conceptual understanding of underlying procedures and knowing why such rules and facts are warranted (e.g., Ball, 1991; Ball et al., 2008). Although teachers' conceptual understanding alone may not guarantee effective teaching, teachers' lack of understanding makes it impossible to create a learning environment in which students can build a meaningful understanding of the concepts they are learning (Ball, 1991; NMAP, 2008). For instance, a teacher cannot explain to students the principles underlying fraction addition if they do not conceptually understand why a common denominator is needed when adding fractions with unlike denominators. In turn, this can lead students to develop incorrect strategies, such as adding across numerators and denominators, because they rely on their prior knowledge of working with whole numbers. Furthermore, the level of teachers' conceptual understanding affects the pedagogical resources teachers employ in their practice (e.g., Borko et al., 1992; Eisenhart et al., 1993).

Thus, the way teachers understand the rules and algorithms shapes students' opportunities to learn these concepts. Several documents published by mathematics education organizations explicitly acknowledge the importance of teachers' deep understanding of mathematics by setting that teachers' robust understanding of school mathematics as a standard for teaching mathematics (Bezuk et al., 2017; National Council of Teachers of Mathematics, 1991). Today's student is not only expected to know the rules and how to execute procedures but also to conceptually understand what the mathematical procedures mean and the connections among mathematical concepts (Common Core State Standards Initiative, 2010). Therefore, a teacher's own understanding of the mathematical concepts underlying the rules and procedures plays a vital role in supporting students' development of a robust understanding of the concepts that meets the expectation of the standards. The importance of teachers' knowledge or lack of knowledge in instruction has also been supported by prior empirical work.

In fact, several studies have documented that teachers' deep understanding of concepts is linked to effective instruction and student learning (Charalambous, 2010; Copur-Gencturk, 2015; Hill et al., 2008; Kersting et al., 2012; Tchoshanov, 2011). Specifically, evidence suggests that teachers' lack of understanding is associated with mathematical errors in instruction (e.g., Borko et al., 1992; Hill et al., 2008) and that strong mathematical knowledge is associated with a higher quality of mathematics instruction, such as making key mathematical ideas more explicit in teaching (Copur-Gencturk, 2015). Although research showing a direct relationship between teachers' understanding of the subject matter and students' learning is mixed (cf. Hill et al., 2005; Kersting et al., 2012), several studies have shown that teachers' subject matter knowledge has an indirect impact on students' learning through instruction (Baumert et al., 2010; Kersting et al., 2012).

2.9 Prior literature on teachers' knowledge of fraction arithmetic

An accumulating body of research has documented that although a significant portion of teachers can perform fraction operations, a smaller portion of the same teachers understand where to use which fraction operations, especially division (Lo & Luo, 2012; Ma, 2010; Newton, 2008). For instance, Son and Crespo (2009) found that all 17 US prospective elementary teachers and 17 US prospective secondary teachers were able to solve a fraction division arithmetic problem (2/9 ÷ 1/3). However, none of the preservice elementary teachers and only 35% of the secondary teachers in their study were able to create a story problem that would illustrate 2/9 ÷ 1/3. Newton's (2008) examination of the fraction knowledge of 85 preservice teachers enrolled in a mathematics course in an elementary education program provided further insight into preservice teachers' struggles with the underpinnings of fraction arithmetic. Participants were asked to compute 3–5 fraction arithmetic problems in each operation (e.g., 21/3 ÷ 9 and 2/4 −3/6).

Analysis of the errors prospective teachers made in computing fraction arithmetic suggested they had a weak understanding of the conceptual underpinnings of fractions and fraction arithmetic, such as the role of the denominator. Specifically, participating teachers used different processes when the denominators of fractions were the same in multiplication and division questions. Although much of the work examining teachers' conceptual understanding of fraction arithmetic has been conducted with preservice teachers, studies conducted with in-service teachers have also painted a picture of U.S.

teachers as having an underdeveloped understanding of fraction arithmetic (Izsák, Jacobson, & Bradshaw, 2019; Ma, 2010). As an example, in an international comparison study of Chinese and US teachers, Ma (2010) interviewed 23 US in-service elementary school teachers who were asked to solve four mathematical tasks, one of which focused on fraction division. She documented that 48% of US teachers in her study were correctly able to divide 1¾ by ½, and none of the teachers were able to come up with a real-word situation or a story problem that would meaningfully represent the division of the two fractions.

When looking at previous studies on fraction arithmetic, one issue stands out: much of the prior work has focused on proxies of teachers' conceptual understanding of fraction arithmetic, such as writing a word problem or drawing or selecting a model to illustrate a situation (Son & Crespo, 2009). Although these studies have delineated problems with teachers' performance, they have failed to detect why teachers have provided these incorrect responses. Explicitly capturing teachers' understanding of an algorithm could reveal potential reasons for their struggles. A case where this was captured is in a study by Borko et al. (1992). Their work with a teacher candidate, Ms. Daniels, found that right after the class computed the answer to a fraction division problem (¾ ÷ ½), a student in her student teaching placement asked her to explain why the invert-and-multiply rule for fraction division worked. Ms. Daniels created a fraction multiplication situation and then realized her model was incorrect. It was clear that the teacher lacked an understanding of the rule that had implications for her practice and student learning. Such in-the-moment struggles (i.e., not being able to correctly model or create a real life example to illustrate the mathematical situation) have been reported in other studies where similar methods have been used (i.e., asking participants to model or create a real-world problem).

Yet, because Borko et al. (1992) collected data from Ms. Daniels regarding her understanding of the invert-and-multiply algorithm, they were better able to gain insights to her struggle. Respectively, when Ms. Daniels was asked to explain why the invert-and-multiply algorithm worked during an interview, before taking a methods course on teaching mathematics, her response focused on multiplication being an inverse operation of division:

> *… you turn [it] into a multiplication problem and since multiplication is the inverse operation of division, then you have to take that second number you see or your divisor and turn it over because you're doing the inverse to it as you would with the division sign. (p. 208, Borko et al., 1992).*

Even though Ms. Daniels had completed several courses in advanced mathematics and was enrolled in an elementary mathematics methods course, she did not seem to understand why the algorithm worked. Yet when she was reviewing the algorithm with students in her placement, she explained the algorithm the same way she had explained it during the interview. Thus, I contend that asking teachers why an algorithm works may reveal how they explain it to their students. Nevertheless, revealing teachers' understanding of a concept by asking why a procedure works is not an easy task because of the linguistic difficulty surrounding the meaning of why sayings like "invert-and-multiply" work for fraction division algorithms (Borko et al., 1992). One way of addressing such a difficulty is to capture teachers' understanding of the key concepts underlying the algorithms for fraction arithmetic. For example, given that the algorithm for fraction addition and subtraction is based on the idea of partitioning a whole into equal-sized pieces, capturing teachers' understanding of the role of the denominator throughout the process of adding or subtracting fractions could provide insight into their more nuanced understanding of the algorithm.

