UNIVERSITY OF EDUCATION, WINNEBA

PRE-SERVICE TEACHERS' CONCEPTUAL UNDERSTANDING OF THE ARITHMETIC MEAN

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MASTER OF PHILOSOPHY

2022

UNIVERSITY OF EDUCATION, WINNEBA

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A thesis in the Department of Mathematics Education, Faculty of Science Education, submitted to the School of Graduate Studies, in partial fulfilment of the requirements for the award of the degree of
Master of Philosophy
(Mathematics Education)
in the University of Education, Winneba

NOVEMBER, 2022

DECLARATION

Candidate's Declaration

I, Godfred Kofi Osei, declare that this thesis except quotations and references contained in published works which have all been identified and duly acknowledged, is entirely my own original work and has not been submitted, either in part or whole for another degree elsewhere.

Signature:……………………….

Date:…………………………….

Supervisors' Declaration

I hereby declare that the preparation and presentation of this work were supervised in accordance with the guidelines for supervision of thesis as laid down by the University of Education, Winneba.

Name of Supervisor: Dr. (Mrs.) Gloria Armah

Signature:…………………………

Date:………………………………

DEDICATION

This work is dedicated to my mother, Lydia Aboagye, and my uncle, Mr Bright Wilson Fordjour. God bless you for all that you have done for me. I am grateful.

ACKNOWLEDGEMENT

What shall I say unto the Lord for all His mercies? Great is your faithfulness! Thank you, Father. The realization of this thesis is the fruit of a total commitment to multiple months of hard work and determination. However, it would not have been possible without the guidance of my supervisor, support from sponsor, research participants, friends, and my family. Firstly, this research received financial support from the Ghana National Petroleum Corporation Foundation (GNPC_Foundation). For this support, I am most grateful. Special thanks to the participants in the research, as without them there would have been no study.

I am particularly very grateful to Dr. Mrs. Gloria Armah my supervisor, for the continuous unflinching support, patience, motivation, and immense knowledge. She repeatedly scrutinized the pages of this thesis making corrections and suggestions, her insightful comments, valuable suggestions, and hard critique motivated me to widen my research from various perspectives which helped in writing this thesis. God richly bless her and her family. I also owe profound gratitude to all the Lecturers at the Mathematics Education Department, University of Education, Winneba (UEW) whose tuition and great thoughts have brought me this far on my academic ladder. I am grateful to Prof. S. K. Aseidu-Addo, Dr. Mohammed Ali, and Mr. Robert Armah whose great thoughts and tremendous support have brought me this far.

I am grateful to the HODs of the three departments that were used for this study. Special heartfelt thanks go to my Pastor, Solomon K. Kyei and my dear friends, Stephen Appiah, Bright Boampong, Jemima Saah, Rachael Ockling and Isaac Mireku, for their support, encouragement, suggestions and prayers. The heartiest of thanks goes to my mum, Lydia Aboagye and my uncle Bright Fordjour. You have always been there for me whenever I need your guidance, love and support. God bless you.

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ABSTRACT

This study explored pre-service teachers‟ (PST) performance and conceptual understanding of the arithmetic mean after going through the Ghanaian Junior and Senior High School's core and elective mathematics curriculum. It also sought to determine their level of conceptual understanding of the arithmetic mean with respect to the Structure of the Observed Learning Outcome (SOLO) taxonomy. The explanatory sequential mixed method was employed. The purposive, stratified and the simple random sampling techniques were employed to select 370 PST sampled from the Departments of Mathematics, Chemistry and Physics Education of the University of Education, Winneba. Statistical Understanding Test of Arithmetic Mean (SUTAM) was used as data collection tool. The data was analysed descriptively by using percentages and inferentially by the t-test and ANOVA. Results revealed that PST‟ performance on the SUTAM was low. There was no significance difference between the mean scores of the Trained teachers and the direct applicant. Findings also revealed PST have conceptualized the arithmetic mean as an average, as a computational act and as a non-zero measure. Also, the findings indicated that majority (55.1%) of the participants were at the multi-structural level of the SOLO Taxonomy with few (14.6%) at the Relational level. It was recommended that teachers teach the concept of the arithmetic mean with its properties to students before teaching the computation of the arithmetic mean.

CHAPTER ONE

INTRODUCTION

1.0 Overview

This opening chapter sets the study in context. It presents the background of the study, statement of the problem, the purpose of the study, objectives of the study as well as the research questions guiding the study and the educational significance and. The chapter further highlights the delimitations and limitations and concludes by outlining the organization of the dissertation.

1.1 Background to the Study

Education plays a critical role in today's world of accelerating technological change. It is a critical tool for the complete development and advancement of each individual and society in dealing effectively with environmental issues. Many people including statesmen, politicians and educators, just to mention but a few, have often said that education leads to national development. Everywhere, there is a strong belief in education as the instrument of change and development. An aspect of this is mathematics education.

Tella (2008) sees mathematics as an abstract subject whose significance for scientific and technological development in any society is undisputed. The usefulness of the subject in the sciences, mathematical and technological activities, commerce, economics, education and even the humanities is almost as equal as that of education as a whole. Mathematics as a subject is important for the development of the reasoning faculties of the human mind. As such, famous educationists, Herbert, Froebel, and Maria recognized its importance as they contended that the intellectual and cultural development of any individual cannot take place without studying mathematics (Yasoda, 2009). It is used by every individual in daily life and provides the foundation for the study of the sciences.

Nabie (2004) emphasized that mathematics is an interactive action-oriented subject, learned through active interaction with the source. However, Anamuah-Mensah and Mereku (2005) observed that majority of students do not have the opportunity to learn a substantial proportion of the content of the mathematics curriculum and that, most students lack the conceptual understanding of the mathematics they learn which is needed to make informed judgment and applications to other related context or problems.

In recent years, there has been a growing recognition that students should learn mathematics in such a way that they can see its relevance to the world in which they live and be able to use it to gain a better appreciation of the world. Often, mathematics has been learned as a set of routines to be carried out blindly in response to stereotype examination questions (Bolt & Hobbs, 2005). The result of such teaching and learning of mathematics is that learners are unable to apply their knowledge outside the standard textbook sum. Furthermore, the motivation for learning becomes largely dependent on getting the ticks corresponding to the right answers and has little to do with any intrinsic interest in the subject or whether or not the answers are meaningful (Bolt & Hobbs, 2005). According to Özdemir (2006), in this modern era, mathematics education has aimed to move away from rote learning and memorization toward providing more challenging, complex work with an emphasis on deeper thinking; and having an interdisciplinary, rather than a departmentalized focus.

In Ghana, mathematics at the Senior High School (SHS) builds on the knowledge and competencies developed at the Junior High School (JHS) level. The student is expected at the SHS level to develop the required mathematical competence to be able to use his/her knowledge in solving real-life problems and secondly, be well equipped for further study and associated vocations in mathematics, science, commerce, industry and a variety of other professions (CRDD, 2010). At this level, mathematics is categorised into two: Core mathematics and Elective mathematics. Areas covered under elective mathematics include Algebra, Trigonometry, Coordinate Geometry, Calculus, Vectors and Mechanics, Matrices and Transformation, Logic, and Probability and Statistics. According to CDRR (2010), Core Mathematics and English Language are the prerequisites for Elective Mathematics. It was also added that subjects like Physics and Technical Drawing may enhance the study of this subject.

Mathematics has many characteristics such as its peculiar language and symbols that distinguish it from other subjects. As a subject, it has discovered its way in all aspects of the human undertaking and it is required in learning other subjects. Subsequently, students who are not solidly grounded in mathematics consistently have issues using mathematical concepts, standards and abilities throughout their education. The learning of mathematics involves abstraction in concepts and as a result of its abstract nature, unique language, and symbols, students sometimes face difficulties in learning it. One of the branches of mathematics that students face difficulties in learning is statistics (Garfield & Ben-Zvi, 2008).

Statistics is a branch of science that deals with the collection, organisation, analysis of data and drawing of inferences from the samples to the whole population (Winters, Winters & Amedee, 2010). A large portion of the arrangements is expected to utilize data and statistical procedures to help in decision-making.

The two main categories of the statistical method are descriptive statistics which deals with all the methods used to describe data. This includes graphical methods and the computation of numerical characteristics such as central tendencies and dispersion measures. The other category which is inferential statistics deals with making inferences about the whole population based on results obtained from samples from the population.

Statistics is becoming such a necessary and important area of study (Ben-Zvi & Garfield, 2008). The fuse of statistics into most mathematics educational programs has required two principal objectives for statistics instruction all around: the planning of certain students for additional investigation of formal statistics and the arrangement of all students to utilize statistics in their everyday life (Watson, 2006).

However, a great deal of concerns has been raised about the degree of students' comprehension of statistical concepts in Ghanaian schools. My experiences both as a student and a Teaching Assistant to one lecturer who teaches introductory statistics at the University of Education, Winneba show that numerous pre-service teachers hold negative discernments about the teaching and learning of statistics. These negative discernments are confirmed in the expostulating remarks regularly made about statistics. My observations of pre-service teachers" disposition towards statistics are that of frenzy, worry and absence of fearfulness The International Commission on Mathematical Instruction (ICMI), together with the International Association for Statistics Education (IASE), reported that while most mathematics teachers recognize the practical significance of statistics and, are happy to give more pertinence to the instructing of statistics, numerous mathematics teachers do not consider themselves well prepared to teach statistics because of their negative perception towards it (Batenero, Burrill & Reading, 2011) If prospective teachers see statistics as troublesome, there is the likelihood that their negative discernments may impact their tendency to utilize or teach statistics in the future or reality transfer this discernment to their students (Lester, Mccormick & Kapusuz, 2004; Pan & Tang, 2005; Estrada & Batanero, 2008).

Introductory statistics courses aim to prepare students to be proficient clients, consumers and communicators of statistics (Qian, 2011). Furthermore, within the statistics community, there has been a significant emphasis on bridging the theory-practice gap by making statistics more relevant and practical (Songsore & White, 2018). A core value of the Statistics Society of Canada (2016) is to help develop public awareness of the value of statistical thinking and the importance of statistics. Similarly, the American Statistical Society (2016) acknowledges the importance of statistics in public policy and human welfare. Statistics transcends the classroom walls and holds value in the daily lives (Pierce & Chick, 2011). Statistics educators are therefore working to develop students who will "take what they learn and apply it to the real world" (Wroughton, McGowan, Weiss, & Cope, 2013, p.50). All things considered, sound statistical thinking aptitudes are not achieved in one training statistics course, rather should be developed and created over one's whole instructive experience (Green & Blankenship, 2013).

Be that as it may, concentrates in recent decades have featured on the issue that students couldn't relate statistics viably in their day-by-day lives (Garfield & Ben-Zvi, 2007). As residents of a quantitative world, all school graduates are clients of statistics instead of makers and with the end goal for them to work appropriately as residents; they should be able to interpret data. A good understanding of statistical concepts supported by adequate mathematical skills is not sufficient without the ability to read and interpret statistical data. This ability is known as statistical literacy skills. Students must master statistical literacy skills well to compete in the 21st-century. Being the future generation of our community, students need to acquire statistical knowledge and its applications to prepare for their future careers as well as to make more rational decisions in daily endeavours (Masfingatin & Suprapto 2020). Students who have statistical literacy skills will be able to think critically about the information or data they read. Statistical literacy carries many interpretations or definitions.

Masfingatin & Suprapto, (2020) interpreted that "statistical literacy is the ability to interpret information or arguments related to statistical data; understand statistical concepts, vocabulary, and symbols; critically evaluate the statistical information or arguments related to data (based on the results of in-depth analysis); and present the results of statistical data processing” (p. 274).

Meanwhile, Watson (2006) viewed statistical literacy as the meeting point of the statistics curriculum and the daily world. Watson believed that when a statistically literate person comes across statistical information in a daily life context, he can make a spontaneous decision based on his ability to apply statistical tools, general contextual knowledge and critical literacy skills. However, Garfield and Ben-Zvi (2007) called attention to the fact that there exists a significant issue concerning school graduates not having the option to relate statistics adequately in their day-by-day lives. The issue is seen as disturbing in light of the fact that it is accepted that students get all their statistical literacy abilities as a feature of their school encounters (Garfield & Ben-Zvi, 2008). They are supposed to graduate from school and enter society prepared with these skills.

Several studies have also highlighted the issue of teachers in the enhancement of statistical literacy (i.e.Watson, 2006; Watson, 2011; Chick & Pierce, 2012). Watson (2006) argued that apart from teachers being the “…big frontier in bringing statistical literacy to all students to prepare them to leave school and enter society” (p.

271), she also firmly believed that teachers should offer their students productive experiences using real-world examples that demonstrate the utility of statistical concepts involved in the enhancement of statistical literacy.

Statistics have consistently been seen as one of the most testing branches of knowledge among school students. As indicated by Garfield and Ben-Zvi (2008), statistics is considered a troublesome subject to learn because of the multifaceted nature of statistical ideas. Comprehension of statistical ideas is not quite the same as understanding the mechanics of statistics which includes connecting numbers to the right equation. Students who can comprehend the statistical ideas can peruse and utilize tools, for example, percentage, ratio, measures of spread, central tendency and variability, as well as tables, charts and maps (Saidi & Siew, 2019).

Among the statistical ideas that students frequently experience issues learning are the measures of the central tendency concept. The term central tendency, according to Blashfield (1976), dates from the late 1920s. It is one of the statistical ideas in descriptive statistics, alongside the measure of variability, which includes the arithmetic mean (simply called the mean), mode, and median. Woldemicheal (2015) explains that the abstractness of the statistical ideas of measures of central tendency contributes to difficulties in students" understanding.

A measure of central tendency describes the central or typical value(s) of a distribution. It may also be called a centre or location of the distribution. Colloquially, measures of central tendency are often called averages. In Ghana, measures of central tendency in the SHS curriculum typically include the mean, mode, and median. Beyond the SHS, they are widely used tools in statistics and research (Adamson & Prion, 2013). The arithmetic mean is often known as the actual centre or balancing point of a set of data as a result of its computation. The mode is the most occurring value in a data set, whereas the median is the middle value of a set of arranged numbers. Among all the measures of central tendency concepts, the mean is the most popular and extensively used and researched, particularly regarding K-12 contexts (Groth & Bergner, 2006; Silver, 2007).

The average of a set of data is probably the most common statistical concept encountered in everyday life. We read about average income, average monthly temperatures, average speed etc., consequently, we make different conclusions and decisions. We also hear phrases like an average student, and an average family, and students learn on average 3 hours per day.

The word average is in everyday life and mathematics instruction usually used and understood as a synonym for the arithmetic mean. Many studies have shown that the teaching and learning of this concept are apparently easy, but the understanding causes difficulties.

Literature reviews and historical analyses indicate that students at various levels (i.e., middle-grade, high school, and undergraduate) have a less effective understanding of the mean (Zazkis (2013), Emmioğlu & Capa-Aydin (2012)). Concurrently, research focusing on pre-service mathematics teachers (PSMT), who are predominately undergraduate students, suggests they lack an understanding of arithmetic mean (Groth, 2009; Jacobbe & Carvalho, 2011; Leavy & O'Loughlin, 2006; Armah (2017). "The conceptual understanding demonstrated by the undergraduate mathematics students in the study indicated that they have an incomplete process conception of the arithmetic mean with respect to the APOS Theoretical Framework" (Armah (2017) p.156).

Teacher knowledge such as subject matter knowledge and pedagogical content knowledge has had a critical influence on students" mathematical understanding and achievement (Ball, Thames, & Phelps, 2008). The equivalent applies to statistical literacy. Teachers' conceptual knowledge of statistical topics influenced their capacity to teach these topics in a manner that improves students' statistical literacy (North, Gal, and Zewotir, 2014)

One explanation numerous students battle to get a handle on the idea of measures of central tendency is because numerous teachers don't give enough context-based guides to assist students with understanding the fundamental ideas (Groth & Bergner, 2006; Jacobbe & Carvalho, 2011). Groth (2009) alleged that not all teachers think about students' intellectual capacity before teaching statistical ideas, for example, mean, mode, and median. Frequently, teachers with less experience give fewer guides to students and don't evaluate students' understanding before moving to different ideas (Groth, 2009; Jacobbe & Carvalho, 2011).

Pre-service mathematics teachers (PSMT) may have two types of knowledge of the mean (Groth & Bergner, 2006): procedural and conceptual knowledge. Procedural knowledge guarantees aptitude to take care of numerical issues algorithmically. The conceptual understanding of arithmetic mean requires recognizing the context where the arithmetic mean has an appropriate measure of central tendency (Watson & Moritz, 2000), interpreting that arithmetic mean as a value that represents a data set (Bütüner, 2020), and possessing the visual and kinaesthetic understandings of arithmetic mean (Cai, 2000;). Conceptual understanding can be used to test the level of student responses that represent the importance of mathematics in the problem of symbolizing and applying algorithms (Strowbridge, 2009).

Differences in students" conceptual understanding due to different student abilities and thinking, student responses in solving problems can be known by using the classification of levels contained in the Structure of Observed Learning Outcomes (SOLO) taxonomy. SOLO taxonomy was first developed by Biggs and Collis in 1982. Generally, it has five levels. Measurement of the level used in the SOLO taxonomy is very appropriate for the advancement of competency, where the hierarchical and linear arrangement makes it a good taxonomy for the field of analysis (Brabrand & Dahl, 2009). The development of students from the beginning could not become expertise formed from a very complex understanding, in SOLO there are levels at each level that students are expected to achieve (Potter & Kustra, 2012). The way students solve problems contained in the answer sheets can be used as material for classifying the quality of student responses in the SOLO taxonomy (Lian & Yew, 2012).

SOLO taxonomy provides the means to make a point quickly and spontaneously from students' conceptual understanding and to be able to see a view of progress in learning (Hodges & Harvey, 2003). The following are the five levels described in the SOLO taxonomy (Caniglia & Meadows, 2018):

1) **Pre-structural**: Students have very little information that is not even interconnected, so it does not form a unified concept at all and does not have any meaning,
2) **Uni-structural**: Students can simply respond to the questions given but cannot understand the responses given by students,
3) **Multi-structural**: Students who have the ability to respond to problems with several separate strategies. Many relationships that they can make, but the relationships are not right,

4) **Relational**: Students can break a unit into parts and determine how the parts are linked to several models and can explain the equality of those models, and

5) **Extended Abstract**: Students have mastered the material and understood the questions given so well that students can realize the concepts that exist.

Mereku (2004) is of the view that the students‟ mathematical working processes are as important as the outcomes of such working processes and their applicability. As a researcher, I also think that analysing pre-service teachers‟ working processes is one of the ways to know how they are thinking. In the context of Ghana, concepts of the mean are covered quite extensively in the school mathematics curriculum at various levels. Thus, Ghanaian pre-service mathematics teachers need to have an adequate foundation related to the conception of the mean involved in the enhancement of statistical literacy. However, the question is at what level should the foundation be related to the conception of the mean being? It is against this background that prompted the researcher to explore pre-service teachers‟ conceptual understanding of mean based on the SOLO taxonomy.

1.2 Statement of the Problem

Although the arithmetic mean is a significant concept encountered in our daily life and the domain of learning statistics, previous studies have demonstrated that students preferred its algorithm (solution in the algebraic or arithmetic form) for solving questions on the arithmetic mean (Cai, 1998, 2000; Enisoğlu, 2014; Uçar & Akdoğan, 2009). Problems associated with the arithmetic mean have been worked on by students for more than 100 years (Watson & Moritz, 2000). However different research studies have shown that students‟ comprehension involves various types of difficulties, showing that it is not so easy for students to understand basic notions associated with this concept (Enisoğlu, 2014). These difficulties are related to different aspects: understanding of the algorithm, understanding of the concept and its properties, use of representations and language, and ability to put forward arguments, among others. For example,

The foundation of the difficulties that students have with arithmetic mean lies in the implementation of algorithm-oriented teaching before enabling them to develop a conceptual understanding of this concept (Cai, 1998, 2000). Most students understand the mean as an "add-them-all-up-and-divide" algorithm (Zazkis, 2013). Moreover, many elementary and middle school mathematics textbooks have defined the mean as the way it is computed (Bremigan, 2003). It is also supported by the exercises and the examples elaborated which do not allow students to develop their understanding of the concept of the mean. Most of them are procedural problems where the students only use the formula when the data are given. Unfortunately, in Ghana, most teachers teach the concept of the mean traditionally, focusing on the computation but not the understanding of the concept. They tend to follow the computational definition and the problems provided in the textbooks without developing students‟ understanding of the concept.