Similarly, attending to the units or wholes the fractions refer to in fraction multiplication and division situations is vital for understanding fraction multiplication and division.

For instance, conceptualizing fraction division by using the divisor as a referent unit (whole) and making sense of division as the number of groups that can be made by the dividend could help teachers develop a foundation for fraction operations. Thus, a fraction arithmetic problem such as 3/2 ÷ ¼ could be conceptualized as the number of groups of ¼ that can be made from 3/2. Generating a fraction equivalent to 3/2, such as 6/4 (i.e., creating a common denominator) is also needed to see how many groups of ¼ can be made from 3/2. Thus, the solution to 3/2 ÷ ¼ is the same as the solution to 6/4 ÷ ¼, which makes it easier to see that there are six groups of ¼ in 3/2. Focusing on the measurement meaning of division and referent units can be applied to any fraction division problem (i.e., a/b divided by c/d) and provides a conceptual explanation for the invert-and-multiply algorithm (see Fig. 2). Specifically, a/b ÷ c/d can be conceptualized as the number of groups of the divisor, c/d that can be made from the dividend, a/b.

Because fractional representations depend on equal-size pieces, creating a common denominator for these two fractions, a/b and c/d, is needed for the fractional representation of the quotient. As shown by the blue rectangle in Fig. 2c, one group of a divisor, c/d, can be made by c × b equal-sized parts. The dividend, a/b, has a × d same-sized parts (see the green rectangle in Fig. 2c). The quotient (i.e., the number of groups of the divisor) can then be found by dividing the total number of parts, a × d, by the number of parts in one referent unit, b × c. This approach provides a conceptual explanation for the division algorithm in that a/b ÷ c/d = (a × d)/(c × b), and by focusing on the fact that the referent unit for the quotient is the divisor, it also creates a meaningful explanation for the denominator of the quotient.

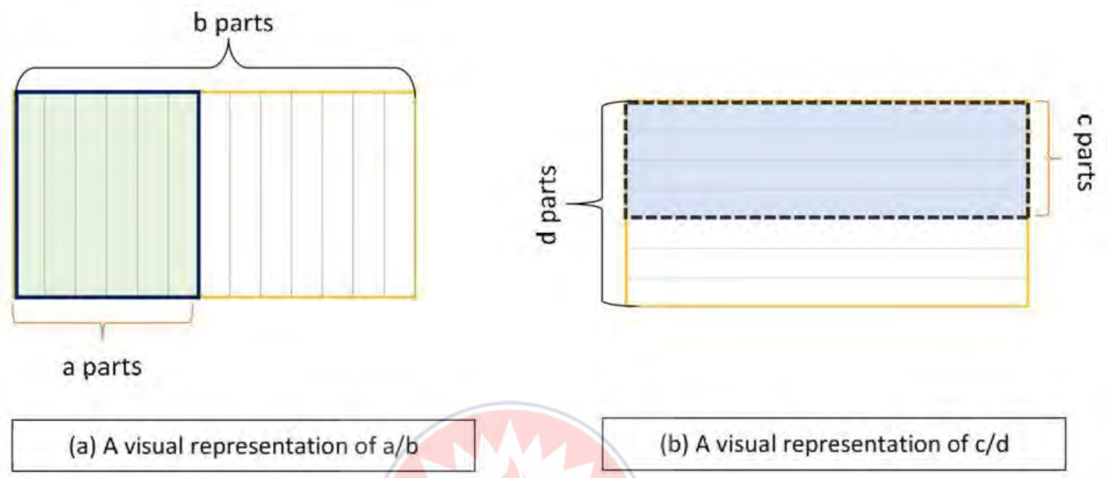

Figure 2: An explanation for the division algorithm based on the measurement meaning of division and the referent units

Thus, the measurement interpretation of division along with the use of the common denominator approach could conceptually explain the division algorithm. That being said, there are other mathematical explanations for the division algorithm (Beckmann, 2012; Tirosh, 2000). For instance, using the same example, the division problem of 3/2 ÷ 1/4 can be found by using the knowledge that division and multiplication are inverse operations. The division of 3/2 by 1/4 can be thought of as an unknown factor multiplication problem (i.e., 3/2 ÷ 1/4 = q is equivalent to q × 1/4 = 3/2). Using the fact that the product of a number and its reciprocal is 1, multiplying both sides of the equation (q × 1/4 = 3/2) by the reciprocal of 1/4 (i.e., 4/1) will then lead to 3/2 × 4/1 = q. Note that both expressions, 3/2 ÷ 1/4 and 3/2 × 4/1, equal q. Thus, the division of 3/2 by 1/4 can be found by multiplying the dividend by the reciprocal of the divisor. This approach is valid for the division of any fraction in that:

$$\frac{a}{b} \div \frac{c}{d} = q$$

$$\frac{a}{b} = q \times \frac{c}{d}$$

$$\frac{a}{b} \times \frac{d}{c} = \left(q \times \frac{c}{d}\right) \times \frac{d}{c}$$

$$\frac{a}{b} \times \frac{d}{c} = q \times \left(\frac{c}{d} \times \frac{d}{c}\right)$$

$$\frac{a}{b} \times \frac{d}{c} = q \times 1 = \text{q; hence}$$

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$$

Similarly, the knowledge of complex fractions in which the denominator, numerator, or both can be fractions, the knowledge of equivalent fractions, and the knowledge that the product of a number by its reciprocal equals 1 can be used to explain how division of the fraction algorithm works. Using the same division example, 3/2 ÷ 1/4 can also be thought of as a complex fraction (3/2)/(1/4). A fraction equivalent to (3/2)/(1/4) can be created by multiplying both the numerator (i.e., 3/2) and the denominator (i.e., 1/4) by the reciprocal of the divisor (i.e., 4/1). This results in (3/2 × 4/1)/(1/4 × 4/1). Because the product of a number by its reciprocal equals 1 and the division of a number by 1 is equal to itself, the expression is (3/2 × 4/1)/(1/4 × 4/1) = (3/2 × 4/1)/1 = 3/2 × 4/1. This method could also be used to prove the division algorithm for the division of any fraction pair:

$$\frac{a}{b} \div \frac{c}{d} = \frac{\frac{a}{b}}{\frac{c}{d}} = \frac{\frac{a}{b} \times \frac{d}{c}}{\frac{c}{d} \times \frac{d}{c}} = \frac{\frac{a}{b} \times \frac{d}{c}}{1} = \frac{a}{b} \times \frac{d}{c}$$

Understanding the key concepts behind these rules is essential not only for developing a robust understanding of fraction arithmetic, but also for understanding fraction concepts. For instance, Izsák, Orrill, Cohen, and Brown (2010) collected data from a convenience sample of 201 middle-grade teachers through multiple-choice items targeting fraction knowledge. As a result of using a mixture-Rasch model to analyze the data, they identified two latent groups of teachers that differed in their identification of the appropriate referent units in problem situations (Izsák, Jacobson, de Araujo, & Orrill, 2012). Furthermore, Copur-Gencturk and Olmez (2020) found that teachers' understanding of referent units was associated with their overall performance on other fraction concepts. Thus, capturing teachers' knowledge of the conceptual underpinnings of fraction arithmetic could delineate their understanding of other fraction concepts. Building off the work of other scholars, this study captures teachers' understanding of the mathematical underpinnings of two fraction operations, fraction addition and fraction division, to investigate the extent of their conceptual understanding of each operation as well as what concepts and representations they used in their explanations.