According to Stohl (2005), better teaching of statistics requires better training for the teachers involved, because with no specific training they are likely to fall back on what are often erroneous beliefs and intuitions, which would then be passed on to their pupils, as was demonstrated in the study by Ortiz, Mohamed, Batanero, Serrano and Rodríguez (2006). This study is therefore designed to investigate pre-service mathematics teachers‟ conceptual understanding of the arithmetic mean.

1.3 Purpose of the Study

The purpose of the study was to investigate pre-service teachers‟ conceptual understanding of the arithmetic mean in relation to the SOLO taxonomy.

1.4 Objective of the Study

This study was guided by the following objectives: To:

1. determine pre-service teachers" performance on mathematical tasks on the Arithmetic mean.
2. investigate pre-service teachers" conceptual understanding of the Arithmetic mean.
3. determine pre-service teachers" level of conceptual knowledge of arithmetic mean with respect to the SOLO taxonomy.

1.5 Research Questions

Based on the objectives of the study, the study sought answers to the following research questions:

1. What are pre-service teachers, performance on mathematical tasks on the Arithmetic mean?
2. What conceptual understanding of the Arithmetic mean do pre-service mathematics teachers have?
3. What level of conceptual understanding with respect to the SOLO taxonomy do pre-service teachers have about the Arithmetic mean?

1.6 Significance of the Study

This study would contribute to the field of mathematics education and statistics education in several ways. First, it builds on and refines existing research aimed at modelling the development of statistical reasoning. Secondly, knowing the conceptual understanding of the arithmetic mean, which is a basic statistical concept, will better inform instructional decisions about teaching the concept.

The outcome of the present study will also inform school textbook writers to incorporate activities that go beyond the computation of the arithmetic mean if found necessary. Third, the study will provide insight into the connection between statistical knowledge (arithmetic mean) and its application in our everyday life

1.7 Delimitations of the Study

Even though the study focused on first-year students (Pre-service teachers) admitted into the faculty of science education of the UEW, only trained teachers in the departments of Mathematics, Chemistry and Physics and 250 students who came directly after SHS were selected for the study. The researcher believes that the trained teachers have already gone through some training and have done some further mathematics after their SHS and are in the field teaching this concept of the arithmetic mean already.

1.8 Limitations of the Study

Out of the sixteen (16) regions in Ghana, the research was conducted in only the Central Region. Three (3) departments in the faculty of science in UEW were selected for this study and this has limited the scope of the research. The consequence of this was that the generalization of the research findings was limited. This limitation was alleviated when students from SHS all over the 16 regions of Ghana were admitted into these 3 departments. This has enriched the sample used for the study in terms of Pre-service teachers" abilities, cultural and social backgrounds. The sample used, therefore, represents the characteristics of Ghanaian Pre-service teachers in any part of the country who had spent at least learned Arithmetic mean in core all elective mathematics at SHS.

Furthermore, the non-existence of information regarding the SOLO taxonomy in the Ghanaian Mathematics curriculum was also a limitation. The researcher was unable to draw from local examples and knowledge, therefore, limiting the number of questions at each level.

Finally, the present study looked at only performance and conceptual understanding of the Arithmetic Mean and did not include the connections between the ideas of Mean with other statistical ideas such as measures of dispersion. Hence, the findings of the present study were confined to conceptual understanding of the Arithmetic mean.

1.9 Organization of the study

This study is organized into five chapters. Chapter one motivates the research, including a discussion on the relationships between mathematics and statistics, and the importance of the arithmetic mean as a subject of study. The research objectives that guided the study are discussed in this chapter. Chapter Two reviews literature that relates to the concept of the mean and discusses the theoretical framework (SOLO taxonomy) that guided this study. Chapter Three is about the research methodology. It discusses the design for the study, population and sample, instruments, and the entire procedure of data collection and analysis. Chapter Four presents the results of the analysed data. Chapter five offers a discussion of the research findings, recommendations based on the research findings, and areas of the future study revealed by the literature review and research study.

CHAPTER TWO

LITERATURE REVIEW

2.0 Overview

This chapter primarily focuses on the literature review and the varied views on what other authors have written concerning the topic under study. The following are discussed: the theoretical framework, Statistics Education as a Discipline, Students" Understanding of Statistical Concepts, Pre-service teachers" Mathematics Education, Understanding Measures of central tendencies, and the concepts of the Arithmetic mean.

2.1 Theoretical Framework SOLO Taxonomy

The theoretical framework that underpinned this study is the "Structure of the Observed Learning Outcome" (SOLO) Taxonomy. The SOLO taxonomy provides systematic steps in describing how students perform when they grow in structural complexity when faced with various tasks (Silwana, Subanji, Manyunu, & Rashahan, 2021). According to Hasan (2017), this framework has been widely applied in various disciplines, especially in the field of Mathematics.

The SOLO Taxonomy describes levels of progressively complex understanding, through five general stages that are intended to be relevant to all subjects in all disciplines. In SOLO, understanding is conceived as an increase in the number and complexity of connections students make as they progress from incompetence to expertise. Each level is intended to encompass and transcend the previous level. The SOLO model was developed by Biggs and Collis in 1982. It was designed mainly as a means to measure students" cognitive ability in an academic learning context (Biggs

& Collis, 1982; Vallecillos & Moreno, 2002). It has been used to analyse the structure of students" mathematical thinking, understanding of mathematical concepts and problem-solving ability over a wide educational span from primary to tertiary levels (Reading, 1999; Vallecillos & Moreno, 2002; Panizzon, Callingham, Wright, & Pegg, 2007; Callingham, Pegg, & Wright, 2009).

The SOLO taxonomy can be used as a learning intervention to help students achieve various learning outcomes by incorporating thinking skills, strategies, rubrics, and tools. This intervention can demonstrate how students' learning has changed. According to Hasan (2017), the SOLO taxonomy can be used because;

- Simulates the stages of competency development in the cognitive domain,
- Formulating or releasing the results of learning,
- Determining objectives in teaching,
- Enabling the achievement of results, and
- Assessing learning

SOLO taxonomy enables students to make a point quickly and spontaneously based on their conceptual understanding, as well as to see a view of their learning progress (Claudia, Kusmayadi, & Fitriana, 2020). The SOLO taxonomy, which comprises thinking skills, methods, rubrics, and tools, can be utilized as a learning intervention to help students accomplish various learning outcomes. This intervention can demonstrate how pupils' learning abilities have changed (Hook & Mills, 2011). Pre-structural, uni-structural, multi-structural, relational, and extended abstract are the five major levels of the taxonomy. The first level of SOLO, pre-structural, is technically an exterior feature of the taxonomy, although the remaining four levels are divided into two categories: surface and deep comprehension. Surface understanding, which focuses on the accumulation of knowledge in quantity, is divided into Uni-structural and Multi-structural levels. The last two levels, relational and extended abstract levels, are categorized as deep understanding where integration and connection of knowledge occur during this stage and lead to a qualitative change in knowledge and ideas (Biggs, & Tang, 2011). These levels are further elaborated as follows:

- **Pre-structural**: Because the knowledge they are supposed to learn has not been taught, students do not yet grasp it. Students at this level have no understanding of how to obtain information about key concepts, organize those ideas in a meaningful way, or what those ideas' content or objective is. Students are considered ineffective at the pre-structural level because they have no prior knowledge of the subject (O'Neill & Murphy, 2010). Atherton (2013) claims that children only learn pieces of information with no logical connection or order. Students don't understand the information and use it in insensitive or irrelevant ways. They may have gathered bits and pieces of information, but it is still jumbled, unstructured, and unrelated to the issue or situation at hand (Brandbrand & Dahl, 2009).
- **Uni- structural**: Students have gained knowledge of an important part of the entire. They may be able to make easy and obvious connections, but they may struggle to develop connections between the idea's meaning, value, and significance, and their comprehension is incoherent and unsophisticated. Students can respond at this point by providing or identifying a relevant fact, but only in isolation. There is no extra background, explanation, or examination of other important aspects for that specific relevant element. According to Hattie and Brown (2004), children might understand one part of a task sequentially but see no relationship between facts or ideas. Only one

relevant component, such as listing, naming, and memorizing, is known, according to O'Neill and Murphy (2010). Students can choose one part of a work to focus on, but their understanding of the task remains fragmented and limited (HookED, 2014). According to Brandbrand and Dahl (2009), pupils might focus on a specific aspect and draw evident connections. Reciting, performing simple instructions/algorithms, paraphrasing, identifying, naming, and counting are some of the operations they can execute. According to Atherton (2013), students are capable of making easy and obvious connections, but they do not understand the relevance of these relationships.

- **Multi-structural**: Students grasp numerous important components of a larger concept or a group of related concepts. Despite their inability to comprehend the arrangement and relevance of the ideas, they are able to draw connections between them, albeit in a less integrated fashion. They can recognize a variety of facts and characteristics, but they are unable to build a deeper connection between these concepts. There is no extra explanation, background, or consideration of other aspects related to that particular relevant factor, as there is at the unistructural level. Students can pick up two or more components of a task or understand them sequentially, according to Hattie and Brown (2004), but they do not see any correlation between them. Atherton (2013) argued that several aspects of the task are known but their relationships to each other and the whole are missed. HookED (2014) argued that students may be able to make a number of connections but they have yet to identify the meta-connections between these aspects and their significance as a whole.
- **Relational**: Students can put concepts together as a whole, understanding relationships and connecting them. They can recognize certain connections

between theory and practice, as well as the importance of ideas. They can also recognize patterns in thoughts and are likely to be able to apply these concepts to new contexts. Students can now present explanations that connect and integrate essential elements, as well as convey specific facts under abstract ideas that may include prior knowledge that provides context and support for their explanations. According to Brandbrand and Dahl (2009) students may grasp relationships between various components and how they relate to one another as a whole. Their understanding can be analogically compared to how the many trees form a forest. Students may have gained the ability to compare, relate, analyse, apply theory, explain cause and effect relationships, and perform a variety of other tasks at this level. Students may integrate multiple parts as a whole and generate a coherent structure and meaning, according to Hattie and Brown (2004). According to O'Neill and Murphy (2010), information is organized into a structure that includes skills like analysing, explaining, and integrating. According to Atherton (2013), pupils can now comprehend the importance of the pieces in connection to the whole. Students link and integrate the aspects which then contribute to a deeper and more coherent understanding of the whole (HookED, 2014)

- **Extended Abstract**: Students can apply and adapt their information in different settings by organizing, judging, and generalizing their entire learning. They can make connections between different ideas and use these connections to better understand them. They may analyse surface and embedded assumptions, consider alternative options, and develop their academic learning as they connect with the world by combining it with real experience. Students can perform a variety of abilities at this level, including reasoning ahead, anticipating possibilities, generating multiple connections, and bringing in (or developing) principles to apply their knowledge in new contexts (Potter & Kustra, 2012). Students generalize information into a new area, according to O'Neill and Murphy (2010), which includes multiple skills such as forecasting, reflecting, and speculating. According to Hattie and Brown (2010), pupils abstract the coherent whole to a higher level of abstraction. Students may be able to generalize structure outside the existing context, perceive it from numerous perspectives, and finally transfer concepts to new areas, according to Bradbrand and Dahl (2009). The leaners might be able to generalize, hypothesis, criticize, theorize, and so on. According to Atherton (2013), students may make connections both within and outside of the subject area, as well as generalize and transfer concepts and ideas from one situation to another. According to Hook and Mills (2011), students can rethink and view the understanding they have obtained on a different conceptual level, and utilize it as the basis for prediction, generalization, reflection, or the production of new understanding.

SOLO taxonomy, according to Potter and Kustra (2012), can scaffold learning by allowing students to delve deeper into their understanding consistently and sequentially until they have a thorough understanding of what they are learning. They can efficiently progress to a higher level of thinking skills with clear outcomes thanks- to the model's interconnected and well-structured levels. According to Hattie and Brown (2004), the SOLO taxonomy allows students to focus on specific target knowledge and expand their perception of content knowledge to the point where they can apply higher-order thinking skills at the end of the SOLO level. The SOLO taxonomy has been widely used as a diagnostic tool for evaluating learning outcomes, but its potential as a teaching tool has yet to be discovered. It can be used to clarify expectations and mechanisms for interpreting materials, as well as to provide opportunities for these skills to be developed to foster a learning environment (Clear, Whalley, Lister, Carbone, Sheard, Simon and Thompson, 2008).

Using the SOLO taxonomy to create specifically intended learning outcome statements is greatly aided by using verbs that correspond to the SOLO taxonomy. Figure 2.1 shows a visual representation of the levels, along with some verbs that are typical of each level. The verbs in the stairwell are broad, indicating what students must be able to do to demonstrate achievement at each level. When teaching a course, SOLO is extremely helpful in determining the levels of understanding we want our students to achieve. In doing, the verbs are extremely important.

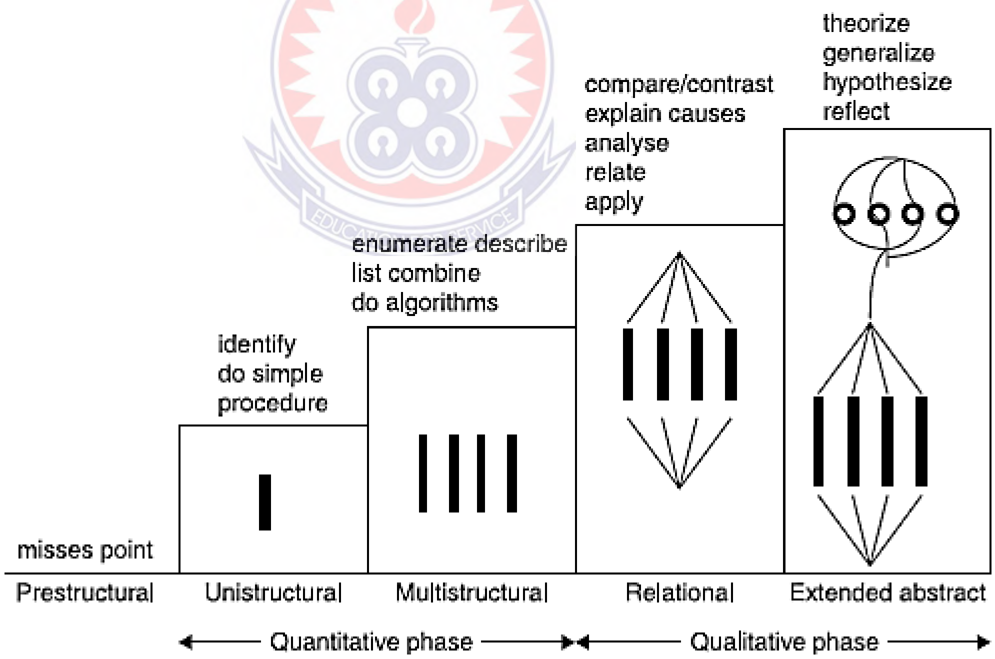


Figure 2.1 *A hierarchy of verbs that may be used to form intended learning outcomes*

2.1.1 SOLO in Teaching and Learning Activities

Although the original purpose of SOLO taxonomy is to determine the cognitive level of students" responses to questions based on the five levels, it has slowly gained prominence as a model for designing activities and questions to assist deep learning (Hunt, Walton, Martin, Haigh & Irving, 2015). Other approaches have been developed from SOLO taxonomy to promote deep learning such as SOLO based approach to teaching (Martin, 2011; Hunt et al., 2015), Learning Challenge (Nottingham, 2017) and SOLO Taxonomy based teaching strategies (El Farra & Rashid, 2013). The approach used in this study focused on the role of SOLO taxonomy in designing questions and analysing students" responses that nurture deep understanding and critical thinking skills on the arithmetic mean.

2.1.2 SOLO Based Approach to Teaching

Martin (2011) proposed the use of SOLO taxonomy in science class where teachers can plan activities that help students attain target levels of learning which he categorized into three manageable levels:

i. One/many ideas: an example of this criterion is the act of identifying and describing different types of heat transfer (uni-structural/multi structural),

ii. Relate: the process of analysing data and presenting it in the graph is classified as "relate" (Relational),

iii. Extend: an example of this criterion is the process of justifying how the use of natural materials in a building reduces certain types of heat transfer. (Extended abstract).

Martin (2011) applied SOLO in e-learning where teachers designed activities according to SOLO levels that can be accessed online by the students. It was argued that this approach increased students" motivation and autonomy as well as teachers" opportunity to provide individual attention especially when the approach could possibly cater to the diverse needs of students through differentiated tasks that were aligned with SOLO levels.

Hunt et al. (2015) conducted a study on the use of SOLO in e-learning on two subjects, science and social study at a school in New Zealand. The purposes of the research were to explore teachers‟ perceptions of the training course they took to increase their competence in using SOLO taxonomy and to study the combined effects of SOLO and e-learning.

The survey revealed that most of the teachers agreed that SOLO could help students develop deep thinking and many of them believed that SOLO could be applied in any subject. The majority of them thought that SOLO taxonomy was useful in their teaching practice. According to the survey and interview with the teachers regarding the approach, the most common forms of e-learning that the teachers were able to apply comprised PowerPoint presentations, video clips, still images and Webquest. Other software such as Edmodo, Microsoft OneNote and Google Docs were also used but to a lesser extent.

This study showed that the delivery of SOLO taxonomy through an online platform was effective. The variety of web tools used to support the application of SOLO taxonomy in the classroom allows for differentiation in the learning taking place. Thus, this study incorporated the element of the digital tool by using Google. Form as a platform for the intervention to take place. The students were assigned to one computer each for them to read the text and answer the SOLO questions presented on Google-Form. The application of computers and the availability of the internet allowed students to search for information that could help them gain a better understanding of what they read.

The focus of the approach was to design activities that enabled students to reach particular levels of learning in SOLO. It indicated the potential use of SOLO in developing and designing tasks to help students progress from their current level to the target level of learning. The use of SOLO is not just limited to assessments. This study aimed to explore the plausibility of SOLO-based questions in enabling students to develop better-thinking skills, particularly in statistics.

2.1.3 Critics of SOLO Taxonomy

Those opposed to the SOLO taxonomy argue that such a structuralist view of human behaviour is an oversimplification, according to a UNICEF report from 2007. By questioning the underlying assumptions, O'Reilly (cited in UNICEF 2007 report) casts doubt on the legitimacy of identifying a "particular pathway through Mathematics." The SOLO taxonomy, he explained, starts with the assumption that mathematical understanding is hierarchical, and the model's job is to reveal those hierarchies. Biggs and Collis (1982) argue, however, that the assumption is explicit and should be tested within the context of a study (cited in UNICEF, 2007).

In addition to the UNICEF 2007 report, O‟Reilly argues that hierarchies are products of "contextual, temporal and societal factors," and that small changes in the items used in the study may affect the identification of hierarchies. The UNICEF (2007) report agrees with this argument but states that SOLO taxonomy cannot be seen as a "Grand theory", as it has been evolving since it was originally published in 1982. Therefore despite the critics of SOLO taxonomy the researcher used it because it has several features which make it attractive as a theoretical framework. Its usage in Denmark, Malaysia and Pakistan and various disciplines shows that it is useful for developing instructional and assessment tools to assist teachers.

2.1.4 Teachers' Perception Regarding SOLO Taxonomy

Kayani, Ajmal, and Rahman (2010) investigated teachers' perceptions of their examination system using the SOLO taxonomy in thirty districts across Punjab (Pakistan). This was after five years of using the SOLO taxonomy to assess their grade five students (2005-2010). The study's sample included 360 teachers, and the results revealed that their SOLO taxonomy-based examination system improved the grade five examination system's reliability and validity, as well as students' creative thinking, reading, writing, and comprehension skills. Teachers were also pleased with the use of SOLO taxonomy as an assessment tool for the grade five examination system. The teachers stated unequivocally that:

1. SOLO approach increased students" learning.
2. SOLO approach increased the students" reading skills.
3. SOLO increased students" creative thinking.

In 2007, all Danish university curricula were reformulated to explicitly state course objectives due to the adoption of a new Danish national grading scale which stipulated that grade were to be given based on how well students meet explicit course objectives. The Faculties of Science at the University of Aarhus and the University of Southern Denmark interpreted „course objective" as „Intended Learning Outcomes (ILO) and systematically formulated all such competencies using the SOLO Taxonomy. Brabrand and Dahl (2009) investigated how the formulation of ILOs using the SOLO Taxonomy gives information about competence progression, educational traditions, and the nature of various science subjects.

Also, Brabrand and Dahl (2009) specifically looked at the verbs used to describe the competencies in the curriculum and via the SOLO Taxonomy. This helped them to investigate how much of what level of the SOLO taxonomy the curriculum stresses at the university. Their investigation revealed that Science, History and Education are indeed the subjects with the most level five (5) of SOLO competencies attained and, that mathematics is the subject with the least level 5 of the SOLO taxonomy. About students‟ progression, the use of the SOLO Taxonomy showed that competency progression in terms of SOLO does indeed exist, except for mathematics, from undergraduate to graduate level.