CHAPTER THREE

METHODOLOGY

3.1 Introduction

The study aimed to explore mathematical knowledge for teaching JHS 1 teachers in the teaching of fractions at the various schools in Atwima Nwabiagya North District. Chapter three presents the methodology based on research design, research site, population and sampling, data collection techniques/methods, analytic sample, tasks, ethical consideration, confidentiality.

3.2 Research Design

The study was mainly supported by a qualitative research approach. A qualitative research approach is suitable for this study since it researches the actual practice of the intermediate phase, teachers from the identified schools when teaching fractions. Through a qualitative research approach, the researcher can explore the mathematical knowledge for teaching on how to introduce, unpack, develop, and define fractions to JHS 1 learners of the intermediate teachers. McMillan and Schumacher (2010) highlighted that the qualitative research approach focuses on exploring, understanding, and determining significance and describing a phenomenon through the participant's practices and viewpoints. This approach is subjective as the researcher cannot detach herself from the issues discussed. The mixed research approach is used because it will give the researcher an opportunity to gain insight into the inner experience of the participants.

3.3 Research Site

The research study took place in the Ashanti Region of Ghana, at the various primary schools in Atwima Nwabiagya North District. The primary schools where the research was conducted have a foundation, intermediate and senior phases. Both sampled schools were public schools.

3.4 Population

The population of the study was the twenty neighbouring Basic schools Mathematics teachers, in the Atwima Nwabiagya North District. The population consist of 80 Mathematics teachers at the Basic schools in the Atwima Nwabiagya North District.

3.5 Sample and Sampling Technique

The research study's sample was 12 mathematics teachers who were teaching JHS 1 from neighbouring schools selected to be part in the study. According to Cohen, et al (2011), convenience sampling involves choosing the nearest individuals to serve as participants. In contrast, Creswell and Clark (20110) state that convenience sampling involves identifying and selecting individuals or groups of individuals that are especially knowledgeable about or have experience with a phenomenon of interest. Furthermore, Leedy and Ormrod (2010) emphasise that sampling is convenient if it is dependent only on the accessibility and availability of participants. According to Farrokhi and Mahmoudi-Hamidabad (2012), convenience sampling is a kind of non-probability or non-random sampling in which participants are selected for the purpose of study if they meet specific criteria. The researcher sampled JHS 1 mathematics teachers at Atwima Nwabiagya North District because of their accessibility, and their willingness to participate. The public schools researched were sampled purposefully because of their accessibility, and their willingness to participate.

3.6 Data Collection Techniques/Methods

The data for this study was collected from JHS 1 teachers who were teaching at Atwima Nwabiagya North District. Participants were recruited through an education survey. Given that recruitment was done through an intermediary and data was collected through an online survey, several precautionary steps was taken to ensure that the data was collected from the targeted population in an appropriate manner. Specifically, teachers received an e-mail with a link to a screening survey, which began with general questions regarding the participant's career and followed by specific questions about their time working as a classroom teacher. Following the screener survey, those who were eligible to participate (i.e., teachers currently teaching mathematics in JHS 1) were allowed to respond to the items used in the study.

3.6.1 Analytic sample

The analytic sample included mathematics teachers who provided background information, and at least one of the items used in the analysis. However, the analytic sample seemed to include more teachers with 3 to 9 years of teaching experience and fewer teachers with more than 20 years of teaching experience. In general, teachers in the study sample had, on average, 9.8 years of mathematics teaching experience. Although the participating teachers were currently teaching mathematics in JHS 1, more than one-third of them (34.7%) ever taught mathematics in Primary 5 or 6.

3.6.2 Tasks

Two tasks were developed to capture teachers' conceptual understanding of two fraction operations: addition and division (see Appendix A for the two tasks). Because this study aimed to capture teachers' understanding of the mathematical ideas behind these two operations, the wording of the tasks used in the study specifically focused on revealing their understanding of the key concepts. Item development followed several iterative processes. First, the two tasks were adapted from two problems developed by Van de Walle, Karp and Bay (2010), one of the most widely used elementary teacher education resources. The problems was shared with mathematics education scholars for feedback, and revised accordingly. Next, in-service teachers in JHS 1 was interviewed using these tasks (see Appendix A). During this process, both tasks were revised to capture teachers' understanding of the mathematics behind these procedures.

3.7 Ethical Consideration

The research study considered ethical issues. A formal request to do research at the identified schools were prepared and forwarded to the Head Mistress/Master of those schools. Over and above this request, another formal request was directed to the identified teachers to get their consent for them to participate in the research study. All participants were well informed that the whole exercise is intended to gather information for the researcher to advance her studies at one of the country's recognised universities. The information was giving willingly as the results of the study might be used to benefit the participants and ultimately improve performance in the teaching and learning of fraction in their respective schools.

3.7.1 Confidentiality

The participating schools were labelled by means of the letters of the alphabet from A to C to ensure confidentiality during the process of observation and interview. All the schools' real identities relating to the alphabet letters remained confidential, only known to the researcher. Information provided by the participants, particularly personal information, was protected and not made available to anyone other than the researcher to ensure confidentiality of the participants' personal information. All participants were assured of confidentiality in writing. The participants were assigned pseudonyms to protect their identities and to ensure confidentiality, e. g. names like John, Emma and Fiifi were used. The researcher reassured the participants that their real names would be kept anonymous, and all data gathered would be kept confidential. The researcher introduced herself before the start of the research to gain the trust of the participants.

3.8 Data Analysis Procedure

The responses obtained from the task were coded in two phases, first was according to the correctness of their explanation, and secondly in sub-scales indicating common representation the teachers used. In analysing the response from the interview, themes were generated as well.

CHAPTER FOUR

RESULTS AND DISCUSSION

4.0 Introduction

This study aims to explore JHS1 teachers' knowledge in teaching fractions in three schools in the Atwima Nwabiagya North District. Chapter four presents the results and analysis of the study.