2.1.5 Application of SOLO Taxonomy

The Structure of Observed Learning Outcomes (SOLO) Taxonomy has not been thoroughly explored despite its growing popularity as an approach that can facilitate the development of critical thinking skills (Lloyd & Mukherjee, 2013). There are four main applications of SOLO taxonomy: classification of thinking levels (Callingham et al., 2009; Laisouw, 2013), specification of target content knowledge, facilitating the development of critical thinking skills (Byrnes, 2001; Potter & Kustra; 2012), delivery of differentiated instructions (Tomlinson, 2003) and increase the explicitness of learning (Hook & Mills, 2011).

The application of the SOLO Taxonomy to know the quality of student responses and fault analysis in accordance with Collis with several advantages of the SOLO Taxonomy are as follows; (a) easy and simple tool to determine the student's response levels to math questions; (b) easy and simple tool for categorizing errors in solving mathematics problems; (c) easy and simple tool to prepare and determine the level of difficulty or complexity of mathematics problems Mallisa (2015).

2.2 Statistics Education as a Discipline.

Research in statistics education is a relatively new field (Rossman & Shaughnessy, 2013). It has focused in recent years on the increase of statistical reasoning in one cognitive area (Slauson, 2008; Zieffler & Garfield, 2013; Reading &

Reid, 2006;). Garfield (2002) stated that statistical thinking and reasoning should be the desired outcomes for a course; however, "no one has yet demonstrated that a particular set of teaching techniques or methods will lead to the desired outcomes" (p.10).

The Guidelines for Assessment and Instruction in Statistics Education (GAISE) written by Franklin, Bargagliotti, Case, Kader, Scheaffer, Spangler, (2015) stated that statistics has "become a key component of the K-12 mathematics curriculum" (p. 3). Society is becoming more and more data-driven, and students" fundamental understanding of statistics is integral if they are to be informed decision-makers in a democratic society and prepared to enter the new statistical driven workforce (Franklin et. al, 2015). The difficulty students have in dealing with variation and context in real-world data makes the attention of statistics education starkly different from mathematics (Delmas, 2004).

2.2.1 Learning Statistics

NCTM"s (2014) Principles to Action: Ensuring Mathematical Success for All (PtA) stated that setting clear learning goals sets the stage for everything within mathematics instruction. NCTM"s (2000, 2014) vision for the learning of mathematics requires students to understand and actively build new knowledge from experience and prior knowledge. NCTM (2014) reiterated support for this vision of learning through the principles of *Adding It Up* (Kilpatrick, Swafford, & Findell, 2002). This vision of mathematical learning proficiency has five intertwined strands: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition (Kilpatrick, et al 2002). Each of these strands is interconnected and develops upon and with one another. These strands are all key ingredients in a mathematically proficient student.

Just as each of these strands is essential for mathematical proficiency (Kilpatrick, et al 2002; NCTM, 2014), the learning of statistics with literacy, reasoning, and thinking should be critical elements within the statistics classroom. Literacy, reasoning, and thinking are all key components of what makes a statistically proficient student. The following sections look at how the concepts of statistical literacy, reasoning, and thinking defined in statistics education research play a fundamental role in the learning experiences in statistics classrooms

2.3 Distinguishing between Statistical Literacy, Reasoning and Thinking

Although statistics is now viewed as a unique discipline, statistical content is most often taught in the mathematics curriculum (at elementary and secondary school level) and in departments of mathematics (tertiary level). This has led to exhortations by leading statisticians, such as Moore (1998), about the differences between statistics and mathematics. These arguments challenge statisticians and statistics educators to carefully define the unique characteristics of statistics, and in particular, the distinctions between statistical literacy, reasoning and thinking (Ben-Zvi & Garfield, 2004). Garfield & Ben-Zvi, 2007 present the following definitions:

Statistical literacy is a key ability expected of citizens in information-laden societies and is often touted as an expected outcome of schooling and as a necessary component of adults‟ numeracy and literacy. Statistical literacy involves understanding and using the basic language and tools of statistics: knowing what basic statistical terms mean, understanding the use of simple statistical symbols, and recognizing and being able to interpret different representations of data (Rumsey, 2002). There are other views of statistical literacy such as Gal‟s (2002), whose focus is on the data consumer: Statistical literacy is portrayed as the ability to interpret, critically evaluate, and communicate statistical information and messages. Gal (2002) argues that statistically literate behaviour is predicated on the joint activation of five interrelated knowledge bases (literacy, statistical, mathematical, context and critical), together with a cluster of supporting dispositions and enabling beliefs.

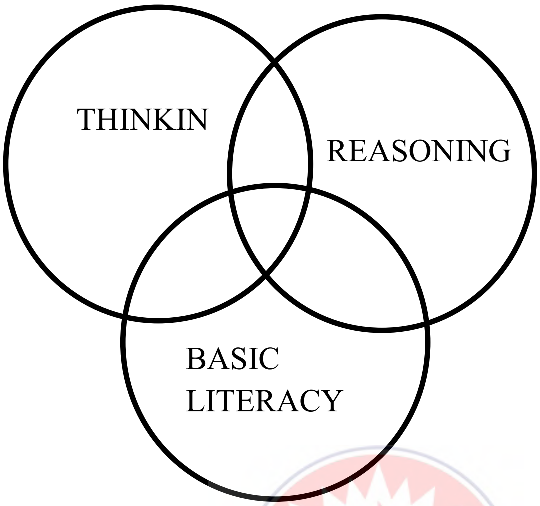


Figure 2.2. *The overlap and hierarchy of statistical literacy, reasoning and thinking*

Statistical reasoning is the way people reason with statistical ideas and make sense of statistical information. Statistical reasoning may involve connecting one concept to another (e.g. centre and spread) or may combine ideas about data and chance. Statistical reasoning also means understanding and being able to explain statistical processes, and being able to interpret statistical results (Garfield, 2002).

Statistical thinking involves a higher order of thinking than statistical reasoning. Statistical thinking is the way professional statisticians think (Wild, 2006). It includes knowing how and why to use a particular method, measure, design or statistical model; a deep understanding of the theories underlying statistical processes and methods as well an understanding of the constraints and limitations of statistics and statistical inference. Statistical thinking is also about understanding how statistical models are used to simulate random phenomena, understanding how data are produced to estimate probabilities, recognizing how, when, and why existing inferential tools can be used, and being able to understand and utilize the context of a problem to plan and evaluate investigations and to draw conclusions (Chance, 2002).

Statistical literacy, reasoning and thinking are unique areas but there is some overlap and a type of hierarchy, with statistical literacy providing the foundation for reasoning and thinking (see Figure2 .2). A summary of additional models of statistical reasoning and thinking can be found in Jones, Langrall, Mooney, & Thornton (2004).

Statistics is vigorously gaining importance and recognition in today‟s society. Statistics is a central tool in moving science, economics, politics, schools, and universities forward. Quantitative information is omnipresent in media and in the everyday lives of citizens worldwide. Data are increasingly used to add credibility to advertisements, arguments, or personal and professional advice. Therefore, there is a growing public and policy consensus that being able to provide reliable and persuasive evidence-based arguments and critically evaluate data-based inferences are crucial skills that all citizens of the twenty-first century should have.

All students consequently must become statistically literate (Gal, 2002) and be able to reason statistically even at an informal level as part of their compulsory and lifelong education (Watson, 2006). It is not surprising, therefore, that attention has accelerated over the last decade to the development of data-based reasoning by the statistics, science, and mathematics education communities, as well as policymakers and the general public worldwide. Reflecting this essential need to improve students‟ ability to think statistically, statistical literacy and reasoning are becoming a necessary and important area of study that involves distinctive and powerful ways of thinking in nearly every field (Watson, 2006); however, the challenges of teaching and learning statistics are numerous.

2.4 The Challenge of Learning and Teaching Statistics

Despite the increase in statistics instruction at all educational levels, historically the discipline and methods of statistics have been viewed by many students as a difficult topic that is unpleasant to learn (Watson, 2006). Statisticians often joke about the negative comments they hear when others learn of their profession. It is not uncommon for people to recount tales of statistics as the worst course they took in college. Many research studies over the past several decades indicate that most students and adults do not think statistically about important issues that affect their lives. Researchers in psychology and education have documented the many consistent errors that students and adults make when trying to reason about data and chance in real-world problems and contexts (Garfield & Ben-Zvi, 2008). In their attempts to make the subject meaningful and motivating for students, many teachers have included more authentic activities and the use of new technological tools in their instruction.

However, despite the attempts of many devoted teachers who love their discipline and want to make the statistics course an enjoyable learning experience for students, the image of statistics as a hard and dreaded subject is hard to dislodge. Currently, researchers and statistics educators are trying to understand the challenges and overcome the difficulties in learning and teaching this subject so that improved instructional methods and materials, enhanced technology, and alternative assessment methods may be used with students learning statistics at the school and college level (Garfield & Ben-Zvi, 2008).

Garfield and Ben-Zvi (2007) listed some of the reasons that have been identified to explain why statistics is a challenging subject to learn and teach. Firstly, many statistical ideas and rules are complex, difficult, and/or counterintuitive. It is therefore difficult to motivate students to engage in the hard work of learning statistics. Secondly, many students have difficulty with the underlying mathematics (such as fractions, decimals, proportional reasoning, and algebraic formulas) and that interferes with learning the related statistical concepts. A third reason is that the context in many statistical problems may mislead the students, causing them to rely on their experiences and often faulty intuitions to produce an answer, rather than select an appropriate statistical procedure and rely on data-based evidence.

Finally, students equate statistics with mathematics and expect the focus to be on numbers, computations, formulas, and only one right answer. They are uncomfortable with the messiness of data, the ideas of randomness and chance, the different possible interpretations based on different assumptions, and the extensive use of writing, collaboration and communication skills. This is also true of many mathematics teachers who find themselves teaching statistics.

2.5 Mathematics Proficiency

The mathematics curriculum in Ghana has been categorized into core mathematics and elective mathematics. The core mathematics is offered to all the senior high school students while the elective mathematics is offered to selected students offering particular programs. Statistics is a topic treated in both core and elective mathematics. The statistics taught under elective mathematics cover a broader perspective and it is much more detailed, compared to that of core mathematics (CRDD Teaching syllabus core mathematics, 2010; & CRDD Teaching Syllabus elective mathematics, 2010).

The core mathematics curriculum has been organized around eight broad areas/topics and two profile dimensions. A dimension is a unit for describing a particular learning behaviour. In other words, the measure of students‟ actions during learning and the use of content. More than one such action description constitutes a profile dimension (CRDD Teaching syllabus core mathematics, 2010). The student‟s ability to recall what has been taught and apply the knowledge acquired which is termed Knowledge and understanding (KU) and application of knowledge (AK) form the main components of the profile dimensions in the curriculum. National Research Council, & Mathematics Learning Study Committee. (2002) and Findell (2002) stated that conceptual understanding, procedural fluency, logical reasoning, ability to formulate and represent mathematical problems enable students to become good mathematics learners.

2.6 Mathematical Understanding

Skemp (1978) posited the existence of two types of mathematical understanding that could be generated by mathematics learning and teaching in schools: instrumental and relational. For Skemp (1978), instrumental understanding was the product of rote learning through rules and theorems and specific applications. Conversely, relational understanding was the product of a learner's involvement with mathematical objects, situations, problems, and ideas.

At each stage in a relational learning cycle, the learner is personally involved with the available data. The data are products of the learner's investigations. In contrast, the data available in instrumental learning are given to the learner to memorize by some external source (usually the teacher, textbook, or computer). Skemp (1978) believed many students possessed an only instrumental understanding of numerous mathematical concepts, having a collection of unrelated procedures for retrieval rather than an appropriate conceptual schema.

A significant portion of PSTs‟ methods curriculum at this university involves examining and discussing Skemp‟s (1978) seminal work of relational and instrumental understanding. As such, Skemp‟s theory was examined with PSTs as part of their participation in the study. Furthermore, PSTs were asked to develop (or find) and implement tasks they believed would engage their students in relational understanding during student teaching.

To provide an analysis of mathematical tasks that PSTs use as they plan their lessons and select problems, the authors required students to utilize Smith, Bill, & Hughes (2008) "Thinking Through a Lesson Protocol" and Smith and Stein‟s (2011) "Five Practices for Orchestrating Mathematical Discussions." Using these resources, PSTs were to implement the following criteria in designing tasks:

- Lesson activities should provide opportunities for all students to be engaged in the exploration, discovery, application, practice, and/or discussion of the mathematical ideas in the lesson. Some lesson activities should provide opportunities for students to make sense of mathematical ideas, procedures, theorems, etc.
- PSTs‟ should justify that the cognitive demands of tasks are appropriate for achieving goals/objectives by giving attention to ensuring the learning opportunities are developmentally appropriate. PSTs should use support their choice of tasks with appropriate outside resources.
- The PSTs should use problem-solving and provide solution strategies for the lesson task(s). They should identify possible student strategies and how they connect to the mathematical goals/objectives for the lesson. Through this

process, the PST‟ should pay attention to students‟ conceptual understanding and help students develop and test conjectures.

2.7 Conceptual Understanding

One of the main aims of mathematics is to solve a problem in a systematic way so that similar problems can be solved more easily in the same way. Mathematics is very important every day, it‟s used to solve problems in such areas as astronomy, business, computer science, economics, navigation, physics, and statistics. Mathematics equips students with a uniquely powerful set of tools to understand and change the world. These tools include logical reasoning, problem-solving skills, and the ability to think in abstract ways. The common core standards in mathematics stress the importance of conceptual understanding as a key component of mathematical expertise. Conceptual mathematics understanding is the knowledge that involves a thorough understanding of underlying and foundation concepts behind the algorithms performed in mathematics. Thus, it involves a situation where students are allowed to make choices and apply their understanding through active engagement (Andamon, & Tan, 2018).

Conceptual understanding has been described as the ability of one to know the facts and the why of it (Frederick & Kirsch, 2011). Conceptual understanding goes beyond just responding to the test items. The essence of it is to probe into students‟ results more than just the correct answer. Wiggins (1998) explained conceptual understanding as the acquisition of enough concepts and skills to reflect, reassess and reformulate the already acquired knowledge (that is when knowledge acquired is linked up in a rightful way to already existing knowledge). According to Hiebert (2013), a student‟s ability to establish a relationship between pieces of information is an indication of attaining conceptual understanding. He further explained that conceptual understanding can be developed through the student‟s ability to establish a relationship between the old knowledge acquired and the new knowledge being acquired. According to Bruner (1961), conceptual understanding is developed through discovery learning. Kilpatrick et al, (2002) explanation outlines and summarises what other researchers have described conceptual understanding to be. According to them, it constitutes (a) comprehension of mathematical concepts (b) operations or processes and (c) relations. According to Skemp (1978), the likelihood of a concept becoming part of students with clear understanding is certain than those who memorized a procedure. In other words, developing a conceptual understanding of a concept is better retained and applied than memorizing it.

NCTM describes that one has conceptual understanding when they provide evidence that they can:

a) recognize, label, and generate examples of concepts;
b) use and interrelate models, diagrams, manipulative, and varied representations of concepts;
c) identify and apply principles;
d) know and apply facts and definitions;
e) compare, contrast, and integrate related concepts and principles;
f) recognize, interpret, and apply the signs, symbols, and terms used to represent concepts

2.8 Performance

A nation's educational quality may be determined by the quality of its teachers. Employing experienced qualified teachers in all schools is the most important factor in improving student performance in mathematics (Abe and Adu, 2013). The interlinking processes used in mathematics classroom instruction determine math performance. Aside from the processes, how students perceive Mathematics and how the teacher presents it always has an impact. This is usually compared to how the teacher instructs rather than how the students learn. This conflict is one of the issues that must be resolved because mathematics performance is a result of both.

As stated by Andamon and Tan, 2018, the teachers" role in the teaching and learning process is very vital in the sense that students" mathematics performance will depend on how the teacher makes the instruction meaningful and interesting. No matter how abstract and difficult Math is, making the instruction dynamic and open for communication will make it simpler. This focuses on the teaching concerns such as method or strategies, educational tools used and even the environment created by the teacher. Mathematics performance, as claimed, is affected by how the students perceived the classroom instruction.

According to Wiggins (1998), performance is how a student did in the light of what he attempted in a test. Performance measures/gauges what the student has absorbed against the outcome and the real output of a student against the benchmark. In assessing the student"s performance, there must be a line of distinction between an optional and what is mandatory in a student"s work.

2.9 Interest

The empirical evidence from some mathematics research emphasizes that interest is one of the important elements and prerequisites for students to develop a conceptual understanding of a subject (Shabani, 2006). Shabani (2006) described interest as an incentive that instigates students" activity power. The interest developed by students enhances the understanding of the material learned and its application (Shabani as cited in Khayati, & Payan, 2014). A study conducted by Swarat, Ortony and Revelle (2012 revealed that genuine interest is a vital component of scientific literacy. The results from the study indicated that practical activities arouse students‟ interest in learning. Students‟ interest developed is not only needed for a career but also an important constituent of scientific knowledge.

2.10 Mathematics Education in Ghana

The importance of mathematics in the development of a country cannot be underestimated as it plays a major role in the economy and the social life of its people. Due to its importance, the government of Ghana is committed to ensuring the provision of high-quality mathematics education. Despite government efforts, learning mathematics has not undergone much change in terms of how it is structured and presented and among other reasons has resulted in consistently low achievement levels among mathematics students in high schools (Mullis, Martin, & Foy, 2008; Ottevanger, Van den Akker, & de Feiter, 2007). Ottevanger et al. (2007) indicated that the most frequently used strategy in mathematics classrooms is the teacher-centred (chalk and talk) approach in which teachers do most of the talking and intellectual work, while students are passive receptacles of the information provided. According to Ottevanger et al. (2007), this type of teaching is heavily dominated by teachers (while students are silent), involves whole-class teaching, lots of notes being copied, and hardly any hands-on activities. In most instances, teachers rush to cover all the topics mechanically to finish on time for examinations rather than striving for in-depth student learning (Ottevanger et al., 2007).

2.10.1 Pre-service Teachers' Mathematics Education in Ghana

Asante and Mereku (2012) have indicated that "the low standard of Mathematics proficiency among pupils and students alike has persisted for decades and test information has consistently indicated problems in the way students learn" (p. 23). They argue that teachers need to know and understand the topics and procedures that they teach because it is for this reason that Teacher Education and policymakers of Teacher Education have designed the Diploma in Basic Education (DBE) Mathematics course to include Content Knowledge. It is, therefore, necessary to look at the kind of training and preparation being given to PTs in Mathematics. This is because "the issue of providing quality education to pupils is directly related to the quality of teachers in the system" (Ampiah, 2010, p.3).

The Senior High School (SHS) mathematics curriculum in Ghana focuses on attaining one crucial goal: to enable all Ghanaian young persons to acquire the mathematical skills, insights, attitudes and values that they will need to be successful in their chosen careers and daily lives (MOESS, 2007). This curriculum is based on the premise that all students can learn mathematics and that all need to learn mathematics. The student is expected at the SHS level to develop the required mathematical competencies to be able to use his/her knowledge in solving real-life problems and secondly, be well equipped to enter into further study and associated vocations in mathematics, science, commerce, industry and a variety of other professions (MOESS, 2007). The rationale of the curriculum has therefore a lot of implications on teaching strategies and the preparation of mathematics teachers for SHS"s

In Ghana, the University of Education, Winneba (UEW) and the University of Cape Coast (UCC) are among other institutions that offer mathematics teacher education for SHSs. These two universities are institutes for higher education that have the specific task to prepare teachers for the SHSs.

The main route in teacher education at both UEW and UCC is the Bachelor of

Education qualification of 4 years duration. Three main components are present in these programs: subject content courses, education courses and teaching practice. The education courses are further sub-divided into general ones and subject-specific ones (i.e. for individual school subjects, or categories of subjects like science). The latter is taught in the science and mathematics education departments and denoted as science or mathematics pedagogy courses.

A major difference in the programs between the two universities lies in the fact that most content in UCC is taught by the Faculty of Science, whilst at UEW this takes place in the Department of Mathematics Education. The mathematics content courses (which cover the SHS curricula) at the first and second-year undergraduate levels are the main basis for teacher education students.

Two main problems can be distinguished that put the quality of the programs under pressure: reduced opportunities for interaction between lecturers and individual students (as a result of the fast expansion of student numbers in universities) and lack of practical orientation. The latter has roots in the educational tradition of the Ghana education system which emphasizes teacher-centred exposition as a main educational method (Adu-Gyamfi & Smit, 2007). This research was conducted within the context of the teacher education program at UEW. The content aspect of Arithmetic Mean for Pre-service teachers offering Mathematics program is taught in the first-year first semester under the course titled "Probability and Statistics I" (MATD 113).