4.2 Analysis of Demographic Characteristics of Respondents

The demographics of the respondents covered the gender, age, qualification, and work experience in table 4.1 below. The analytic sample included mathematics teachers who provided their background information, and at least one of the items used in the analysis (N = 12). However, the analytic sample seemed to include more teachers with 3 to 9 years of teaching experience and fewer teachers with more than 20 years of teaching experience. In general, teachers in the study sample had, on average, 9.8 years of mathematics teaching experience. All the sampled teachers hold Mathematics certificate with the majority holding degree certificate. Female respondents were the majority.

Table 1: Demographic Distribution of Respondents

S/N of Interviewee	Characteristics of Interviewees
Respondent 1	This respondent is a female and serving as a teacher. She is 33 years of age and have worked for 9 years. She holds Bed in Mathematics as her qualification.

Respondent 2	This respondent is a female and serving as a teacher. She is 33 years of age and have worked for 13 years. She holds Bed in Mathematics as her qualification.
Respondent 3	This respondent is a female and serving as a teacher. She is 37 years of age and have worked for 8 years. She holds Masters' Degree in Mathematics as her qualification.
Respondent 4	This respondent is a female and serving as a teacher. She is 27 years of age and have worked for 7 years. She holds Diploma in Mathematics as her qualification.
Respondent 5	This respondent is a female and serving as a teacher. She is 28 years of age and have worked for 8 years. She holds Diploma in Mathematics as her qualification.
Respondent 6	This respondent is a female and serving as a teacher. She is 35 years of age and have worked for 6 years. She holds First Degree in Mathematics as her qualification.
Respondent 7	This respondent is a male and serving as a teacher. She is 33 years of age and have worked for 5 years. She holds First Degree in Mathematics as her qualification.
Respondent 8	This respondent is a female and serving as a teacher. She is 30 years of age and have worked for 9 years. She holds First Degree in Mathematics as her qualification.
Respondent 9	This respondent is a female and serving as a teacher. She is 29 years of age and have worked for 6 years. She holds First Degree in Mathematics as her qualification.

Respondent 10	This respondent is a male and serving as a teacher. He is 31 years of age and have worked for 9 years. He holds First Degree in Mathematics as his qualification.
Respondent 11	This respondent is a male and serving as a teacher. He is 27 years of age and have worked for 5 years. He holds First Degree in Mathematics as his qualification.
Respondent 12	This respondent is a male and serving as a teacher. She is 28 years of age and have worked for 4 years. He holds First Degree in Mathematics as his qualification.

Source: Field study, (2022)

4.3 Differences in the mathematical accuracy of teachers' explanations for the addition and multiplication algorithms.

Data were coded in two major phases. Teachers' responses were first coded according to the correctness of their explanations (i.e., incorrect/no, partially, or correct). Then, sub-codes (Saldana, 2013) were developed to identify common representations the teachers used (e.g., real-world examples or visuals) or the kinds of explanations they gave in their responses (e.g., the measurement meaning of division). In the first phase, teachers' explanations were evaluated based on the extent to which their explanations focused on key ideas underlying the algorithms. The correctness of teachers' explanations was rated using three categories: incorrect/no explanation, partially correct, and correct explanations (Table 2). The incorrect explanations category included incorrect responses or responses that focused on an algorithm. For instance, one teacher said, "We need a common denominator to make the problem easier to solve." This response was coded as an incorrect explanation because the mathematical reasoning underlying fraction addition is not to, "…make the problem easier to solve." Second, responses coded as partially correct explanations consisted of responses in which the key mathematical ideas underlying these procedures were implied, but not made explicit.

Table 2: Scoring rubric used to categorize teachers' explanations

Category	Operation	Description
Incorrect/no explanations	Addition	The response was either incorrect or teachers simply stated the steps in the algorithm. Teachers who reported they did not know how to explain were also coded in this category.
	Division	The response was either incorrect or teachers simply stated the steps for the algorithm. Teachers who reported they did not know how to explain were also coded in this category.
Partially correct explanations	Addition	Either the key idea (same size or equal partitioning) was not explicitly stated or the examples or visuals provided were not accurate.
	Division	Either the key idea (why the invert-and-multiply algorithm works or the need to attend to the referent units) was not

		explicitly stated or the examples or visuals provided were not accurate.
Correct explanations	Addition	The key idea (same size or equal partitioning) was explicitly stated.
	Division	Why the invert-and-multiply algorithm works was explained conceptually or the teacher attended to the referent whole, or both.

Source: Field study, (2022)

For example, a teacher said, "It's difficult to add fractions that are not equal pieces. So, one must 'cut up' the pieces into pieces that are multiples of both. Can't add apples + oranges." This statement provides a mathematical rationale for creating a common denominator; however, it does not explicitly articulate the mathematical idea behind the operation, and the example provided is not completely appropriate. Although the teacher mentioned that the pieces are not equal, she or he did not explain why an equal denominator is needed (e.g., fractional representation is based on the number of equal-sized pieces that make up the whole). Additionally, examples using apples and oranges could be problematic in that when adding and subtracting fractions, the same referent whole is used. Therefore, using two different fruits could potentially be misleading. The final category, correct explanations, included responses that explicitly focused on the underlying key ideas. Responses such as the following were coded in this manner:

> *This is done because it will become harder to compute and understand in case you are adding or subtracting. The denominator of a fraction tells you the relative size of the pieces. For instance, ½ is bigger than ¼ because it*

> *only takes 2 pieces to make a whole, as opposed to 4 pieces to make the whole. One might connect the need for a common denominator to the need for having common units before adding and subtracting (you wouldn't add 12 inches to 12 feet and get 24 for an answer). Therefore, the reason fractions need a common denominator before adding or subtracting is so that the number of pieces you are adding/subtracting are all the same size. Note that the numerator for a fraction just tells you how many pieces you have of that size.*

As shown in this response, the key mathematical idea is to create equal-sized pieces because a denominator tells the number of equal-size pieces needed to make the whole. Furthermore, the example chosen to illustrate the point is mathematically appropriate in that both inches and feet have the same referent unit. After finalizing an initial version of a rubric to use with the responses teachers provided from the task, a second rater was trained. Together, the researcher coded several responses to establish interrater reliability and refined the criteria for each category. After finalizing the rubric criteria and gaining confidence in using the rubric to code the responses reliably, the researcher independently coded the responses from 10 teachers and discussed the ratings, working through any disagreements and noting exemplars. The researcher continued rating 10 teachers' responses at a time, until reaching 90% exact agreement. Upon reaching the 90% threshold, the researcher coded separately and held a final meeting to review the ratings and settle any disagreements in the ratings. In the second phase, the co-rater and the researcher developed low-inference subtopics to capture noticing from the data. An example of a low-inference subtopic is how, in both tasks, teachers were often using visuals or real-world problems to explain these procedures. Specifically, for the division problem, it was noticed that teachers used certain interpretations of division problems, or it was found that they mixed fraction multiplication with division. These codes required less interpretation than the codes in the first phase because the topics were explicitly derived from the teachers' responses. The co-rater and the researcher then discussed the sub-codes and began coding the dataset again with these sub-codes in mind.