2.11 The Arithmetic Mean

The notion of arithmetic mean is central to the study of statistics. It has important implications in a variety of areas that surround our daily lives, such as meteorology, medicine and agronomics, to mention just a few. It is also an important concept for informed citizens (Zazkis, 2013). Most students encounter this concept in their daily lives before receiving formal training in statistics (such as those in height, age, and score means) (Bütüner, 2020). Bütüner, 2020 emphasize the importance of understanding measures of central tendency, including the arithmetic mean as the most commonly occurring measure and its necessity in shaping a statistically literate society

Different from many of the other descriptions of average, the arithmetic mean has uses in statistics beyond the suggestion of central tendency. It is utilized, for example, in calculating other statistics such as the standard deviation, creating formulas for distributions such as the Poisson and normal, finding confidence intervals, and testing hypotheses (Kilpatrick et al, 2002).

Franklin et al., 2015, are also of the view that the arithmetic mean can also inform or model concepts outside of statistics. In a physical sense, the arithmetic mean can be thought of as a centre of gravity. From the mean of a data set, we can think of the average distance the data points are from the mean as standard deviation. The square of standard deviation (variance) is analogous to the moment of inertia in the physical model.

The cross-disciplinary nature of the arithmetic mean makes it conceptually constructive in many disciplines of study, including statistics, mathematics, and physics, and its use as a statistical tool makes it omnipresent in educational, vocational, and recreational settings. The arithmetic mean"s diversity has fostered research aimed at finding an understanding of how students arrive at their knowledge base for the arithmetic mean and the instructional techniques that promote its conceptual learning (Marnich, 2008).

The arithmetic mean is one of many different kinds of averages used to describe the centre or representative value of a data set. This seemingly simple calculation is a relatively complex concept that is most often developed as an “add-them-up-and-then-divide” mathematical procedure, rather than as a statistically representative concept (Konold &Higgins, 2003).

Although the arithmetic mean is a significant concept that we encounter in the daily life and in the domain of learning statistics, previous studies demonstrated that students preferred the arithmetic mean algorithm (solution in the algebraic or arithmetic form) for solving questions on the arithmetic mean (Cai, 2000; Enisoğlu, 2014; Armah, 2017, Saidi & Siew, 2019 and Bütüner, 2020). The mean is not as simple as the algorithm. It is interrelated with the concepts of centre and spread. Many studies have shown that teaching-learning this concept is easy, but understanding causes difficulties. The understanding of the mean is intimately related to the comprehension of the properties which according to Strauss and Bichler (1988) are:

a) the mean is located between the extreme values ($minimum\ value \leq average \geq maximum\ value$);
b) the sum of the deviation from the mean is zero ($\sum(Xi - \bar{x}) = 0$);
c) the mean is influenced by each and by all the values ($\bar{x} = \frac{Xi}{n}$);
d) the mean does not necessarily coincide with one of the values which are composed by it.
e) the mean maybe a number that does not have a correspondence with the physical reality (for example, the mean number of children per couple can be 2.3);
f) the calculation of the mean takes into consideration all the values including the negative and zero;

g) the mean is a representative value of the data from which has been calculated. In spatial terms, the average is the value that is closer to all the values.

Despite the importance of the arithmetic mean, results of studies dealing with this topic show limited conceptual knowledge of the concept of average or arithmetic mean (Watson, 2006). There are indications that understanding of arithmetic mean increases with students‟ age. Most students seem to know the computational algorithm and manage problems with arithmetic mean by applying algebraic skills. Students are more able to find results with „adding all and dividing‟, but are less successful in the reverse operation when they have to find an unknown value in a set of data where the average is given (Sirnik & Kmetiĉ, 2010).

The common conclusion of studies points out that the concepts of the mean are taught as rules or computational algorithms, while the learning situations related to understanding the concepts of average inappropriate everyday context are less frequent (Sirnik & Kmetiĉ, 2010). Cruz (2006) and Mousoulides (2006) emphasize the importance of solving open problems and suggest that some types of open questions can be used to examine students‟ ideas about the concept of the arithmetic mean. Studies have shown that pre-service mathematics teachers have limited conceptual understanding of the arithmetic mean and are unable to relate it to real-life situations (Estrada, 2007, Zazkis, 2013, Armah, 2017).

2.11.1 Empirical Review

Over the past few years, the arithmetic mean has been widely introduced into educational curricula in various countries, given its importance in various social spheres and the fact that it is a basic concept for the study of other subjects (Assagaf, 2014). Problems associated with the arithmetic mean have been worked on by students for more than 100 years (Watson & Moritz, 2000). However, different research studies have shown that students‟ comprehension involves various types of difficulties, showing that it is not so easy for students to understand basic notions associated with this concept. These difficulties are related to different aspects: understanding of the algorithm, understanding of the concept and its properties, use of representations and language, and ability to put forward arguments, among others. For example, Zazkis (2013) found that even university students can fail to take into account frequencies when solving average problems.

Callingham (1997) surveyed 100 pre-service and 36 in-service teachers regarding four problems involving averages. The results showed that teachers provide relatively good solutions to the first three questions (one about calculating the mean from a set of data and the other two involving the comparison of two data sets) using bar charts. However, they had more difficulties with the fourth problem, which required them to determine the weighted mean from a set of data. In this case, only 58% of the teachers responded correctly. Regarding the problems in which the data were presented graphically, it seems that the teachers based their answers on numerical arguments rather than solely on the appearance of the data.

Begg and Edwards (1999) studied 22 in-service and 12 pre-service teachers and found that the majority of them were not familiar with the mathematical definitions of the terms mean, median and mode. In terms of their understanding of these measures, teachers were clearer about the meaning of the mean than they were about the median and mode.

A study by Leavy and O‟Loughlin (2006) with 263 pre-service elementary school teachers found that while 57% of them used the mean to compare two sets of data, only 21% gave a correct answer to a problem about the weighted mean, and 88%

of them were able to construct a data set that had a predetermined mean. The results also revealed that only 25% of these teachers demonstrated some kind of conceptual understanding of the mean, while the remainder showed a procedural understanding. The authors concluded that to improve the statistical training of future teachers, it was necessary to provide trainees with experiences that would increase their conceptual understanding of the mean, especially the qualitative and quantitative aspects of data representation.

Estrada (2007) in a study to assess the statistical knowledge of 367 pre-service primary teachers, observed that although more than 50% of them produced correct answers to the proposed statistical problems, the results also indicated a lack of knowledge of basic statistical concepts such as the mean, median and mode, as well as mistakes concerning the average; for example: not being aware of the effect on the mean of typical values, not being skilled in inverting the algorithm of the mean, and confusing mean, median and mode. The findings indicate a need to improve the statistical training of pre-service teachers.

The contributions of García, Cruz and Garrett (2008) on 130 secondary education pupils and 97 university students, of whom 31 were studying to be primary maths teachers, showed that participants displayed different types of reasoning about the arithmetic mean and that their answers to the proposed problems could be linked to the five levels of understanding described in the SOLO taxonomy of Biggs and Collis (1991). A further finding was that there were no significant differences between university students and secondary education pupils in terms of the observed levels of interpretation. These results suggest that to address the difficulties and errors that occur when learning arithmetic mean it is necessary to work with real-life problems and to encourage students to be more proactive in developing their knowledge.

In a study to assess the statistical and pedagogical knowledge of 55 pre-service teachers, Godino, Font, Wilhelmi and Lurduy (2011) observed that although many of them had a good idea of probability, they undervalued variability. Specifically, only 29% of participants made use of the mean to compare the results obtained in real and simulated coin-tossing sequences. The authors concluded that significant changes needed to be made to initial teacher training to improve the statistical knowledge of pre-service teachers.

Zazkis (2013) also explored students‟ understanding of the statistical idea of the mean inference from a fixed total. He investigated the way high school students solved three tasks related to the concept of mean as "inference from a fixed total". The idea of „inference from a fixed total‟ means that even though the values of the data are different, as long as the total is the same, the average is also the same. He suggests that this reasoning should be an additional focus for the next study and instructional development since most of the participants still focused on the algorithm rather than on the notion of the mean as a fixed total

Armah (2017) used the APOS Theoretical Framework to determine the levels of conceptual knowledge of 430 undergraduate students who were pre-service teachers about the mean. It was revealed that a majority (70.5% of overall participants) was able to compute the arithmetic mean from a discrete frequency distribution table without being provided with any external cues. Results of the study indicated that participants were not limited to an action conception, the least level of the APOS Framework, of the arithmetic mean, In the study, only 1.6.0% of the participants were able to reverse the process of the computation of the arithmetic mean from a bar chart provided. The conceptual understanding demonstrated by the undergraduate mathematics students indicated that they have an incomplete Process conception of the arithmetic mean. In the study, Armah (2017) reported that participants demonstrated mastery of the computational algorithm of the concept of the mean without any external promptings. They also did not exhibit an understanding of the properties of the arithmetic mean.

2.11.2 Mathematical Knowledge of the Arithmetic Mean

The mathematical knowledge necessary to understand, calculate and utilize the arithmetic mean is a subset of a student‟s complete mathematical knowledge and understanding. According to Ortiz and Font 2014, Mathematical Knowledge specific to the arithmetic mean can be procedural, such as computing the mean using a formula, defining the relevant variables, or knowing the fact that the „sum of the deviations from the mean is zero.‟ This knowledge may also be conceptual, such as mathematically understanding why the mean formula forces the sum of the deviations from the mean to be zero or using the properties of algebra to realize the mean is not a binary operation and therefore does not have an identity element, as such; it is influenced by numbers other than the average.

2.11.3 Mathematical Procedural Knowledge of Arithmetic Mean

Bütüner (2020) reported two types of procedural knowledge. The first type of procedural knowledge includes the awareness of syntactic rules to be familiar with the symbols that represent mathematical ideas and write the symbols in an acceptable format. The second type of procedural knowledge includes the procedures, algorithms, and rules used for solving mathematical problems. For example, while the expression $8x + 2 = 18$ is an acceptable representation, the expression $2 + = x8 + 18$ is an unacceptable representation. This knowledge is the first type of procedural knowledge; knowing algorithms or rules to solve the equation of $8x + 2 = 18$ is the second type of procedural knowledge.

When this view is applied to the concept of arithmetic mean, a student who knows that the symbolic representation of arithmetic mean is $\bar{x}\ or\ \mu$ will have demonstrated the first type of procedural knowledge. Further symbolic knowledge includes recognition of the variable x as representing values of data points and n as the number of data points. The second aspect of procedural knowledge involves the rules, algorithms, and procedures for calculating the mean. This includes using the $\bar{x} = \frac{\sum x}{n}$ formula or using an add-and-then-divide strategy if the symbolic representation of the formula is not yet learned.

2.11.4 Mathematical Conceptual Knowledge of Arithmetic Mean

In discussing conceptual knowledge of mathematics, Hiebert (2013) stated: "Conceptual knowledge is characterized most clearly as the knowledge that is rich in relationships. It can be thought of as a connected web of knowledge, a network in which the linking relationships are as prominent as the discrete pieces of information. Relationships pervade the individual facts and propositions so that all pieces of information are linked to some network". (p. 3)

From this description, the essence of conceptual knowledge is that it involves forming cognitive connections between bits of information that might otherwise be perceived as unrelated.

Understanding the mathematical concept of mean needs to be linked to knowledge of *arithmetic* operations. One such link is a conceptual understanding of addition and division. For example, $\bar{x} = \frac{\sum x_i}{n}$ means taking $\sum x_i$ and separating it into n equal parts of size $\bar{x}$. Furthermore, it is helpful to understand the connection between addition and multiplication, (i.e. $n\bar{x} = \sum x_i$ means that if you add $\bar{x}$ to itself n times you get the same total as summing the individual data points).

The conceptual understanding of the arithmetic mean requires recognizing the context where the arithmetic mean has an appropriate measure of central tendency (Watson & Moritz, 2000), interpreting that arithmetic mean as a value that represents a data set, and possessing the visual and kinaesthetic understandings of arithmetic mean (Cai, 2000; Ginat & Wolfson, 2002). For the conceptual understanding of arithmetic mean, balance and fair-share models are considered to be strong analogies (Van de Walle, Karp, & Bay-Williams, 2013). Bütüner, 2020 reported that the idea of a fair share positively affected students‟ performance in solving problems related to the arithmetic mean.

2.11.5 Developing Students' Understanding of the Concept of the Mean

Since the arithmetic mean is an entity in statistics, it is reasonable to parallel the role of mathematics in statistics to the role of mathematics in the arithmetic mean. That is, the arithmetic mean is a statistical concept defined outside the field of mathematics, but uses mathematics extensively in its calculation. The statistical and mathematical attributes of the arithmetic mean can be uniquely defined and then integrated to understand and thoughtfully apply the arithmetic mean.

Nickerson (1985) defines the understanding as to the ability to build a bridge as a connection between one conceptual domain and another. He also states that understanding always grows if ones know more about the subjects. Barmby (2007) specifically describes that the degree of understanding is determined by the number and the strength of the connection between the conceptual domains.

Meel (2003) describes the development of understanding as a process of connecting the representations to a structured and cohesive network. The connections require the recognition of the relationship between the concept and the elements inside the concept as a whole. Developing also means progress in doing something.

Based on the theories above, understanding in this study is defined as making a connection between existed scheme or information and the new scheme or information. Therefore, students" understanding refers to the ability of students to make a connection between their prior knowledge and the new knowledge that has learned in the classroom.

Developing students" understanding of the concept of the mean refers to the progress in making the connection between the students" prior knowledge with the concept of the mean itself as a measure of central tendency. To make the understanding visible, the present study provides indicators of understanding which refers to indicators of conceptual understanding from NCTM. The indicators are as follows:

a. Distinguishing some interpretations of the word average in daily life
b. Identifying the strategies to describe the data
c. Using the diagram to represent the mean
d. Know and apply the concept of the mean

The research indicates that an understanding of the arithmetic mean is best developed by addressing the statistical ideas associated with the arithmetic mean before presenting the mathematical procedures for calculating it. It is also clear that both the statistical and mathematical concepts about the arithmetic mean are vital in understanding and utilizing it. Because of the symbiotic relationship between statistics and mathematics, it is reasonable to draw upon the extensive research in mathematics education devoted to the development of procedural and conceptual knowledge to help explain the conceptual relationships of the mathematical knowledge associated with the arithmetic mean ((Barmby, 2007).

2.12 Summary of Review

Among averages, the arithmetic mean is unique in that the conceptual basis from which it is developed, representativeness, is not typically developed before the procedure to calculate it is introduced. Without connections to the statistically founded concept of representativeness, one‟s knowledge of arithmetic mean seems limited to computation of the mathematical formula; thus allowing little or no access to mathematically and statistically rich or adaptive problems, including those that arise in our everyday lives.

Statisticians contend that statistics is not a subfield of mathematics, but rather a field that utilizes mathematics, much like physics or economics. A statistician understands the concepts of statistics and the significance of statistical thinking and uses the tools of mathematics to solve or predict within the context of a problem. This idea suggests the need to develop a statistical sense of the arithmetic mean before a procedural technique is introduced.

Studies have shown that pre-service mathematics teachers have limited conceptual understanding of the arithmetic mean and are unable to relate it to real-real-life situations (Estrada, 2007, Zazkis, 2013, Armah, 2017).

The review is thus in the direction of the problem of the study. This will help to compare the results of the current study, from the Ghanaian setting to the majority of studies conducted outside Ghana into students‟ conceptual understanding of the arithmetic mean.

CHAPTER THREE

RESEARCH METHODOLOGY

3.0 Overview

This chapter discusses the research design, population and sample as well as the sampling procedures. It also covers the research instruments used and the procedure for data collection. Finally, the method of data analysis is also discussed.

3.1 Research Design

Research questions, according to Creswell (2003) and Merriam (2009), guide the choice of research methodology. The researcher used a mixed-method approach to accomplish the goals of this study. In a single study or series of studies, the mixed-method methodology focuses on collecting, analysing, and combining both quantitative and qualitative data (Creswell, 2003; Creswell, 2006). Mixed method research is defined as a design in which quantitative and qualitative research methods are combined "to increase understanding and move scientific inquiry forward" (Shank, 2013, p.185). Several reasons informed this decision to adopt a mixed method design. Firstly, quantitative research can reveal generalizable information for a large group of participants but often fails to provide specific answers, reasons, explanations or examples (Creswell, 2009). On the other hand, qualitative research provides data about meaning and context regarding the participants and environments of study but findings are often not generalizable because of the small numbers and narrow range of participants (Creswell, 2009). However, when „mixed", both designs will complement each other to the immense benefit to this study in terms of gaining deeper understanding of the phenomenon (Pampaka, 2014; Creswell, 2003, 2009, 2013). Mixed-method studies have the advantage of being able to show the result (quantitative) and explain why it was obtained (qualitative) (McMillan & Schumacher, 2013). The purpose of this strategy is also to use qualitative data and results to assist in explaining and assigning reasons for quantitative findings (Fife-Schaw, 2012).

Although there exist several variants of mixed methods design and data mixing procedures in the extant literature (see Creswell, 2008), concurrent triangulation (Convergent Parallel) design appeared to resonate well with the purpose of the study. Concurrent triangulation mixed method design is characterized by simultaneously collecting both quantitative and qualitative data, merges them using both quantitative and qualitative data analysis methods, and then interprets the results together to provide a better understanding of a phenomenon of interest (Creswell, 2012). Typically, the purpose of a concurrent triangulation design is that the strengths one method offset the weaknesses of the other and that a more complete understanding of a research problem results from collecting both quantitative and qualitative data (McMillan & Schumacher, 2013). Approximately equal emphasis is given to each method, even though one can follow the other or both can be conducted at the same time

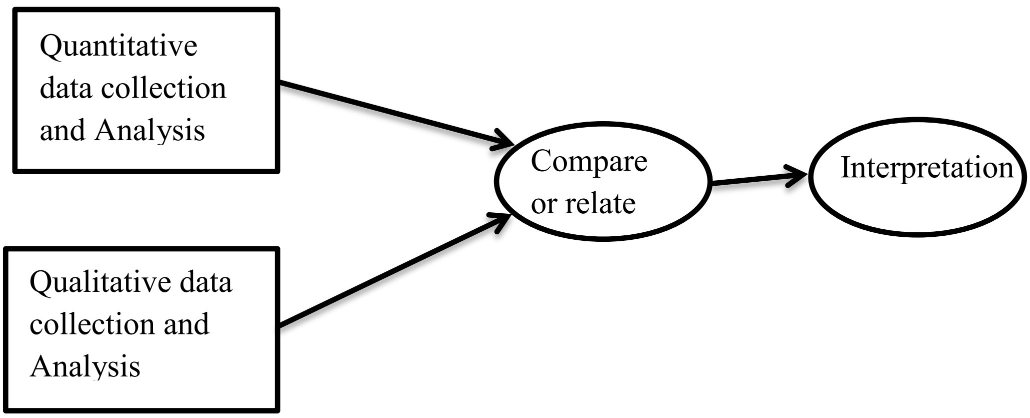


***Figure 3.1**: Schematic diagram representing the various stages of the design*

3.2 Population

The target population for the study was all first-year undergraduate students admitted into the departments of Mathematics, Chemistry and Physics education in UEW in the 2020/2021 academic year. According to Yin (2014*),* researchers must select participants from the perspective of its convenience, accessibility and geographical proximity. These factors largely influenced the decision to conduct the study in UEW. The researcher selected this population because the study was focused on exploring the conceptual understanding of the arithmetic mean of pre-service teachers in the first year. The per-service teachers in the chemistry and physics departments were included in this study because they have studied Mathematics in Ghana at the JHS and SHS level and have all passed the West African Senior School Certificate Examination (WASSCE), which tests among other things their ability in statistics. Students were also selected from two department for the purpose of comparing results. As a matter of fact, all pre-service teachers in the three departments have similar characteristics in terms of multiple ethnicities, age differentials and admission requirements.

3.3 Sample and Sampling Procedures

Due to the large number of respondents involved, coupled with the constraints of time and resources, a sample of the population was selected for the study. The sample consisted of all trained teachers (people who went to College of Education before coming to the university) in the three (3) departments and 216 students who came directly from SHS. The researcher believed the trained teachers had already gone through some training and had done some further mathematics after their SHS and are in the field teaching this concept of the arithmetic mean already.

Sampling procedures for this study were a combination of purposive, stratified and simple random sampling techniques. In purposive sampling, the researcher employs his or her own "expert" judgment about who to include in the sample frame. In other words, it is based on deliberate choice and excludes any random process (Stout, Marden & Travers, 2000). On the other hand, in simple random sampling, each member of the population has an equal chance of being selected. Hence there is a high probability that all the population characteristics would be represented in the sample (McMillan & Schumacher, 2014). A common variation of simple random sampling is called stratified random sampling. In this procedure, the population is divided into subgroups, or strata, based on a variable chosen by the researcher, such as gender, age, location, or level of education (McMillan & Schumacher, 2014). Once the population has been divided, samples are drawn randomly from each subgroup.

All trained teachers were purposively selected based on reasons already assigned in the opening paragraph under this section. However, the 216 direct applicants for the study were sampled through simple random sampling. The use of this procedure was to avoid unfairness in a sampling of the students. The grouping of the population into the various department was done through the stratified sampling. In all 370 pre-service teachers were sampled for this study. The distribution of the sampling procedure is presented in Table 3.1.

Table 3. 1. Stratified Sampling of Participants

Strata	Number of Trained Teachers	Number of Direct Applicants	Total
Mathematics	102	150	252
Chemistry	23	40	63
Physics	29	26	55
Total	**154**	**216**	**370**

3.4 Research Instrument

To examine Pre-service teachers" performance and conceptual understanding of the Arithmetic Mean, Statistical Understanding of Arithmetic Mean (SUTAM) was used to assess students" understanding of the arithmetic mean. Some of the items SUTAM were adapted from reviews of literature, while others were constructed by the researcher. (See Appendix C for a sample of the test).

3.4.1 Description of SUTAM

The SUTAM consisted of two parts. The first part was designed to obtain the demographic characteristics of the participants in the study. As part of the demographic attributes, aspects such as gender, age range, SHS attended by pre-service teachers, program studied at SHS, area of location of SHS attended and year of completion of SHS were requested.

The second part contained items meant to investigate the performance and conceptual knowledge of the arithmetic mean that students have constructed as a result of the learning experiences they have undergone through the Ghanaian JHS and SHS core mathematics and the SHS elective mathematics curricula. It was the intention of the researcher to analyse the test results both quantitatively and qualitatively.

There were twelve (12) test items in the second part of the test, five (5) were solely on students" performance on arithmetic mean and seven (7) on their conceptual understanding. One of the tests items (item 3) which was on the concept of the Arithmetic mean being located between the extreme values was adapted from Armah"s (2017) dissertation test items. Items 4, 6, 7, and 9 were constructed by the researcher. The other items were either retrieved from some statistics test items online or from the assessment builder section of the Assessment Resources Tools for

Improving Statistical Thinking (ARTIST) website. The ARTIST, which is a National Science Foundation (NSF)-funded Web project has developed a web-based assessment resource for introductory statistics courses. The goal is to help teachers in assessing statistical literacy, statistical reasoning, and statistical thinking in first courses of statistics. Few of the items obtained online were adapted. In all, there were four (4) dichotomous-response items, which elicited Yes/No. However, participants were given the chance to give justification for their choice of answer. Spaces were provided for participants to provide their own answers and give explanations for the answers.

The five items (4, 7, 8, 10 and 11) that were used to test their performance on the arithmetic mean, were based on the SOLO taxonomy. Item 7 question was on the pre-structural level. It required students to find the mean from raw data. Question 4 assessed students on the Uni-structural level which asked participants to find the arithmetic mean from a frequency distribution table. Here the students were expected to refer to the table provided in the SUTAM. Question 8 was on the multi-structural level. Here participants were provided with four grades on the test and were asked to find the minimum grade which is the fifth to achieve a certain average given in the question. Question 11 was on the Relational level; students were asked to find the arithmetic mean from the histogram provided. The 10th question which was also the last question on the performance was used to assess students on the Extended Abstract level. The question demanded students find the combined mean of two different groups. All these questions were marked out of 20. All items on the performance were analysis quantitatively.

According to Creswell (2006, 2009, 2013 and 2014), Merriam (2009) and Pampaka, 2014, one of the ways of collection qualitative data is the use of Open-

Ended question on test item or questionnaires. On test item, one may ask some questions that are closed ended and some that are open ended. The advantage of this type of questioning is that predetermined closed-ended responses can net useful information to support theories and concepts in the literature (Creswell 2013). The open-ended responses, however, permit you to explore reasons for the closed-ended responses and identify any comments people might have that are beyond the responses to the closed-ended questions (Pampaka, 2014).

Therefore, in collecting qualitative data, written responses to the open-ended test items were the data sources. Items 1, 2, 3, 5, 6, 9, and 12 were used to investigate students" conceptual understanding of the arithmetic mean. Some of the items asked participants to decide whether the statement provided them is yes or no as well as justify the answer they chose. Items 5 and 6 requested students to construct their responses to the given questions.

3.4.2 Scoring of Test Items

Scoring was done by the researcher. The study employed both the quantitative and qualitative types of analysis. The researcher marked the questions involving calculation based on the marking scheme (see Appendix D). Marks of each participant were recorded. This was done to know their achievement in the SUTAM. The researcher was also interested in whether a participant"s response to an item was correct or wrong. The percentage of students getting an item right or wrong was noted. Furthermore, the responses of students to constructed response items were grouped into themes and percentages of students whose responses fell in a theme were noted.

With regards to the levels of conceptual understanding the researcher considered how participants were able to describe the arithmetic mean and their reasons for selecting an answer. With participants‟ answers to the test items, the researcher tried to look at the nature of the questions participants were able to answer, what they were required to know to be able to answer those test items and also how they answered the test items. This the researcher did, not forgetting the characteristics expected of individuals at the levels of conceptual knowledge of arithmetic mean according to the SOLO taxonomy.

3.5 Validity and Reliability

Reliability and validity of research instruments should be examined and verified to ensure that the instruments are appropriate, useful and effective in identifying and evaluating the relevant data (Wiersma, 2000). Validity refers to the extent to which the research instruments are effectively authentic or truthful. It is a demonstration that a particular research instrument measures what it purports to measure (Mushquash & Bova, 2007; Williams, 2014). According to Cohen, Manion, and Morrison (2007), threats to validity and reliability can never be completely eliminated; however, the effects of these threats can be mitigated by focusing on validity and reliability. According to Cohen et al. (2007), the instrument must demonstrate content validity by showing that it fairly and comprehensively covers the domain or items it claims to cover. These conceptions and notions of validity informed the validity measures used in this study.

To validate my research instruments, the researcher consulted Ghanaian JHS and SHS core mathematics and the SHS elective mathematics curricula and research work patterning to the current studies. The purpose was to gain insight into what learners were expected to learn so that instruments are developed accordingly. The researcher also consulted the supervisor, other senior lecturers, and researcher‟s colleagues for their suggestions before administering the test items. Durrheim (1999) suggests the researcher approaches others in the academic community to check the appropriateness of his or her measurement tools. Finally, the test items were found to be similar to the test items used by other researchers (Armah, 2017; Zazkis, 2013; Marnich, 2008) this was to ensure convergent validity. Vanderstoep and Johnson (2009) stated that to determine to construct validity, the researcher must evaluate convergent validity**,** which is the extent to which other measures of the same behavior are similar to your measure.

The reliability of an instrument measures the extent to which the variation in scores is due to true differences between people, the characteristic being measured, or random measurement error. It is the tendency of obtaining the same result if it is replicated over time, over instruments, and/or over groups of respondents (Cohen et al., 2011). Therefore, to ensure the reliability of the test items, the researcher used the same test items for all participants, the same duration was given to all the participants and the test items were administered before all students started learning measures of central tendency in the University. This is because usually, high reliability is easy to obtain by presenting all subjects with a standardized stimulus; observer subjectivity is greatly eliminated (Owens, 2002). In addition, the researcher checked the inter-rater reliability of the answered scripts with different experts. According to Mackinnon (2000), a two-person assessment can be utilized to eliminate bias and prevalence. Several discussions were made to achieve 97% agreement between the two examiners.

Finally, to ensure trustworthiness of the qualitative data, participants were briefed on the purpose of the study and also made to feel at ease. They were made to understand that results were not going to be part of their continuous assessment. They were also not restricted to any time duration during the test. This was to ensure they had enough time to think and express themselves. Consultation with advisors during the collecting and processing of data helped to control biasedness on the part of the researcher. For this study, the researcher made sure all research procedures were described in detail. Also, the views of supervisors, advisors and experts in the field of qualitative research were sought to ensure the right research procedures were followed to confirm dependability. Also, a detailed description of the processes used in the study, as well as the results have been spelled out to aid replication if need be.

3.6 Pilot Study

A pilot study was conducted on a group of 30 students from group seven of the mathematics department. The subjects for the pilot study were selected from the same target population for this study but they were exempted from the actual study. A pilot study is, "A small-scale test of the methods and procedures to be used on a larger scale …" (Porta, 2008). The fundamental purpose of conducting a pilot study is to examine the feasibility of an approach that is intended to ultimately be used in a larger-scale study (Leon, Davis, & Kraemer, 2011). Lancaster, Dodd and Williamson (2004) explained how a pilot study can be used to test aspects of the research including accumulating information prior to the actual research in order to improve its implementation. The pilot study served as the platform for testing the reliability and validity of the research instruments. As the objective of the pilot study was to ensure that the respondents understood the instructions, the questions being asked, the terminologies used, no misleading questions, clarity was observed, and the instrument was reliable to the study.

Also, the researcher was able to obtain the duration period that should be assigned for the administration of test items and possible responses from students.

This was highly feasible as the students" responses would be evaluated and categorized into the different levels of SOLO taxonomy in order to identify their level of understanding

3.7 Data Collection Procedure

The researcher visited the three departments sampled in the early part of the first semester. The visit enabled the researcher to discuss the purpose of the study and also to seek permission from the heads of departments in these various departments. This mission was made possible through an introduction letter given to me by my department (See Appendix B). The researcher later visited the respondents, introduced himself to them and informed them of the scheduled day for the administration of the SUTAM. On the 19^{th} of February 2021, all fresh undergraduate mathematics students who had reported for the 2020/2021 academic year were assembled at four lecture halls for the administration of the test. In the same way, the Chemistry department and Physics department participants took their tests on the 24^{th} of February 2021 and the 26^{th} of February, 2021 respectively. The researcher administered the test himself. Equal time interval was given to all participants. Students were made to understand that the exercise was not going to be part of their continuous assessment and such it is strictly for research purposes. So, they should feel free to answer all questions as truthfully as they can. (Appendix C).

The researcher then selected some students" work purposely based on their level on the SOLO taxonomy and analyzed their working processes. Purposefully selecting participants" work means that qualitative researchers select individuals who will best help them understand the research problem (Creswell, 2009).

3.8 Data Analysis

Data analysis was done quantitatively as well as qualitatively. There are two types of statistics: descriptive statistics and inferential statistics. Descriptive statistics are methods used to summarize and organize data. Data can be organized through tables or graphs and summarized through certain descriptive values such as the average score. Descriptive Statistics are used to present quantitative descriptions in a manageable form by simplifying and reducing large amounts of data into a simpler and clear summary. (Field, 2005: Seliger & Shohamy, 2012). Inferential statistics are methods that can be used to draw general conclusions about populations based on the available data to more general conditions. Inferential statistics enable conclusions that extend beyond the immediate data to be made. There are several ways of processing the data in a way that allows inference and judgment to be made (Field, 2005: Seliger & Shohamy, 2012). Therefore, the current study used both descriptive (mean, standard deviation and frequency tables) and inferential statistics (paired sample t-test and chi-square and One-way ANOVA) in analysing the data.

This study explored the conceptual understanding of students using the SOLO taxonomy. The research generated both quantitative and qualitative data. Emphasis was placed on students‟ explanations in relation to the answer they selected. Students‟ explanations were analysed using meaning coding. According to Kvale and Brinkman (2009), meaning coding involves attaching one or more keywords to a text segment in order to later permit identification. Coding can be either concept or data-driven (Kvale & Brinkman, 2009). In this research, data-driven coding was used. Data-driven coding is a type of coding where the researcher starts with codes but develops them through the reading of the data collected (Kvale & Brinkman, 2009). In this study, the researcher went through the working process of all students and gave codes to the similar working processes used and these codes were entered in Statistical Package for Social Sciences version 25 (SPSS_v25) and analysed with SPSS.

3.9 Ethical Consideration of the Study

Carrying out research in education often raises ethical concerns, because it involves people other than the researcher. McMillan and Schumacher (2014) state that, research ethics are focused on what is morally proper and improper when engaging with participants or when accessing data. Under ethical issues, participants must voluntarily agree to participate and the researcher needs to safeguard against unwanted exposure and loss of anonymity. The researcher needs to fully reveal the procedures of research to the participants at the onset.

As a result, the University of Education, Winneba requires that research carried out by staff and students is conducted within clear ethical guidelines. Consistent with this policy, this study was conducted within the ambit of the ethical requirements of the University of Education, Winneba and several Ethical Guidelines for educational researchers.

I obtained an introduction letter from the head of the Mathematics Education Department of the University of Education, Winneba which enabled me to seek permission from the Heads of the department of the selected departments that were used for the study (See Appendix B).

Informed consent and voluntary participation: The subjects were informed of the comprehensive nature of the research including the objectives, instruments and the intervention involved in this study. They were given the choice to either willingly accept their roles as research participants or decline the roles. The researcher also assured them that there was no pressure or penalty given to them in whichever choice they decide to make. The results of the study and the implementation of the research would have zero influence on their learning and assessment. They were also notified of their freedom to express their feelings or opinions related to the study throughout the experiment so necessary actions could be taken to make them feel comfortable and safe. Consent forms were distributed to the subjects to formalize their decisions on their participation. See Appendix A for a copy of the consent form.

Anonymity and confidentiality: The researcher understood how important it was for the identity of the subjects to remain anonymous and any information related to them to stay confidential. Thus, the identity of the subjects would not be revealed in any form either in the informal setting such as conversation at the workplace or on social media as well as in the formal setting such as conference presentations and publication of academic papers containing such information.

CHAPTER FOUR

RESULTS AND DISCUSSIONS

4.0 Overview

The study sought to use both quantitative and qualitative analysis to explore Pre-service Teachers‟ (PST) conceptual understanding of the Arithmetic Mean (AM). It also sought to find out the achievement of Pre-service Teachers in problems relating to the AM, and investigate their level of conceptual understanding of the AM in relation to the SOLO taxonomy. In pursuance of the purposes stated above, the following research questions were formulated to guide the study:

1. What is pre-service teachers‟ performance on mathematical tasks on the AM?
2. What conceptual understanding of the AM do pre-service teachers have?
3. What level of conceptual understanding with respect to the SOLO taxonomy do pre-service teachers have about the AM?

This chapter focuses on the results of the analyses of the data and discussions on the major findings. The data were organized and presented using Tables, Figures, and descriptive and inferential statistics. The discussions focused on the following areas:

I. demographic characteristics of respondents,
II. the achievement of pre-service teachers on mathematical tasks on the AM,
III. the pre-service mathematics teachers‟ conceptual understanding of the AM, and
IV. the pre-service mathematics teachers‟ level of conceptual understanding of the AM with respect to the SOLO taxonomy.

4.1 Demographic Characteristics of Respondents

Information about the demographic background of pre-service teachers who were sampled for this study covered a wide range of characteristics such as their gender status, age pattern, program offered at SHS level, region within which respondents attended SHS, and whether the respondent studied elective mathematics at SHS level or not.

Three departments were used as the sample for the study. Table 4.1.1 shows the number of respondents from each of the three departments.

Table 4.1 Number of Respondents from the Departments

Department	N	%
Mathematics	252	68.1
Chemistry	63	17.0
Physics	55	14.9
Total	**370**	**100.0**

From Table 4.1 it can be seen that from the three hundred and seventy (370) participants sampled for the study, two hundred and fifty-two (252), representing 68.1%, came from the Department of Mathematics Education (PME), Sixty-three (63), representing 17.0%, came from Department of Chemistry Education (DCE), with the remaining fifty-five (55), represent 14.9%, coming from the Department of Physics Education (DPE).

Respondents indicated their gender. Table 4.1.2 looks at the gender of the respondents from each of the departments.

Table 4.2 Gender of Respondents

Department	Male		Female		Total	
	N	%	N	%	N	%
Mathematics	182	72.2	70	27.8	252	68.1
Chemistry	51	81.0	12	19.0	63	17.0
Physics	47	85.5	8	14.5	55	14.9
Total	**280**	**75.7**	**90**	**24.3**	**370**	**100.0**

Statistics gathered in Table 4.2 on the gender status of respondents showed that out of the two hundred and fifty-two (252) respondents from DME, 72.2% (182) were males and 27.8% (70) were females. From the DCE, 81.0% (51) were males, and 19.0% (12) were females, and then from the DPE, 85.5% (47) of the fifty-five respondents were males with 14.5% (8) being females.

In all, 75.7% (280) of the total sample were males with 24.3% (90) being females. Since the three departments sampled for this study were all mixed-sex, the low percentage of females suggested that the population of females in these three departments was low as compared to that of their male counterparts. The low percentage of females in the sample could be attributed to the assertion by (Aguele & Agwagah, 2007) that gender differentials in enrolment and achievement in higher education are invariably rooted in inequality at the primary and secondary levels, where the real sorting out of university-bound students do take place. Furthermore, female participation and interest in science, technology, and mathematics diminishes as they move on the educational ladder towards the university level due to a variety of factors that are primarily rooted in their religious and cultural beliefs surrounding the role of women in society (Aguele & Agwagah, 2007). The researcher also looked at the age patterns of the participants. This information is shown in Table 4.3.

Table 4.3 The Age Pattern of Respondents

	Below 20		20-30		Above 30		Total	
Department	**N**	**%**	**N**	**%**	**N**	**%**	**N**	**%**
Mathematics	35	13.9	185	73.4	32	12.7	252	68.1
Chemistry	7	11.1	51	81.0	5	7.9	63	17.0
Physics	4	7.3	42	76.4	9	16.4	55	14.9
Total	**46**	**12.4**	**278**	**75.2**	**46**	**12.4**	**370**	**100.0**

Results in Table 4.3 showed that majority, 73.4% (185), 81.0% (51), and 76.4% (42) of the participants from DME, DCE, and DPE respectively aged between 20 and 30 inclusive. It can also be observed that 13.9% (35), 11.1% (7,) and 7.3% (4) of the respondents from the DME, DCE, and DPE respectively were below 20 years.

The researcher was interested in the programs the respondents read at the SHS. This information is shown in Table 4.1.4

Table 4.4 Programs Studied by Respondents at SHS

Department	Science		Business		Arts		Total	
	N	**N %**	**N**	**N %**	**N**	**N %**	**N**	**N %**
Mathematics	128	50.8	100	39.7	24	9.5	252	68.1
Chemistry	63	100	0	0.0	0	0.0	63	17.0
Physics	55	100.0	0	0.0	0	0.0	55	14.9
Total	**246**	**66.5**	**100**	**27.0**	**24**	**6.5**	**370**	**100.0**

Statistics gathered in Table 4.4 on programs studied at SHS by the PST showed that out of the two hundred and fifty-two (252) respondents from Mathematics 50.8% (128) studied Science, 39.7% (100) offered Business and 9.5% (24) studied General Arts. All participants from chemistry and physics offered science. In all, 66.5% (246)

of the total sample studied science, 27.0% (100) and 6.5% (24) offered business and general Arts respectively. This clearly shows that majority of the participants studied science at SHS. The sciences consisted of those who studied general science, agricultural science, and technical programs. Table 4.5 looks at students who did elective mathematics at SHS.

Table 4.5 Participants who studied elective mathematics in SHS

	N	%
Studied elective mathematics	338	91.4
No elective mathematics	32	8.6
Total	**370**	**100.0**

It can be seen from Table 4.5 that 91.4% (338) of the total respondents studied elective mathematics at SHS with only 8.6% (32) not studying elective mathematics. This means that majority of the PST sampled for this study have comprehensive knowledge of AM since it was both taught in both core and elective mathematics.

In Ghana, the College Educations (CoE) are the institutions that train teachers for early childhood and basic education. Universities like; UEW and the University of Cape Coast (UCC) also train professional teachers for all levels of education. An already trained teacher from CoE can seek study leave to upgrade himself/herself in any of the universities; they are nicknamed “trained teachers” in the university discourse. Of the current study, the researcher was interested in knowing the conceptual understanding of the trained teachers and those that came directly after SHS (direct applicant). Table 4.6, therefore, shows the information.

Table 4.6 Entry Status of Respondents

Department	Trained Teacher		Direct Applicant		Total	
	N	N %	N	N %	N	N %
Mathematics	102	40.5	150	59.5	252	68.1
Chemistry	23	36.5	40	63.5	63	17.0
Physics	29	52.7	26	47.3	55	14.9
Total	**154**	**41.6**	**216**	**58.4**	**370**	**100.0**

From Table 4.6 it can be seen that for the DME 40.5% (102) are trained teachers with 59.5% (150) being direct applicants. Twenty-three (36.5%) and 29 (52.7%) trained teachers were from DCE and DPE respectively. In total 41.6% (154) were trained teachers and 58.4% (216) were direct applicants.