Similar to the first phase, the researcher coded data together until reaching 90% interrater reliability and then coded the remaining teacher responses separately while meeting to discuss and settle any disagreements. In sum, each individual response was coded in two ways, by two independent raters. Once to measure the correctness of the teachers' explanations and a second time based on subtopic codes for the concepts or representations they used in their explanations. Using this rated and coded data, quantitative analyses were utilized to address the four research questions (RQ) guiding this study. To investigate the extent to which teachers correctly explained the conceptual underpinnings of the fraction addition and division algorithms (RQ-1a), frequencies of responses for both tasks are reported separately. Additionally, to test whether teachers had a better understanding of the fraction addition algorithm than the division algorithm (RQ-1b), a paired-sample t test was employed. To examine the relationship between the correctness levels of teachers' explanations for both procedures (RQ-2), a 3×3 contingency table was used for a chi-square test of independence to examine the relationship between teachers' explanations for these two operations. This analysis allowed the researcher to examine whether teachers who understood one algorithm were likely to understand the other.

To answer the extent to which the teachers explained the underpinnings of these operations and what concepts and representations they used in their explanations (RQ-

3), the researcher summarized characteristics of the explanations for each correctness level along with sample responses and reported the frequency of the subtopics and representations used for each operation. Finally, to further investigate how teachers' educational background was related to the correctness of their explanations (RQ-4), an ordered logistic regression, in which teachers' correctness level was predicted by their years of mathematics teaching experience (standardized), credential type (generalist, teaching mathematics, or other, with generalist being the reference category), highest grade level of mathematics being taught (ranging from 4 to 9), and whether they held a regular teaching certificate. The tasks were added as fixed effects, and standard errors were clustered around teachers in this analysis.

4.3.1 Mathematical accuracy of teachers' explanations of fraction addition and division

The purpose of this study was to investigate teachers' conceptual understanding of fraction addition and division. As shown in Figure 3, 19.8% of teachers provided an incorrect or no explanation for the fraction addition procedure whereas this rate was 58.1% for fraction division. About half (55.6%) of the teachers explained the need for a common denominator mathematically and included partially correct explanations for adding fractions with unlike denominators, whereas only 26.1% of the teachers provided a conceptual explanation for the division procedure. About 24.6% of the teachers explained the need for a common denominator mathematically and included correct explanations for adding fractions with unlike denominators, whereas only 15.8% of the teachers provided a conceptual explanation for the division procedure. The difference in mathematical accuracy of teachers' responses indicate that teachers were able to provide a more accurate explanation for the fraction addition algorithm than the division algorithm.

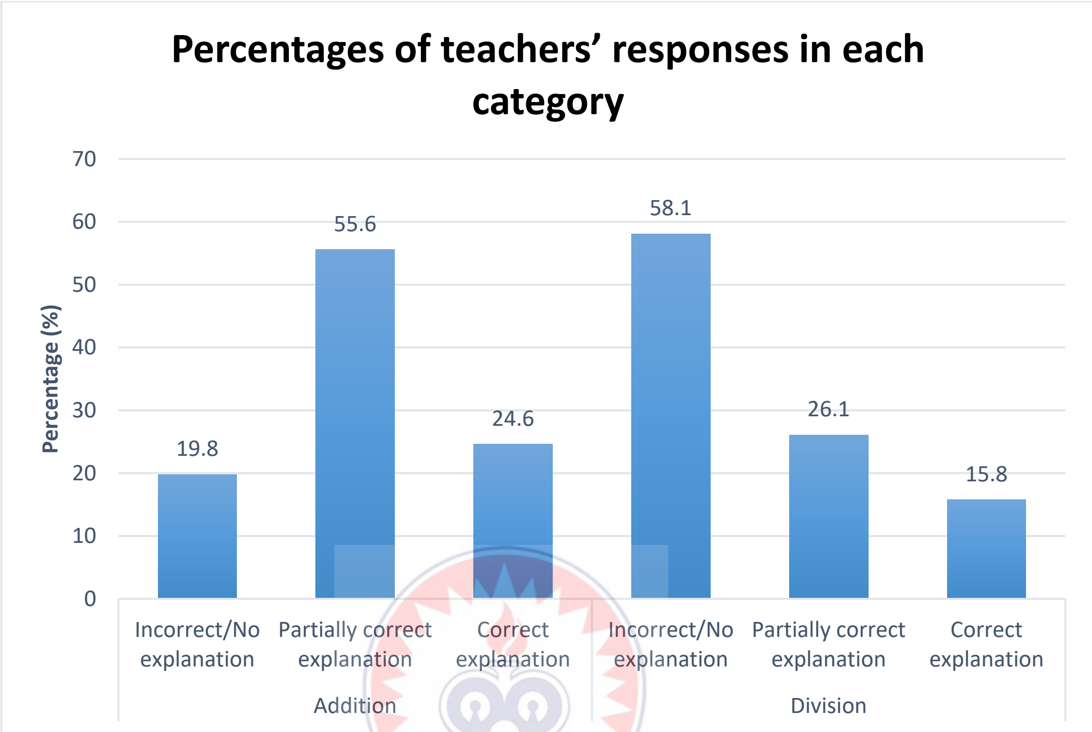

Figure 3: Percentages of teachers' responses

Source: Field study, (2022)

4.4 The relationship between teachers' explanations for fraction addition and multiplication.

There was a statistically significant and moderate relationship between teachers' understanding of the conceptual underpinning of both operations ($\chi 2(4, 276) = 15.85$, $p = .003$; Gamma =.34). Specifically, 15.2% of the teachers provided incorrect explanations for both operations, and 18.9% provided correct explanations for both operations (see Table 3). Half of the teachers who could explain the addition procedure provided incorrect explanations for the division procedure, whereas only 7.4% of the teachers who failed to explain the addition procedure were able to explain the division procedure.

Table 3: Number of teachers in the explanation categories for division and addition

		Division		
		Incorrect/No	Partially correct	Correct
Addition	Incorrect/No	42	8	4
	Partially correct	38	12	17
	Correct	78	25	52

Source: Field study, (2022)

4.5 The characteristics of teachers' explanations with concepts and representations of fraction addition and multiplication?

Here, teachers' explanations of the addition and division algorithms are delineated by overall patterns along with sample responses. For both operations, teachers whose responses were incorrect either reported that they did not know why the procedure worked or they simply stated the steps of the operation (see Figure 4 for sample incorrect explanations).