The researcher wanted to look at the representation of the regions in Ghana within which these students had their SHS education. The request PSTs‟ region of SHS attended by the researcher was indeed necessary for identifying some factors that might have contributed to the difficulties in conceptualizing the AM. Some regions are endowed with facilities such as good social amenities, good schools with necessary learning facilities and the likes. This presupposes that SHS located in such environments are likely to enjoy facilities that enhance teaching and learning. This was also done to find out whether all the sixteen (16) regions in Ghana were represented in the sample. The results are shown in Table 4.7

Table 4.7 Regional Representation of Respondents

Region	N	N%
Ashanti	85	23.0
Western	25	6.8
Volta	55	14.9
Eastern	50	13.5
Upper West	8	2.2
Central	56	15.1
Upper East	12	3.2
Greater Accra	25	6.8
Savannah	1	.3
North East	8	2.2
Bono East	4	1.1
Oti	3	.8
Ahafo	15	4.1
Bono	7	1.9
Western North	10	2.7
Northern	6	1.6
Total	**370**	**100.0**

From Table 4.7, the Ashanti region had the highest representation of 23.0% (85) followed by Central and Volta with 15.1% (56) and 14.9 (55) respectively. The Eastern region was the fourth on the list with 13.5% (50), Savannah region represented the least with 0.3% (1). The remaining region ranged between 1% and 6.9%.

Again, from their responses, it was seen that two hundred and six (206) representing 55.7% of the total respondent were already in the classroom teaching mathematics either at pre-school, primary, or junior high school level for not less than one year. This shows that the majority of respondents have been teaching the concept of the arithmetic mean to others.

4.2 Research Question 1. Pre-Service Teachers' Performances on Mathematical Tasks on the AM

Research question 1 sought to investigate the achievement of PST on mathematical tasks on the AM. To answer this question, items 4, 7, 8, 10, and 11 in SUTAM (See Appendix C) asked participants to compute the mean on a different task. The items were marked and scored out of 20 based on the marking scheme (see Appendix D). The marks were shared depending on the thinking level of the item. The marks distribution is as follows; item 4 – 4marks, item 7 – 2marks, item 8 – 4marks, item 10 – 4marks and item 11 – 6marks. Each response was analysed before looking at the overall achievement. Refer to Appendix E for a sample of students‟ responses to the test.

4.2.1 Analysis of Item 4

The Table below represents the distribution of ages of a group of students taking a course

Age (x)	5	6	7	8	9
Freq. (N)	2	3	3	2	1

Calculate the arithmetic mean for the distribution.

Box 1

Table 4.8 Students' Response to Question 4 Based on Department

	Department							
	Mathematics		Chemistry		Physics		All	
Response	N	N %	N	N %	N	N %	N	N %
Correct	149	59.1	25	39.7	32	58.2	206	55.7
Wrong	99	39.3	32	50.8	17	30.9	148	40.0
No Response	4	1.6	6	9.5	6	10.9	16	4.3
Total	**252**	**100.0**	**63**	**100.0**	**55**	**100.0**	**370**	**100.0**

The results in Table 4.8 shows that 59.1% (149), 39.7% (25) and 58.2% (32) of participants from DME, DCE, and DPE respectively answered the question correctly. It can also be seen that 39.3% (99) of the mathematics students answered the question wrongly and 1.6% (4) did not attempt the question. With the chemistry students 50.8% (32) answered the question wrongly and 9.5% (6) did not attempt the question and with physics, 30.9% (17) answered the question wrongly and 10.9 % (6) did not attempt the question. In total 55.7 % (206), 40.0% (148) and 4.3% (16) answered correctly, wrongly and didn"t attempt the question respectively. The researcher also looked at the performance based on whether the respondent is a trained teacher or a direct applicant. Table 4.9 shows the results.

Table 4.9 Students' Response Based on Entry Status

	Entry Status					
	Trained Teacher		Direct Applicant		Total	
Response	**N**	**N%**	**N**	**N%**	**N**	**N%**
Correct	89	57.8	117	54.2	206	55.7
Wrong	62	40.3	86	39.8	148	40.0
No Response	3	1.9	13	6.0	16	4.3
Total	**154**	**100.0**	**216**	**100.0**	**370**	**100.0**

Majority of both the trained teachers (TT) 57.8% and direct applicants (DA) 54.2% accurately answered the question as shown in Table 4.9. On the other hand, 40.3% and 39.8% erroneously answered the question for TT and DA respectively. Almost two percent (1.9%) of TT didn"t attempt the question and 6.0% of DA did the same.

4.2.2 Analysis of item 7

The ages of 10 chemistry students were recorded as 23, 24, 19, 27, 22, 20, 19, 30, 21, and 31. Compute the mean of this data.

Box 2

Students were asked to compute the mean ages of 10 chemistry students. Their responses are shown in the Tables below.

Table 4.10 Students' Response Based on Department

	Department							
	Mathematics		Chemistry		Physics		All	
Response	N	N %	N	N %	N	N %	N	N %
Correct	191	75.8	49	77.8	43	78.2	283	76.5
Wrong	60	23.8	9	14.3	9	16.4	78	21.1
No Response	1	0.4	5	7.9	3	5.5	9	2.4
Total	**252**	**100.0**	**63**	**100.0**	**55**	**100.0**	**370**	**100.0**

Table 4.11 Students' Response Based on Entry Status

	Entry Status					
	Trained Teacher		**Direct Applicant**		**Total**	
Response	**N**	**N%**	**N**	**N%**	**N**	**N%**
Correct	122	79.2	161	74.5	283	76.5
Wrong	29	18.8	49	22.7	78	21.1
No Response	3	1.9	6	2.8	9	2.4
Total	**154**	**100.0**	**216**	**100.0**	**370**	**100.0**

It is indicated from Table 4.10 that between 75.7% and 78.1% of participants across departments were able to compute it correctly. The non–response rate for this item was reduced drastically to 2.4% and below for all year groups. This implies that it is easier for students to compute the AM from raw data than when in a frequency distribution table. As a combined group, 76.5% of all the respondents were able to compute the AM. Information gathered in Table 4.11 also shows that majority of both TT (79.2%) and DA (74.5%) were able to compute the mean.

4.2.3 Analysis of item 8

A student has gotten the following grades on his tests: 87, 95, 76, and 88. He wants a mean of 85 or better. What is the minimum grade he must get on the fifth test in order to achieve that average?

Box 3

Students were asked to compute the minimum grade needed for a student to get arithmetic mean of 85 or better. Their responses are shown in the Tables below.

Table 4.12 Students' Response Based on Department

	Department							
	Mathematics		Chemistry		Physics		All	
Response	N	N %	N	N %	N	N %	N	N %
Correct	140	55.6	27	42.9	33	60.0	200	54.1
Wrong	98	38.9	24	38.1	15	27.3	137	37.0
No Response	14	5.6	12	19.0	7	12.7	33	8.9
Total	**252**	**100.0**	**63**	**100.0**	**55**	**100.0**	**370**	**100.0**

Table 4.13 Students' Response Based on Entry Status

	Entry Status					
	Trained Teacher		Direct Applicant		Total	
Response	**N**	**N%**	**N**	**N%**	**N**	**N%**
Correct	84	54.5	116	53.7	200	54.1
Wrong	57	37.0	80	37.0	137	37.0
No Response	13	8.4	20	9.3	33	8.9
Total	**154**	**100.0**	**216**	**100.0**	**370**	**100.0**

The results in Table 4.12 show that 55.6% (140), 42.9% (27) and 60.0% (33) of respondents from DME, DCE, and DPE respectively answered the question correctly. In all 37.0% (137) wrongly answered the question. Information gathered in Table 4.13 shows that 54.5% (84) of the TTs were able to find the minimum value whereas 37.0% (57) answered it wrongly and with the DA 53.7 (116) were able to answer it correctly and 37.0% (80) answered it wrongly. Since the reasoning level of this question was high as compared to items 4 and 7, most students couldn"t answer it thereby reducing the number of students solving correctly in the case of item 7.

4.2.4 Analysis of item 10

There are ten people in an elevator, four women and six men. The average weight of the women is 120 pounds, and the average weight of the men is 180 pounds. What is the average of the weights of the ten people in the elevator?

Box 4

Students were asked to compute the combined mean of two different groups. Their responses are shown in the Tables below.

Table 4.14 Students' Response Based on Department

	Department							
	Mathematics		Chemistry		Physics		All	
Response	N	N %	N	N %	N	N %	N	N %
Correct	63	25.0	7	11.1	8	14.5	78	21.1
Wrong	187	74.2	52	82.6	41	74.6	280	75.7
No Response	2	0.8	4	6.3	6	10.9	12	3.2
Total	**252**	**100.0**	**63**	**100.0**	**55**	**100.0**	**370**	**100.0**

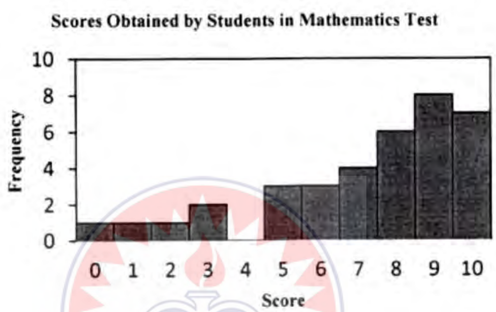
Table 4.15 Students' Response Based on Entry Status

	Entry Status					
	Trained Teacher		**Direct Applicant**		**Total**	
Response	**N**	**N%**	**N**	**N%**	**N**	**N%**
Correct	36	23.4	42	19.4	78	21.1
Wrong	115	74.7	165	76.4	280	75.7
No Response	3	1.9	9	4.2	12	3.2
Total	**154**	**100.0**	**216**	**100.0**	**370**	**100.0**

Although Table 4.14 shows a reduction in the non-response rate, it can be seen that majority of all the departments couldn"t answer the questions. Most of them had the confidence to answer the question but ended up getting it wrong. It is indicated in Table 4.14 that between 74.1 % and 82.7% of participants across departments were unable to compute it correctly. This implies that majority of students have a problem when finding the combined mean. Majority of students who were unable to answer it just added the two means and divided it by two or ten (10) (thus a total number of men and women) (see Appendix E). As a combined group, 75.7% (280) of all the respondents were unable to compute. Information gathered in Table 4.15 shows that majority of both the TTs (68.2%) and DAs (70.8%) were unable to compute the mean. This clearly shows that students are unable to answer questions on the AM on high thinking level.

4.2.5 Analysis of item 11

The scores obtained by students in a mathematics test are shown in the representation below:



Find the mean score obtained by the students.

Box 5

Students were asked to compute the mean from the histogram. Their responses are shown in Tables 4.2.9 and 4.2.10.

Table 4.16 Students' Response Based on Department

	Department							
	Mathematics		Chemistry		Physics		All	
Response	N	N %	N	N %	N	N %	N	N %
Correct	88	34.9	13	20.6	9	16.4	110	29.7
Wrong	157	62.3	41	65.1	38	69.1	236	63.8
No Response	7	2.8	9	14.3	8	14.5	24	6.5
Total	**252**	**100.0**	**63**	**100.0**	**55**	**100.0**	**370**	**100.0**

Table 4.17 Students' Responses Based on Entry Status

	Entry Status					
	Trained Teacher		Direct Applicant		Total	
Response	**N**	**N%**	**N**	**N%**	**N**	**N%**
Correct	54	35.1	56	25.9	110	29.7
Wrong	95	61.7	141	65.3	236	63.8
No Response	5	3.2	19	8.8	24	6.5
Total	**154**	**100.0**	**216**	**100.0**	**370**	**100.0**

Table 4.16 indicates that less than 35% of participants from all the departments were able to answer this item correctly. Majority of each of the departments (between 62.2% and 69.2%) had it wrong. As a common group, 29.7% (110) were able to find the mean from the histogram given to them. Majority of them (63.8%) were not able to find the mean from the histogram.

4.2.6 The Overall Scores of Students on the SUTAM

This section presents the descriptive statistics of the overall scores obtained by students in the SUTAM. See Appendix F for the overall scores of students

The results in Appendix F show that 44.1% of the students obtained scores between 0 and 6 inclusive, while 18.9% obtained scores of 16 and above. The results further showed a score of 0 has the highest frequency with 20 with the least frequency. This indicates that the general performance of students in the SUTAM was weak. The descriptive statistics on the total scores are presented in Table 4.18.

Table 4.18 Descriptive Statistics on the Total Score of Students.

Department	Mean	Std. Deviation	N
Mathematics	9.1071	6.10274	252
Chemistry	7.0952	5.83016	63
Physics	8.3273	6.12238	55
All	**8.6486**	**6.09107**	**370**

The results in Table 4.18 show that out of a total score of twenty (20) marks, the mean score of students in mathematics, chemistry and physics was 9.1071, 7.0952 and 8.3273 respectively with their standard deviations between 5.84 and 6.13. In total the mean score of all participants was 8.6486 and the standard deviation was 6.09107. The high standard deviation is an indication of how diverse the scores were.

The researcher wanted to find out if there is a difference between the mean scores of TT and DA on the SUTAM. The researcher formulated the following null and alternative hypotheses;

H_o: There is no significant difference between the mean scores of TT and DA on the SUTAM.

H_1: There is a significant difference between the mean scores of TT and DA on the SUTAM.

Table 4.18 shows the mean and standard deviation of the two groups.

Table 4.19 Descriptive Statistics

Entry Status	Mean	N	Std. Deviation
Trained Teacher	9.3571	154	5.80250
Direct Applicant	8.1435	216	6.25302

Table 4.19 shows that TT has a mean of 9.3571 and a standard deviation of 5.80250 whiles the DAs has a mean of 8.1435 with a standard deviation of 6.25302. To test for the hypothesis above, the researcher used an independent sample t-test with a significant level of 0.05, the results of the test are shown in Table 4.20

Table 4.20 t-test

t-test for Equality of Means						
T	df	P-Value	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
					Lower	Upper
1.896	368	.059	1.21362	.64016	-.04520	2.47245

The results in the independent sample t-test (as shown in Table 4.20) show a p-value of 0.059 which is greater than the alpha value of 0.05. This indicates a non-significant difference between the mean scores of the TTs and DAs on the SUTAM. It is therefore concluded that there is no significant difference between the mean scores of TT and DA on SUTAM.

In order to know if there is a difference in means score across the three departments as stated in chapter three the following null and alternative hypotheses were formulated;

H_o: There is no significant difference between the mean scores of the three departments on the SUTAM.

H_1: at least one population mean is different from the rest.

To test for the hypotheses above the researcher used a one-way ANOVA test with a significant level of 0.05, the results of the test are shown in Table 4.21

Table 4.21 One-Way ANOVA

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	210.680	2	105.340	2.868	.058
Within Groups	13479.645	367	36.729		
Total	**13690.324**	**369**			

Table 4.21 shows the output of the ANOVA analysis and whether there is a statistically significant difference between the group means. It can be seen that the significance value is 0.058 (i.e., $p = .058$), which is greater than 0.05. and, therefore, there is no statistically significant difference in the mean scores between the three departments.

4.2.7 Discussion of Results on Students' Performance in Mathematics test on the Arithmetic Mean

One of the most important concepts we encounter in daily life and statistics is the arithmetic mean. The concept of arithmetic mean is used in a variety of fields in everyday life, including meteorology, medicine, and agriculture (Zazkis, 2013). Despite the fact that arithmetic mean appears to be a simple concept, studies show that secondary and high school students struggle with it, resulting in poor performance (Bütüner, 2020; Cai, 2000). This research was no exception.

The descriptive statistics analysis of PST's performance on the SUTAM revealed that the mean score was very low, while the standard deviation was greater than one (1), indicating that the mean is not a true representative of students' performance and that most students performed poorly. Students' performance on the SUTAM was generally poor, owing to their inability to apply the properties of the arithmetic mean in their calculations. The majority of them understand the arithmetic Mean as a simple "add and divide" measure, so they were able to find the mean from the raw data provided. However, as the thinking level of the questions rises, the number of students who correctly answer the questions decreases. Examining how this concept is presented in textbooks and what types of problems are included can help explain why students perform poorly in the arithmetic mean. Because textbooks are the primary source of information for teachers in their classes, they have an impact on what and how they teach, as well as the types of problems they ask students to solve (Bütüner, 2020).

The results of the independent-samples t-test also revealed that there was no significant difference between the mean score of the trained teachers and the direct applicant at a significance level of 0.05. Furthermore, the mean scores of students from the three departments do not differ significantly.

4.3 Research Question 2. What conceptual understanding of the AM do pre-service teachers have?

The ability to know the facts and why they are important has been defined as conceptual understanding (Frederick & Kirsch, 2011). The essence of conceptual understanding is to probe into students' results beyond just the correct answer, and it goes beyond just responding to test items. Bremigan (2003) stated that knowing the seven properties of arithmetic mean identified by Strauss and Bichler (1988) is necessary for solving problems and providing examples of problem cases related to these properties in her work. As a result, the second research question sought to determine PSTs' conceptual understanding of the AM. This question employed the qualitative type of analysis. The written responses to the open-ended test items were the data sources for this question. The open-ended items allowed the researcher to categorize responses into various themes. The test items focused on statistical reasoning and conceptual understanding of the concepts of the arithmetic mean. As a result, the researcher was interested in how participants responded to test items than assigning scores to the responses.

- Whether a participants‟ response to an item was correct or wrong and their percentage noted.
- The themes that run through their justifications to Yes/No answers were noted as well as
- The responses of students to open ended items were also grouped into themes and the percentages were noted.

A sample of student responses to the test can be found in Appendix E.

4.3.1 Analysis of Item 5

The weights of a class of 100 sociology students were measured, and the arithmetic mean was found to be 69.5kg. What does it mean to say that the arithmetic mean of all the weights is 69.5kg?

Box 9

Table 4.22 shows the analysis of the responses to item 5 by the participants from the three departments.

Table 4.22 Students' Responses to Item 5

	Department							
	Mathematics		**Chemistry**		**Physics**		**All**	
Response	**N**	**N %**	**N**	**N %**	**N**	**N %**	**N**	**N %**
The Average	150	59.5	14	22.2	21	38.2	185	50.0
Adding their weight and diving by their number will give you 69.5	70	27.8	33	52.4	22	40.0	125	33.8
No Response	32	12.7	16	25.4	12	21.8	60	16.2
Total	**252**	**100.0**	**63**	**100.0**	**55**	**100.0**	**370**	**100.0**

It is discovered from Table 4.22 that, the majority of the DME PSTs (59.5), and few of the others, precisely, 22.2% and 38.2% of the chemistry and PDs respectively, described the AM in the scenario as "an average" whereas majority from the chemistry and PD (52.4% and 40.0% respectively) described it using the computational algorithm: as "Adding their weight and diving by their number". None of the three departments could describe the AM as either a "representative value" or a "typical value" of a data set. The no response increased drastically across the departments.

Bringing all the groups together, exactly 50% of all the participants described the AM as an average with 33.8% of them describing it with the computational algorithm "Adding their weight and diving by their number".16.2% of them did not respond to the item.

The students were further asked to indicate the purpose for finding the AM (item 6). The responses of the PSTs on this item are presented in Table 4.23

4.3.2 Analysis of item 6

What is the purpose of finding the arithmetic mean of a data set?

Box 10

Table 4.23 Students' Responses on Item 6

Response	Mathematics		Chemistry		Physics		Total	
	N	N %	N	N %	N	N %	N	N %
To get a representative value	21	8.3	1	1.6	3	5.5	25	6.8
To get the Average of the Data	174	69.0	44	69.8	27	49.1	245	66.2
For drawing conclusions/making generalizations	8	3.2	1	1.6	3	5.5	12	3.2
To know the Middle Number	7	2.8	2	3.2	3	5.5	12	3.2
Others	17	6.7	7	11.1	10	18.2	34	9.2
No Response	25	9.9	8	12.7	9	16.4	42	11.4
Total	**252**	**100.0**	**63**	**100.0**	**55**	**100.0**	**370**	**100.0**

Information gathered in Table 4.23 shows majority across the three departments (Mathematics (69.0%), Chemistry (69.8) and Physics (49.1%)) indicated that the purpose of finding the AM of a data is to get the average of the data set. It also came out that 2.8% of the MD, 3.2% of the CD and 2.1% of the PD indicated that they find the AM when we want to know the middle number of a data set. Also, between 1.5 and 5.6 across the departments stated that we find the AM in order to draw conclusions and make generalizations. On the other hand, just a handful from all the departments (Mathematics (8.3%), Chemistry (1.6) and Physics (5.5%)) indicated that we find the mean to get a single number to represent the data set (representative value). As many as between 16% and 35% across departments either did not provide any response to the item or gave responses that had no bearing on the AM. Some of those responses included; "to give us the actual or exact data needed for analysis"; "to determine the frequency of each student"; "to determine the maximum and minimum mark each person would get", "for easy analysis" etc.