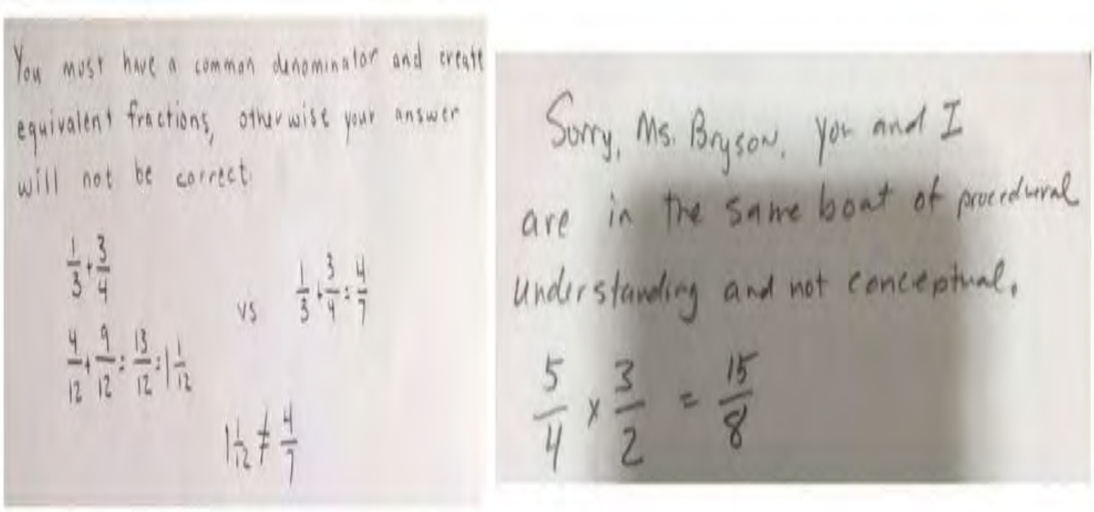

Figure 4: Sample responses for incorrect explanations

Source: Field study, (2022)

The teachers who provided partially correct explanations for the addition algorithm (24.7%) implicitly focused on the key concept underlying the fraction addition procedure (i.e., equal partitioning). However, either their responses did not make the concept explicit or the examples they provided and the visuals they used were not accurate. For instance, as shown in the left panel of Figure 5, the teacher did not state explicitly that equivalent fractions made the fraction pieces equal, but his or her drawing used the same size whole and equal-sized pieces. For the division operation, 15.8% of the teachers' explanations focused on the key ideas involved in the fraction division procedure, such as making the number of groups of the divisor (2 3), but they failed to show it accurately.

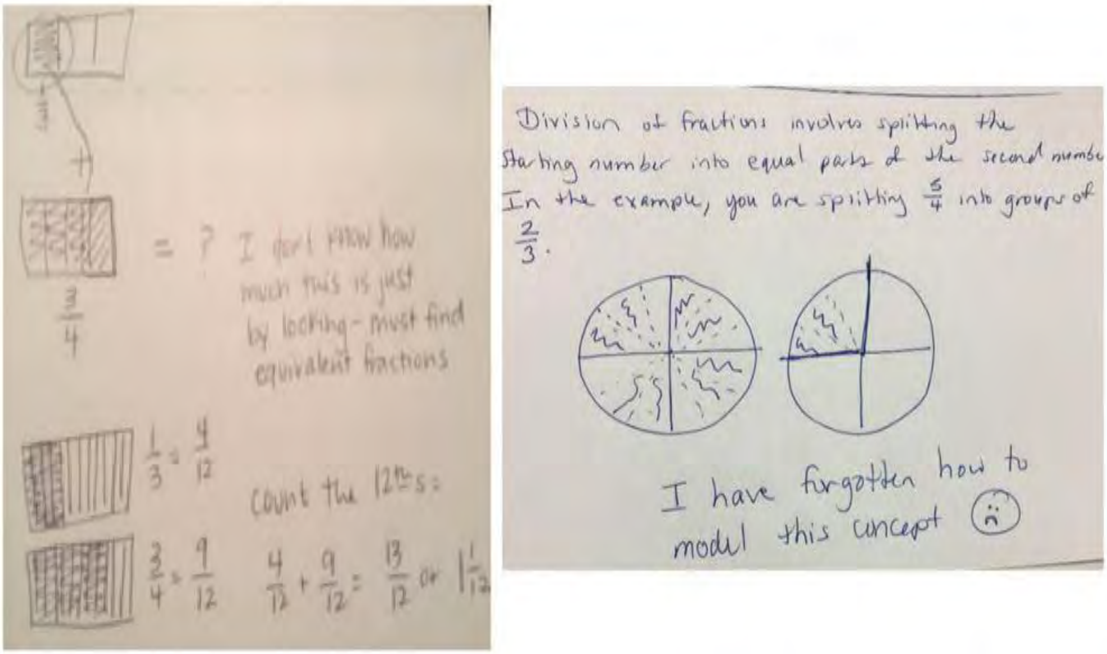

Figure 5: Sample responses for partially correct responses

Source: Field study, (2022)

Again, as shown in the right panel of Figure 6, the teacher attempted to provide an explanation for the division procedure but did not explicitly state how the procedure worked. Among those who provided correct explanations, about half (55.6%) of the teachers explained the need for a common denominator mathematically and included correct explanations for adding fractions with unlike denominators, whereas only 26.1% of these teachers provided a conceptual explanation for the division procedure (see Figure 6). Additionally, for the division algorithm, 11.0% of the teachers provided an explanation for how the division algorithm worked and how to make sense of the quotient.