The overall responses of the respondents indicate that a majority of them, 66.2%, have conceptualized the AM as an average of a data set, with less than 7.0% describing it as a representative value. As many as 20.6% of them either did not respond to the item or gave wrong responses to the item.

4.3.3 Analysis of item 1

During the last five (5) days the temperature in degrees in Dubai were recorded as: Day 1 = 28, Day 2 = 29, Day 3 = 30, Day 4 = 32, Day 5 = 36. Do you think there is a one single number that can represent the temperature in degrees during the last 5 days? Yes: [ ] No: [ ] Explain your answer:…………………………………………………………… …………………………………………………………………………………...

Box 6

The responses of the PSTs from the three departments are presented in the Tables below.

Table 4.24 PTS' Responses on Item 1

	Department							
	Mathematics		Chemistry		Physics		All	
Response	N	N %	N	N %	N	N %	N	N %
Yes	153	60.7	35	55.6	39	70.9	227	61.4
No (Wrong)	96	38.1	22	34.9	14	25.5	132	35.7
No Response	3	1.2	6	9.5	2	3.6	11	3.0
Total	**252**	**100.0**	**63**	**100.0**	**55**	**100.0**	**370**	**100.0**

From Table 4.24 it can be seen that majority of the students across the three departments agreed to the fact that a single number can be used to represent the temperature for the last five days. Between 25.0% and 39.0% responded in the negative. In total 227 representing 61.4% answered correctly the question with 35.7%

going the opposite way, and 3.0% didn"t respond to the question. Students were further asked to justify their answers. Their justifications are shown in Tables 4.3.4 and 4.3.5.

Table 4.25 Students' Justification of Why a Single Number Can Be Used

	Department							
	Mathematics		Chemistry		Physics		All	
Response	**N**	**N %**	**N**	**N %**	**N**	**N %**	**N**	**N %**
By adding all the temperatures and dividing by the number of days	47	30.7	2	5.7	10	25.6	59	26.0
By finding the mean/ average	96	62.7	23	65.7	23	59.0	142	62.6
No response	10	6.5	10	28.6	6	15.4	26	11.4
Total	**153**	**100.0**	**35**	**100.0**	**39**	**100.0**	**227**	**100.0**

Table 4.26 Students' Justification of why a single number cannot be used

	Department							
	Mathematics		Chemistry		Physics		All	
Response	**N**	**N %**	**N**	**N %**	**N**	**N %**	**N**	**N %**
The Temperature is not the same	46	47.9	12	54.5	6	42.9	64	48.5
Finding the mean doesn't give a single number	46	47.9	8	36.4	8	57.1	62	47.0
No Response	4	4.2	2	9.1	0	0.0	6	4.5
Total	**96**	**100.0**	**22**	**100.0**	**14**	**100.0**	**132**	**100.0**

Explaining their choice of selecting yes as shown in Table 4.24, it can be seen that more than 58% across the three departments justified their answer as finding the mean (average). Of the 30.7% of participants from the DME, 5.7% of the Chemistry and 25.6% of the Physics who responded affirmatively, indicated that their justification was based on "adding all the temperatures and dividing by the number of days". A handful across the three departments couldn"t justify their responses. 47.9%, 54.5% and 42.9% from mathematics, chemistry and physics respectively said "the temperature is not the same" therefore a single number cannot be used as shown in Table 4.26. "Finding the mean doesn"t give a single number" was the justification given by 47.9% of the mathematics, 36.4% of the chemistry and 57.1% of the Physics who responded negatively. These students were confused about the difference between single digits and single number

The overall responses of the respondents indicate that, though two hundred and twenty-seven (227) representing 61.4% (Table 4.25) correctly indicated that a single number can represent the temperature during the last five days, 88.6% of them gave a correct justification thus: finding the mean or adding the all the temperatures and dividing by the number of days (which is also the mean). The rest didn"t give any justification for their answer. From Table 4.26 it can be seen that majority of them didn"t know the property of the mean which states that "the mean does not necessarily coincide with one of the values which are composed by it" (Strauss & Bichler, 1988). Also, they didn"t know the mean as a representative value

Research indicates most students view the arithmetic mean as a procedure (McGatha, Cobb, & McClain, 2002), and often do not understand it as a fair-share distribution of the data (Sirnik & Kmetič, 2010). In finding out the conceptual understanding of the AM, the researcher wanted to find out if students understand the mean as a fair share notation. Item 2 tested students on it. Their responses are presented in Tables 27, 28 and 29.

4.3.4 Analysis of item 2

Friends decided to share the cookies they brought to their party. Each one brought a different number of cookies, but Dela brought the biggest number (6 cookies). When they were given the shared cookies, each one received 8 cookies. Do you think this could happen?

Yes: [ ] No: [ ]

Explain your answer:

……………………………………………………………………

Box 7

The responses of the participants for the three departments represented in Table 4.27 and their reasons for answering in the affirmative or not are shown in Tables 4.28 and 4.29 respectively.

Table 4.27 Students' Responses on Item 2

	Department							
	Mathematics		Chemistry		Physics		All	
Response	**N**	**N %**	**N**	**N %**	**N**	**N %**	**N**	**N %**
Yes (Wrong)	62	24.6	19	30.2	7	12.7	88	23.8
No	189	75.0	42	66.7	39	70.9	270	73.0
No Response	1	0.4	2	3.2	9	16.4	12	3.2
Total	**252**	**100.0**	**63**	**100.0**	**55**	**100.0**	**370**	**100.0**

Table 4.28 Students' Justification for Selecting Yes

Response	Department							
	Mathematics		Chemistry		Physics		All	
	N	N %	N	N %	N	N %	N	N %
The biggest number of cookies was 6	19	30.6	2	10.6	1	14.3	22	25.0
The number of friends and the number of Cookies they brought was not given	21	33.9	5	26.3	2	28.6	28	31.8
Others	20	32.3	7	36.8	3	42.8	30	34.1
No response	2	3.2	5	26.3	1	14.3	8	9.1
Total	**62**	**100.0**	**19**	**100.0**	**7**	**100.0**	**88**	**100.0**

Table 4.29 Students' Justification for Selecting No

Response	Department							
	Mathematics		Chemistry		Physics		All	
	N	N %	N	N %	N	N %	N	N %
The biggest number of cookies was 6 so no one can receive more than 6	102	54.0	24	57.1	24	61.5	150	55.6
The number of friends and the cookies they brought was not given	32	16.9	11	26.2	3	7.7	46	17.0
Other	46	24.3	5	11.9	9	23.1	60	22.2
No Response	9	4.8	2	4.8	3	7.7	14	5.2
Total	**189**	**100.0**	**42**	**100.0**	**39**	**100.0**	**270**	**100.0**

It can be seen from Table 4.27 that majority (between 66.6% and 75.1%) of the participants across the department answered negatively. Of the MD 24.6, answered affirmatively likewise 30.2% of the CD and 12.7% of the PD. However, when they had to justify their response for correctly selecting no, as shown in Table 4.29, majority (MD (54.0%), CD (57.1) & PD (61.5)) of them attributed their justification to the fact that since the highest cookies brought to the party was 6 no one can receive 8. Among the other responses, between 7.7% and 26.3% across the departments said, this cannot happen because the number of friends and the cookies they brought was not given. The rest either could not give any justification for their answer or gave other reasons, some of which are; “Dela may think that he has been cheated since he brought bigger than others. Therefore, he would think he should get bigger share”, “because the probability of the cookies to be received is half of the one Dela brought to the party but not more than”, “the reason is that the cookies are to be shared according to the number each presented”

For the justification of those who erroneously selected yes, as indicated in Table 4.28, between 35.5% and 63.2% of the participants across the departments could not give any justification for their answer or gave other reasons, some of which are; “ because the individual items were gathered and then shared equally among them”, “this could happen because, maybe the cookies they brought were more than the friends that gave shared cookies”, “ adding a different number of cookies together including Dela‟s and striking an average could result in each one receiving 8 cookies”, “this is because, in statistics, the mode which is the biggest is always smaller than mean in a given data”. Explaining their choice of selecting yes as revealed in Table 4.28, it can be seen that between 7.6% and 26.3% of all the departments pointed out that since the highest number of cookies was six (6) each one can receive 8. Of participants of the DME, 30.6%, 10.6% of the CD and 14.3% of PD said the number of friends and the cookies they brought was not given so it could be possible.

With the overall responses to question 2, it can be concluded that though two hundred and seventy (270), representing 73.0%, of the respondents, correctly indicated that it cannot happen (Table 4.27), only 150 (representing 55.6%) of them gave a correct justification that "the highest number of cookies was 6 so no one can receive more than 6". Nevertheless, quite a number from each department could not justify their responses.

Participants were also given discrete frequency distribution, and asked whether or not it could be possible for the AM of the distribution to be 10, without doing any calculation. This item was adapted from Armah‟s (2017) dissertation. This was to assess whether students are familiar with the property of the AM which states that "the AM of a data set can only take on values between the extremes" (Strauss & Bichler, 1988).

4.3.5 Analysis of item 3

The table below represents the distribution of ages of a group of students taking a course.

Age (x)	5	6	7	8	9
Freq. (N)	2	3	3	2	1

3.A student said the arithmetic mean of the data is 10. Without calculating, can this be true?

Yes: [ ] No: [ ]

Give reason(s) for your answer?

…………………………………………………

Box 8

The responses of the participants for the three departments when asked whether the AM of a distribution can be 10, which is greater than the highest age in the distribution is represented in Table 4.30. Their reason for answering in the affirmative or not are shown in Tables 4.3.10 and 4.3.11 respectively.

Table 4.30 Students' Responses on Item 3

Response	Mathematics N	Mathematics N %	Chemistry N	Chemistry N %	Physics N	Physics N %	All N	All N %
Yes (Wrong)	13	5.2	3	4.8	4	7.3	20	5.4
No	239	94.8	54	85.7	47	85.5	340	91.9
No Response	0	0.0	6	9.5	4	7.3	10	2.7
Total	**252**	**100.0**	**63**	**100.0**	**55**	**100.0**	**370**	**100.0**

Table 4.31 Students' Justification of Why the AM Can Be 10

Response	Mathematics N	Mathematics N %	Chemistry N	Chemistry N %	Physics N	Physics N %	All N	All N %
The next number in the series is 10	10	76.9	1	33.3	1	25.0	12	60.0
From Calculation	2	15.4	0	.0	1	25.0	3	15.0
No Response	1	7.7	2	66.7	2	50.0	5	25.0
Total	**13**	**100.0**	**3**	**100.0**	**4**	**100.0**	**20**	**100.0**

Table 4.31 Students' Justification of Why the AM Cannot Be 10

Response	Mathematics N	Mathematics N %	Chemistry N	Chemistry N %	Physics N	Physics N %	Total N	Total N %
The AM of a set of data is always between the highest and the lowest value of a set of data	18	7.5	5	9.3	2	4.3	25	7.4
10 is greater than the highest value in the data set.	42	17.6	10	18.5	13	27.7	65	19.1
From Calculation	162	67.8	34	63.0	27	57.4	223	65.6
No Response	17	7.1	5	9.3	5	10.6	27	7.9
Total	**239**	**100.0**	**54**	**100.0**	**47**	**100.0**	**340**	**100.0**

The responses show that just a handful thus between 5.1% and 7.4% across the three departments responded in the affirmative, whiles a majority of all the departments, more than 85% responded in the negative (Table 4.30). 76.9% of the mathematics, 33.3% of the chemistry and 25.0% of the PDs, who answered in the affirmative, saw the ages as arithmetic progression (AP) series and as such their justification was "10 was the next number in the series. Fifteen percent (15%) and 25.0% of participants from mathematics and PDs respectively, who answered affirmatively, indicated that their justification can be attributed to the outcome they obtained after computing the AM of the data. Nevertheless, between 7.6% and 66.8% as can be seen in Table 4.31 could not justify their responses.

Though the participants were not to do any computation, from Table 4.31, between 57.3% and 67.9% of those who responded in the negative also attributed their response to the answer they had after computing the AM for the data. Of the participants from the DME, 17.6%, 18.5% of the CD and 27.7% of the PD could point out that "10 was greater than the highest value (9) of the distribution" hence it could not be the AM of the data set. Between 4.2% and 9.4% of the respondents pointed out that "The AM of a set of data is always between the highest and the lowest value of a set of data" therefore 10 cannot be the mean. The rest could not give any justification for their answer.

With the overall responses on item 3, it can be concluded that though three hundred and forty (340), representing 91.9%, of the respondents, correctly indicated that the AM cannot be 10 (Table 4.30), only 90 (representing 26.5%) of them gave a correct justification that either "10 is greater than the highest age in the data" or "10 is not in the middle of the data".

To further investigate participants" conceptual understanding of the AM, participants were asked if the AM of data can be zero (0). Their responses are shown in Table 4.32.

4.3.6 Analysis of Item 9

Can the arithmetic mean of a data be zero? Yes: [ ] No: [ ] When can that be? ……………………………………………………….

Box 11

The responses of the participants for the three departments when asked whether the AM of a data can be zero are represented in Table 4.32. Their reason for answering in the affirmative or not are shown in Tables 4.3.13 and 4.3.14 respectively.

Table 4.32 Students' Responds on Whether the AM Can Be Zero

	Department							
	Mathematics		**Chemistry**		**Physics**		**All**	
Response	**N**	**N %**	**N**	**N %**	**N**	**N %**	**N**	**N %**
Yes	108	42.9	17	27.0	12	21.8	137	37.0
No (Wrong)	136	54.0	42	66.7	37	67.3	215	58.1
No Response	8	3.2	4	6.3	6	10.9	18	4.9
Total	**252**	**100.0**	**63**	**100.0**	**55**	**100.0**	**370**	**100.0**

Table 4.33 Students' Response on When the AM Can Be Zero

Response	Mathematics		Chemistry		Physics		Total	
	N	N %	N	N %	N	N %	N	N %
When all data values are zero (0)	29	26.9	8	47.1	3	25.0	40	29.2
When data values comprise both positive and negative numbers	24	22.2	2	11.8	4	33.3	30	21.9
When the total Frequency is Equal to Zero	43	39.8	1	5.9	4	33.3	48	35.0
No Response	12	11.1	6	35.3	1	8.3	19	13.9
Total	**108**	**100.0**	**17**	**100.0**	**12**	**100.0**	**137**	**100.0**

Table 4.35 Students' Justification on Why the AM Cannot Be Zero

Response	Department							
	Mathematics		Chemistry		Physics		All	
	N	N %	N	N %	N	N %	N	N %
Unless there is no data/ Impossible	45	33.1	7	16.7	10	27.0	62	28.8
The mean is always greater than zero (0)	48	35.3	11	26.2	9	24.3	68	31.6
No justification	43	31.6	24	57.1	18	48.6	85	39.5
Total	**136**	**100.0**	**42**	**100.0**	**37**	**100.0**	**215**	**100.0**

It can be seen from Table 4.32 that majority (between 53.9% and 68.0%) of the participants across the department answered negatively when asked if the AM can be zero. More than forty percent (42.9%) of the MD, 27.0% of the CD and 21.8% of the PD correctly answered in the affirmative. However, when they had to justify their response for correctly selecting yes, as shown in Table 4.33, majority (between 49 and

59) across the department gave some correct justifications thus; “AM can only be zero when all data values are zero” and “the AM can only be zero when data values comprise of both positive and negative numbers”. “When total Frequency is Equal to Zero” was the justification given by majority (39.8%) of the MD, 5.9% of the CA and 33.2% of the PD. Quite a number also failed to justify their answer (Mathematics (11.1%), Chemistry (35.3%) and Physics (8.3%)).

For the justification of those who erroneously selected no, as indicated in Table 4.35, between 41.5% and 57.2% of the participants across the departments could not give any justification for their answer. Explaining their choice of selecting no as revealed in Table 4.35, it can be seen that between 24.2% and 35.4% of all the departments pointed out that the AM is always greater than zero (0) therefore it cannot be zero. A little bit over thirty-three percent (33.1%) of the MD, 16.7% of the CD and 27.0% of PD said the AM can‟t be zero unless there is no data.

With the overall responses on item 9, it can be concluded that though one hundred and thirty-seven (137), representing 37.0%, of the respondents, correctly indicated that the AM of a set of data can be zero (Table 4.32), only 70 (representing 18.9%) of them gave a correct justification that either “When all data values are zero (0)” or “When data values comprise of both positive and negative numbers”. Majority of the respondents (58.1%) have conceptualized the AM to be a non-zero number.

4.3.7 Discussion of Results on Students’ Conceptual Understanding of the Arithmetic Mean

The arithmetic mean's connection to other knowledge spaces that help cultivate its understanding can be obscured by the relatively simple calculation for quantifying it. The concepts of fair share, the centre of balance, and typical or representative value are three such knowledge spaces. According to research, most students regard the arithmetic mean as a procedure (McGatha et al 002), and do not recognize it as a fair-share distribution of the data (Mokros & Russell, 1995), the data set's centre-of-balance (Pollatsek, Lima, & Well, 1981), or a typical or representative value (Pollatsek, Lima, & Well, 1981). Furthermore, even for those with a strong understanding of statistics, articulating a link between the concepts of fair-share, the centre of balance, and a typical value can be difficult (MacCullough, 2007). By connecting the three conceptualizations of the arithmetic mean to each other and mathematical concepts, we can build a web of understanding for the arithmetic mean. The researcher was able to conduct a detailed analysis of pre-service teachers' output using the theoretical categories provided by Biggs and Collis, capturing the complexity of the common content knowledge (representations, concepts, properties, etc.) that were activated when they were asked to solve problems involving the arithmetic mean.

In the case of item 1, while the majority of departments correctly answered yes, only a few provided a valid justification. The arithmetic mean was not recognized by some as the representative value for the temperatures given. Students' primary solution strategies for arithmetic mean problems, according to previous research, are based on the arithmetic mean formula (Cai, 1998; Groth & Bergner, 2006; Groth, 2009; Armah, 2017). This previous finding was confirmed in the current study. It was revealed that most students have conceptualized the mean as a computational act with the “add them up and divide algorithm.”

Only a handful of the participants, described the arithmetic mean as either a typical or a representative value of a data set as described by some researchers. Again, when participants were asked whether the AM can be zero, few were able to point out that the AM can be zero but less than half were able to give a correct justification. Most of the participants have conceptualized the AM as always, a non-zero value.

According to the test results, PST have conceptualized the arithmetic mean to be an average as well as a computational act using the "add them up and divide algorithm" after going through the JHS and SHS syllabi. They were able to describe the algorithm for computing the arithmetic mean without any external prompts, but they were unable to demonstrate a conceptual understanding of the algorithm's result. They also didn't see the arithmetic mean as a distinct entity with any properties, and they couldn't reverse the algorithm's process.

They defined the arithmetic mean as the "sum of numbers in a data set divided by the number of values in the data set" as a computational act. Despite the fact that they are all skilled at this act or skill, they have no idea what the result of this act or skill represents or how it relates to the set from which it was computed. They simply switch back and forth between "the arithmetic mean is an average" and "the sum of data values divided by the number of data values" (Groth & Bergner, 2006, Armah, 2017).

4.4 Research Question 3. Pre-Service Teachers' Level of Conceptual Understanding of the AM with respect to the SOLO Taxonomy

To respond to research question 3, students' responses were categorized. The classification is based on Biggs and Collis' theory (1982; 1991). SPSS was used to code the students' working processes. The various techniques used by students to solve the test items were given codes. Meaning coding, according to Kvale and Brinkman (2009), entails attaching one or more keywords to a text segment in order to allow identification later. As a result, the codes were organized into themes, with frequency and percentages reported in this study.

The Part B of SUTAM consisted of 12 items, of which five (5) (items; 4, 7, 8, 10 and11) were based on the SOLO taxonomy (see Appendix C). Item 7 was on the Pre-structural level, question 4 assessed students on the Uni-structural level, questions 8, 11 and 10 assessed students on the Multi-structural level, Relational level and Extended Abstract level respectively. The number of students getting each of the questions correct and wrong together with their corresponding percentages were captured as shown in section 4.2 (Tables; 4.2.3, 4.2.1, 4.2.5, 4.2.9 and 4.2.7 for the five levels respectively). Therefore, Table 4.36 is a summary of the number of students getting each question at each level correct or wrong.