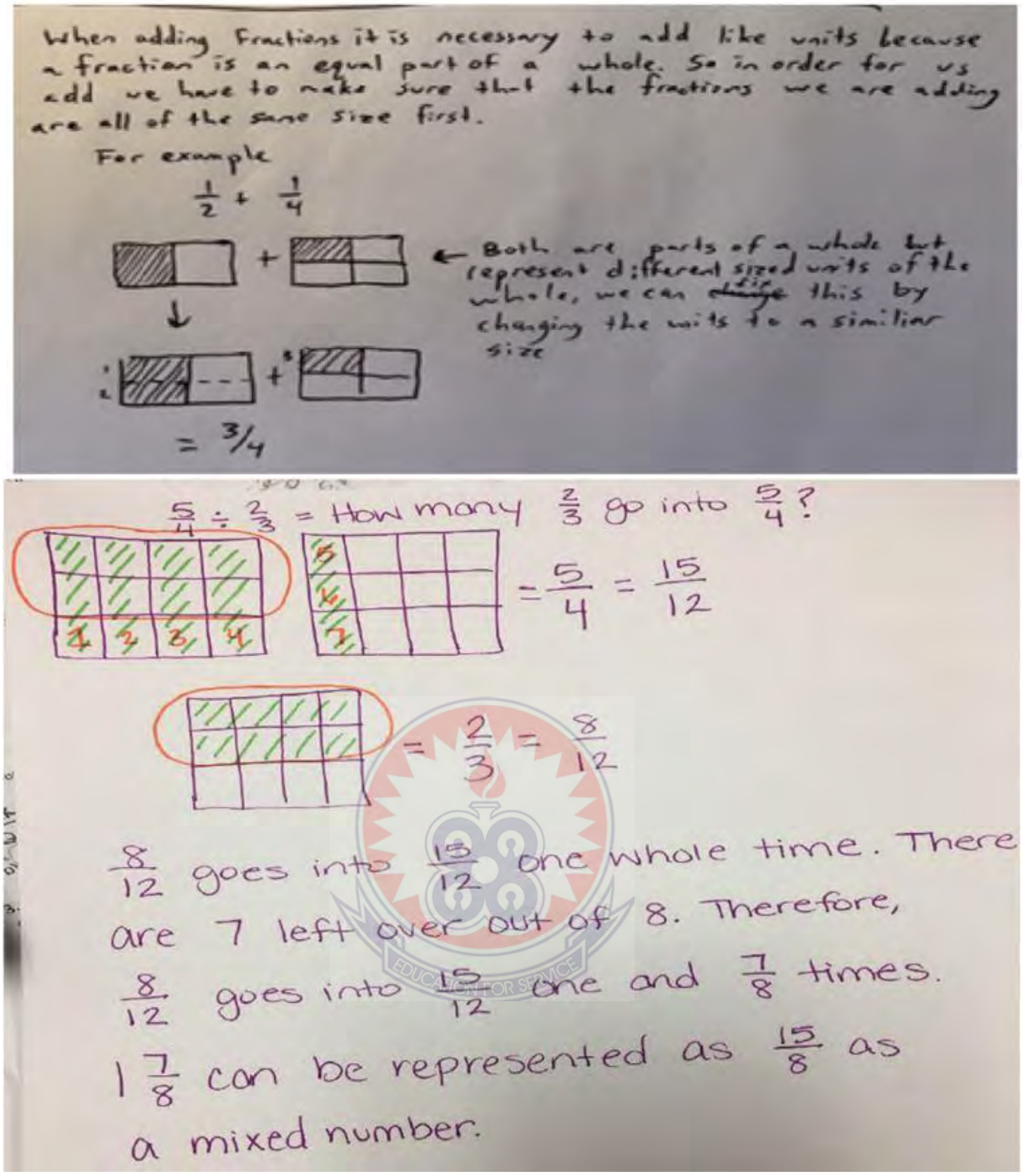

Figure 6:Sample responses for correct explanations

Source: Field study, (2022)

4.5.1 Representations and concepts used in teachers' explanations

Teachers heavily used visuals and real-world examples in their explanations (see Figure 7). In their responses, 74.0% of the teachers used drawings to explain the addition procedure, whereas 52.6% used visuals in their explanations for fraction division. Additionally, teachers used more real world examples to partially explain the addition procedure than the visual procedure (26% vs. 24%), and teachers who provided real-world examples generally used examples such as "you cannot add apples and oranges" to explain why a common denominator would be needed. As mentioned, this example could be problematic, given that apples and oranges are different fruits, yet addition and subtraction can be done if both fractions refer to the same unit.

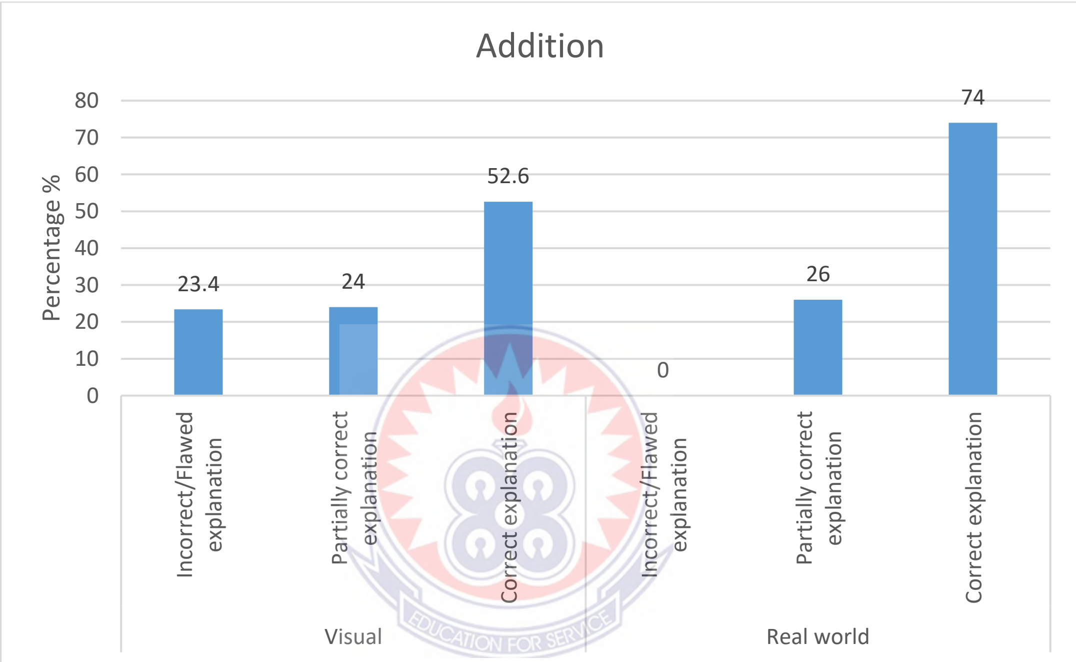

Figure 7: Percentage of teachers using concepts and representations at each level of explanation

Source: Field study, (2022)

In addition to teachers' use of visuals, three patterns emerged in teachers' explanations of fraction division (see Figure 8). First, teachers used easier fraction pairs, such as a whole number divided by a unit fraction, to explain fraction division correctly (89%). In fact, 8% of the partially correct responses included easier fractions (e.g., 2÷ 1/2). Second, the measurement meaning of division (how many groups of a divisor could be made with the dividend) seemed to be used for more than half of the partially correct and correct explanations (45% and 54%, respectively). Third, the real world meaning of division seemed to be used for partially correct and correct explanations (26% and 74%, respectively. Fourth, the visual meaning of division seemed to be used for incorrect/flawed explanation, partially correct and correct explanations (23.4%, 24% and 52.6%, respectively). 4% of the teachers mixed division with multiplication. In fact, these teachers' drawings were modeling fraction multiplication rather than fraction division.

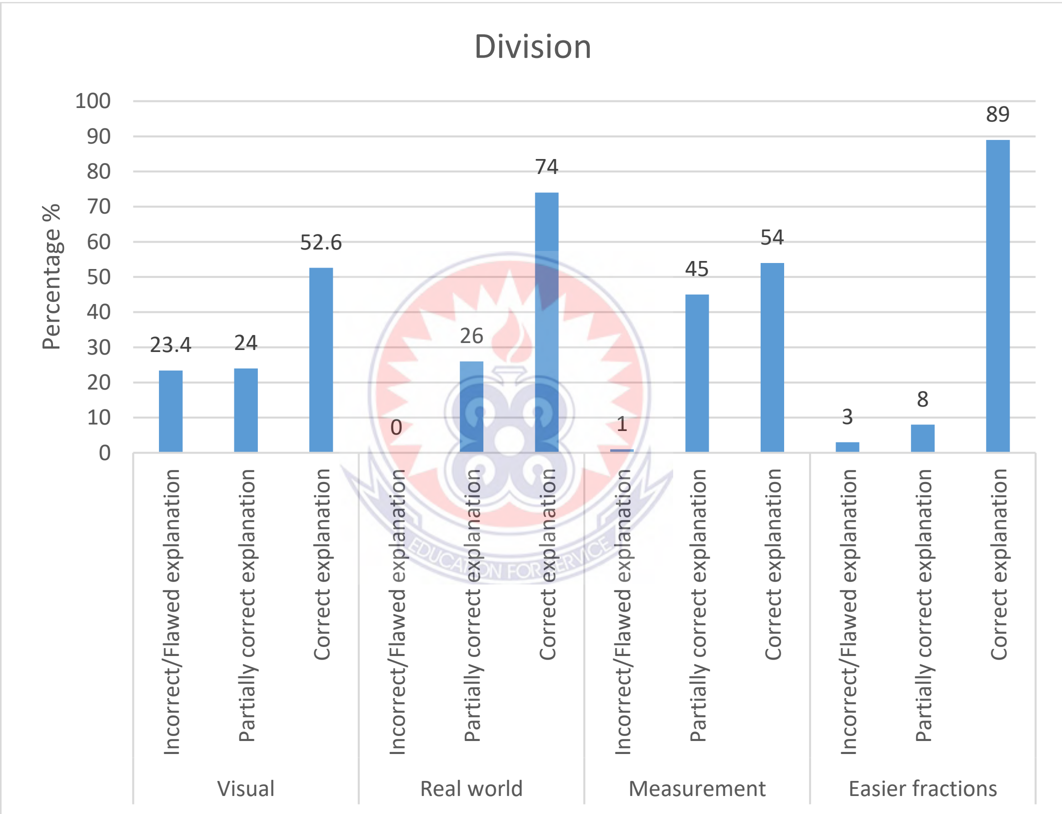

Figure 8: Percentage of teachers using concepts and representations at each level of explanation

Source: Field study, (2022)

CHAPTER FIVE

SUMMARY, CONCLUSION, AND RECOMMENDATION

5.0 Introduction

This study aimed to explore JHS1 teachers' knowledge in teaching fractions in three schools in the Atwima Nwabiagya North District. This chapter presents the summary of the study and major findings. It also presents conclusions and recommendations.

5.2 Summary

The study was hinged on the theory of Mathematics Teachers Knowledge (MTK) and Pedagogical Content Knowledge (PCK). The Transformative learning theory (TLT) and the theory of Cognitive. The study was guided by the following questions:

1. Is there a difference in the mathematical accuracy of teachers' explanations for the addition and multiplication algorithms?
2. What is the relationship between teachers' explanations for addition and multiplication?
3. What are the characteristics of teachers' explanations with concepts and representations of addition and multiplication?

A qualitative research approach was used to explore the mathematical knowledge of teachers on how to introduce, unpack, develop, and define fractions to JHS 1 learners of the intermediate teachers.

Tasks were used to obtained the teachers conceptual understanding of two fraction operations: addition and division. The following were the major findings of the study. It was revealed there teachers had a difference in their mathematical accuracy. This was an indication that teachers were able to provide a more accurate explanation for the fraction addition algorithm than the division algorithm. It was also revealed that half of the teachers could explain the addition procedure, however, they provided incorrect explanations for the division procedure. Teachers also were found to have used visuals and real-world examples in their explanations of addition procedures and explanations for fraction division.

5.3 Conclusion

The study's findings show that even though in-service teachers appeared to know more about the conceptual foundations of fraction arithmetic than did the preservice teachers portrayed in previous studies, a sizable portion of the teachers still did not appear to be able to demonstrate a profound understanding of fraction procedures. As a result, teachers require more learning opportunities if they are to fully grasp the logic behind the laws and processes.

5.4 Recommendation

This study also calls attention to the need to more precisely capture teachers' robust understanding of the facts and procedures of the subject matter they are expected to teach so that we can better explore the role that teachers' conceptual understanding plays in the development of students' knowledge and their misconceptions. It is recommended that:

1. A teacher must have a deep understanding [emphasis added] of the mathematics being taught in order to affect student accomplishment.
2. A teacher must have the ability to impart knowledge to students, regardless of how much content knowledge she possesses.

3. I believe a teacher should grasp how to solve problems, but not necessarily memorize formulas and algorithms.

The findings of this study not only confirm that "Teachers' mathematical knowledge is important for students' achievement," as stated by the National Mathematics Advisory Panel, but they also go further in identifying the kind of teacher content knowledge that is essential for middle school student success. This study's recommendations for practice include emphasizing teachers' conceptual and relational knowledge development while offering content-focused professional development that is especially created to raise student accomplishment. Future research on integrated teacher knowledge and how it affects student achievement is required.

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APPENDIX

TASKS USED IN THE STUDY

Mrs. Johnson is planning a lesson on fraction addition. She understands that she needs to find a common denominator when adding two fractions. For example, when she adds $\frac{1}{3}+\frac{3}{4}$, she finds a common denominator of 12: $\frac{4}{12}+\frac{9}{12}=\frac{13}{12}$. However, she does not understand why she needs a common denominator or how to interpret the need for a common denominator when adding fractions. Can you explain to Mrs. Johnson why we need to use a common denominator to add fractions? Please feel free to use any method to explain the key concepts (e.g., visual representations, real-world examples, etc.). Ms. Bryson is planning a lesson on fraction division. She understands that sometimes when we divide fractions, the answer includes a fraction that has a different denominator than either the divisor or the dividend. For example,

$\frac{5}{4}\div\frac{2}{3}=\frac{15}{8}$. She knows the denominator of the answer is different because of the invert-and-multiply algorithm ($\frac{5}{4}x\frac{2}{3}=\frac{15}{8}$) but she does not understand why it happens (e.g., where does $\frac{1}{8}$ come from?) or how to interpret the denominator of the answer (what does $\frac{15}{8}$ mean?). Can you explain to Ms. Bryson why the denominator of the answer is different? Please feel free to use any method to explain the key concepts (e.g., visual representations, real-world examples, etc.).