Table 4.36 Summary

Levels		Department							
		Mathematics		Chemistry		Physics		Total	
		N	N %	N	N %	N	N %	N	N %
Pre-structural	Correct	191	75.8	49	77.8	43	78.2	283	76.5
	Wrong	60	23.8	9	14.3	9	16.4	78	21.1
	No Response	1	.4	5	7.9	3	5.5	9	2.4
Uni-structural	Correct	149	59.1	25	39.7	32	58.2	206	55.7
	Wrong	99	39.3	32	50.8	17	30.9	148	40.0
	No Response	4	1.6	6	9.5	6	10.9	16	4.3
Multi-structural	Correct	140	55.6	27	42.9	33	60.0	200	54.1
	Wrong	98	38.9	24	38.1	15	27.3	137	37.0
	No Response	14	5.6	12	19.0	7	12.7	33	8.9
Relational	Correct	88	34.9	13	20.6	9	16.4	110	29.7
	Wrong	157	62.3	41	65.1	38	69.1	236	63.8
	No Response	7	2.8	9	14.3	8	14.5	24	6.5
Extended Abstract	Correct	63	25.0	7	11.1	8	14.5	78	21.1
	Wrong	187	74.2	52	82.6	41	74.6	280	75.7
	No Response	2	.8	4	6.3	6	10.9	12	3.2

Question 7 in the SUTAM was used to assess students on the Pre-structural level; it asked students to find the AM from raw data. The quantitative results in Table 4.36 shows that majority (more than 75%) across the three departments got the question correct. This implies that if the students have raw data, majority of them can compute the arithmetic mean. Between 14.2 and 23.9 across the department attempted this but got it wrong due to wrong computations.

Question 4 was used to assess students on the uni- structural level; participants were also asked to compute the AM of the data in the frequency table. Statistics from Table 4.36 show that 59.1% of MD, 39.7% of CD and 58.2% of the PD were able to compute it correctly. The remaining percentage failed to attempt or got it wrong.

Question 8 asked students to compute the minimum grade needed for a student to get 85 or better overall. This was used to assess students on a multi-structural level. The quantitative results in Table 4.36 show that between 42.8 and 60.1 across were able to calculate the minimum grade needed to get 85 or better.

Question 11 was used to assess students on the Relational level. Students were asked to compute the AM from a histogram. Results from Table 4.36 show that minorities (less than 35) across the three departments were able to compute the mean from the histogram presented. Between 62.2% and 69.2% got it wrong due to wrong computation or lack of understanding.

Question 10 asked students to compute the combined mean of two different groups, this was used to assess students on the last level of the SOLO taxonomy thus Extended Abstract. The number of students getting each question correct across the department reduces drastically as shown in Table 4.36. The majority (more than 74.1) across the departments were unable to compute the mean from this question. The students demonstrated a lack of understanding of the solutions they presented.

As a combined group, 76.5%, 55.7%, 54.1%, 29.7% and 21.1% were able to compute the AM on each of the five (5) levels respectively. This shows that as the level increases the number of students answering questions correctly at that level reduces.

Marks were allocated to each question at each level depending on the demand of the question (See Appendix D). Table 4.37 present the overall scores of students.

Table 4.37 Overall Scores of Students On SUTAM Based on SOLO

	Department							
	Mathematics		Chemistry		Physics		Total	
Scores	**N**	**N %**	**N**	**N %**	**N**	**N %**	**N**	**N %**
0	33	13.1	7	11.1	9	16.4	49	13.2
2	39	15.5	16	25.4	13	23.6	68	18.4
6	63	25.0	15	23.8	9	16.4	87	23.5
10	65	25.8	16	25.4	14	25.5	95	25.7
16	37	14.7	8	12.7	9	16.4	54	14.6
20	15	6.0	1	1.6	1	1.8	17	4.6
Total	**252**	**100.0**	**63**	**100.0**	**55**	**100.0**	**370**	**100.0**

The results in Table 4.2.2 show that 55.1% (204) of the students obtained less than half of the total score, while 19.2% (71) obtained more than half. The results further showed a score of 10 has the highest frequency with 20 with the least frequency. This indicates that the general performance of students in the SUTAM was weak.

4.4.1 Levels Reached by Students on the SOLO Taxonomy

The researcher categorized the number of students who reached each level of the SOLO taxonomy in terms of both frequency and percentages. Pre-service teachers' abilities to compute the AM from five different questions were found to have five distinct levels of thinking. The first two levels mirrored the type of concrete-symbolic thinking that elementary school students might use to learn the concepts. The levels of the SOLO taxonomy as demonstrated by students are listed in Table 4.38. This analysis is similar to the one used to classify pre-service teachers' common content knowledge of the arithmetic mean in Spain. The frequency and percentage of students who reached each level of the SOLO taxonomy are shown in Table 4.38.

Table 4.38 Levels of the SOLO Taxonomy as Demonstrated by Students

	Department							
	Mathematics		**Chemistry**		**Physics**		**Total**	
Level	**N**	**N %**	**N**	**N %**	**N**	**N %**	**N**	**N %**
Pre-structural	72	28.6	23	36.5	22	40.0	117	31.6
Uni-structural	63	25.0	15	23.8	9	16.4	87	23.5
Multi-structural	65	25.8	16	25.4	14	25.5	95	25.7
Relational	37	14.7	8	12.7	9	16.4	54	14.6
Extended Abstract	15	6.0	1	1.6	1	1.8	17	4.6
Total	**252**	**100.0**	**63**	**100.0**	**55**	**100.0**	**370**	**100.0**

From Table 4.38, 28.6% (72), 36.5% (23) and 40.0% (22) of the participant from MD, CD and PD respectively reached the Pre-structural level, while 25.0% (63)

of students from MD, 23.8% (15) from CD and 25.5% (9) from PD reached the Uni-structural levels of the SOLO taxonomy. The students who reached the Pre-structural level were able to solve only question 7 or were not able to solve any of the questions. However, the students who reached the Uni-structural level were able to answer only two (2) questions (4 & 7) out of five (5) questions correctly. At the Uni- structural stage, the subject (PTS) can use a clear and straightforward piece of information from the problem This implies that the majority of the PST who reached these two levels were either incompetent or could only follow simple procedures such as finding the mean from a raw data or a given frequency distributions table.

Also, at the multi-structural level, PST can use two or more pieces of information from the given question, and the subject can understand the question asked. Therefore, from Table 4.38 between 25.3% and 25.9% across the department reached the Multi-structural level. This indicates that the number of students reaching this level could find the minimum grade needed to get 85 or better overall. However, the students who reached the Multi-structural level were able to answer pre-structural (Question 7), Uni-structural (Question 3) and Multi-structural correctly (Question 8) but couldn‟t solve the remaining two questions (10 &11). They lacked the concepts of finding the mean from a histogram.

The item on the Relational level of the taxonomy was on finding the AM from a given histogram. At the relational level, the subject can think by using two pieces of information or more of the given problem so that from the information the subject can determine the AM. 14.7% (37), 12.7% (8) and 16.4% (9) of the participants from the MD, CD and PD respectively were able to reach the Relational level.

Finally, 6.0% (15) of the MD, 1.6% (1) of the CD and 1.8% (1) of the PD students reached the Extended Abstract level of the taxonomy. At the extended abstract level, the subject can relate the information and conclude to build new concepts and apply them for possible application to other issues.

As a combined group, 31.6% (117) of the students reached the Pre-structural level, while 23.5% (87) reached the Uni-structural levels of the SOLO taxonomy. This leaves the number of students reaching the lowest levels of the SOLO taxonomy at 55.1% (204). This implies that the majority of the PST who reached these two levels were either incompetent or could only follow simple procedures. Also, 25.7% (95) of the students reached the Multi-structural level. I4.6% (54) were able to reach the relational level of the SOLO taxonomy. Finally, only 4.6% (17) reached the last level of the SOLO taxonomy thus the Extended Abstract. This indicates less than 5% of the students were able to answer all questions about the AM in the SUTAM

Figures 4.1 and 4.2 present in percentages the proportion of students reaching the five original levels of the SOLO taxonomy as suggested by Biggs and Collis.

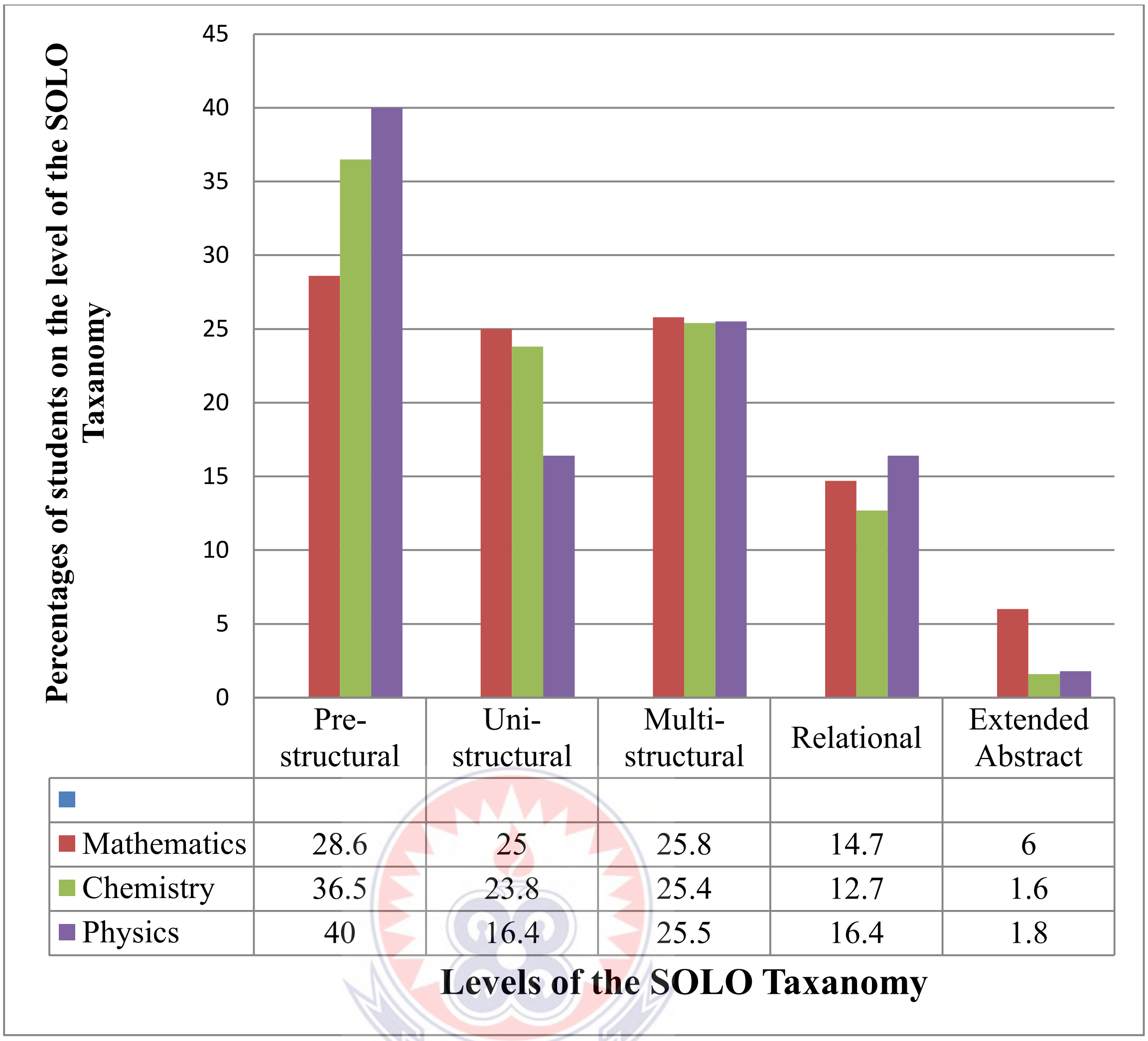


Figure 4.1: *Bar Chart Showing Levels Reached by Students on SOLO Taxonomy for the three departments.*

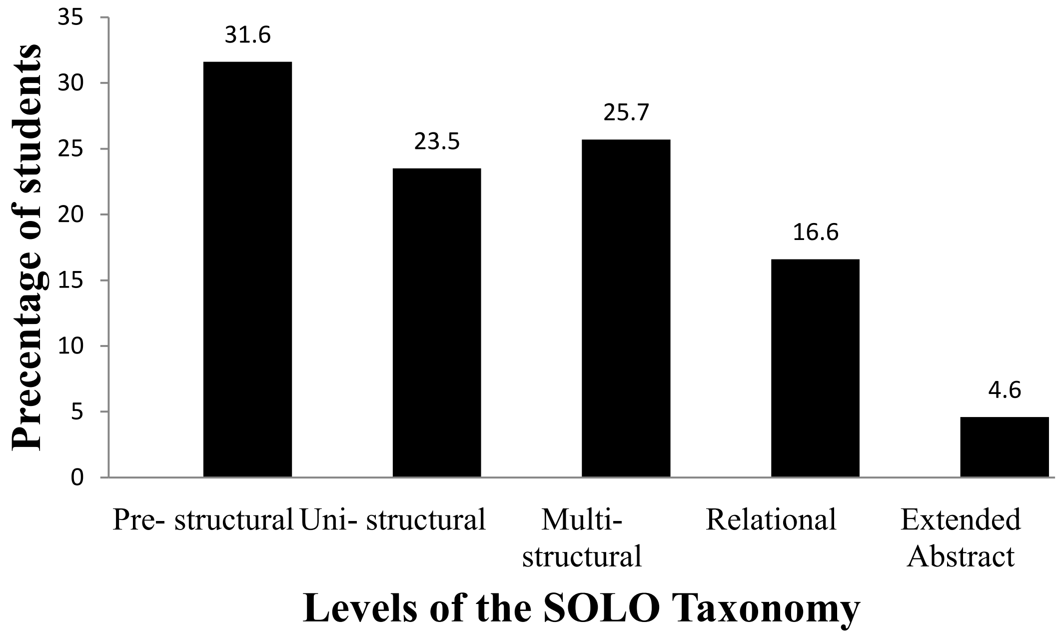


Figure 4.2 *Bar Chart Showing Levels Reached by all Students on SOLO Taxonomy*

4.4.2 Discussion of Results on Students' Level Conceptual Understanding of the Arithmetic Mean

The researcher was able to conduct a detailed analysis of pre-service teachers' output using the theoretical categories provided by Biggs & Collis (1982; 1991), capturing the complexity of the common content knowledge (representations, concepts, properties, etc.) that were activated when they were asked to solve problems involving the arithmetic mean. Indeed, the cognitive configurations of primary mathematical objects derived from this analysis show that the PST solved these statistical problems using a wide range of common content knowledge.

Students provided a variety of responses based on the 370 students who were chosen. Starting with the students who responded at the lowest level, or pre-structural level, it was discovered that these students fall into the following categories based on their answers: (1) The student is unable to process information correctly, (2) has misconceptions, (3) the steps used have no meaning, and (4) does not consistently give the answers to the problems. When the problem does not lead to the desired solution, the path chosen by students does not lead to the desired solution, and the algorithm does not correspond to a higher level, student answers are insufficient (Ozdemir & Yildiz, 2015).

The next uni-structural level, which can be defined as the students who fall into the uni-structural level category based on the results of the answers that have been analysed: (1) Students do not make the most of available information, (2) Students only focus on what will be sought without understanding the value or meaning of the table, (3) Students only connect information rationally, and (4) The results found are less precise. Students who only focus on problems and use relational steps do not understand the value of existing data or the relationships between data and others, resulting in inconsistency in student responses (Ozdemir & Yildiz, 2015). Students can do the initial process that begins with an example until the selection of steps is used, but students do not understand exactly how the real problem is (Caniglia& Meadows, 2018).

The third level is the multi-structural level, from the results of answers that have been analysed it can be said to the students included in the category of multi-structural level: (1) Students understand one concept that is an example with variables and can model mathematics, (2) But still do not understand the concept of the target function, (3) The steps that are used are appropriate, (4) It is less precise in finding points. Students have been able to apply some data that can lead to ideas in the use of rare steps to find solutions, but have not been able to understand the relationship contained in existing information, consequently, student answers become inconsistent (Ozdemir & Yildiz, 2015). Students can understand what is asked by the problem, but students have not been able to connect information with existing concepts (Caniglia& Meadows, 2018).

The fourth level is the relational level, from the results of answers that have been analysed, it can be said that the students are included in the relational level category: (1) Students can process information appropriately, (2) Chose steps algorithmically and systematically, (3) Students understand concepts that exist, (4) The answers generated are already fulfilling and correct, and (5) Unfortunately, students have not been able to conclude these questions. Students can use all information so that they can provide answers to problems and students understand the meaning of overall information and can understand the relationship between data, consequently, students can build structures consistently (Caniglia& Meadows, 2018). Students can connect concepts that exist as a whole with a harmonious and meaningful understanding (Caniglia & Meadows, 2018).

The last level is extended abstract, from the results of answers analysed, it can be said to be in the extended abstract level category: (1) Students have a good understanding of concepts, (2) the information in the questions is put to good use, (3) the relationship between concepts and the application of information is properly organized, (4) the steps used are algorithmic, (5) the answers given have answered the problem's question and are correct, and (6) students draw conclusions from the problem story. Students' ability to generalize and create new ideas is very high when it comes to using steps to solve these problems. Overall, information and concepts are abstracted to a higher level (Caniglia & Meadows, 2018).

The quantitative analysis of levels reached by students on the SOLO Taxonomy showed that the majority of the students reached the lowest levels of the taxonomy that is the pre-structural and the Uni-structural level, 25.7% reached the multi-structural level. Also, 14.6% reached the Relational level and only 17(4.6%) out of the 370 students sampled for this study reached the last level which is the Extended abstract. These results were not different from previous research (Groth& Bergner, 2006; Randall & Jennifer, 2006; Laisouw, 2013; Ozdemir & Yildiz, 2015; Hasan, 2017; Caniglia & Meadows, 2018; Claudia, Kusmayadi, & Fitriana, 2020) that also used the SOLO taxonomy to assess students‟ level of thinking.

CHAPTER FIVE

SUMMARY, CONCLUSION AND RECOMMENDATIONS

5.0 Overview

This chapter provides a summary of the study and the major findings. It highlights the conclusion of the study and its implications for practice. It further outlines some recommendations and avenues for future research.

5.1 Summary

The study explored undergraduate mathematics, chemistry and physics students" conceptual understanding of the arithmetic mean at their entry stage into the university, based on their mathematical learning experiences at the JHS and SHS. It also sought to investigate their achievement in questions relating to the arithmetic mean, it further sought to use SOLO taxonomy as a theoretical framework in an attempt to investigate the level of conceptual understanding of the students. In pursuance of the purposes stated above, the following research questions were formulated to guide the study:

1. What is pre-service teachers" performance on mathematical tasks to the arithmetic mean?
2. What conceptual understanding of arithmetic mean do pre-service teachers have?
3. What level of conceptual understanding with respect to the SOLO taxonomy do pre-service teachers have about the arithmetic mean?

The general approach used in this research was an explanatory sequential mixed method. The population was made of all first-year students admitted into the

Department of Mathematics, Chemistry and Physics in UEW in the 2020/2021 academic year cohort. In all, three hundred and seventy (370) students from three departments were used as a sample for the study. They comprised two hundred and fifty-two (252), sixty-three (63) and fifty- five (55) students from the mathematics, chemistry and physics departments respectively. Statistical Understanding Test of Arithmetic Mean (SUTAM) was used as a research instrument for this study. The results from the SUTAM were used to answer the research questions. In particular, each research question was looked at from all relevant data sources.

5.2 Major Findings

The findings of the study are summarized and presented under the three sub-headings in line with the research questions.

5.2.1 Research question 1: What is pre-service teachers' performance level on mathematical tasks on the arithmetic mean?

According to the results of the SUTAM, students' overall performance was poor. The majority of students can easily find the arithmetic mean when data is in its raw form or in a frequency distribution table, but only a small percentage can solve practical questions involving the arithmetic mean, resulting in poor performance. The results of this study also revealed that there was no significant difference in mean scores between trained teachers and direct applicants. Furthermore, there was no distinction in performance between the three departments.

5.2.2 Research question 2: What conceptual understanding of the arithmetic mean do pre-service mathematics teachers have?

Students in undergraduate mathematics, chemistry, and physics have conceptualized the arithmetic mean as an average and as a computational act using the "add them up and divide algorithm," according to SUMTAM responses. It was also discovered that the participants had a misconception that the arithmetic mean must always be positive and also cannot be zero. Undergraduate mathematics, chemistry, and physics students were found to have a poor understanding of the mean as a fair-share distribution, the data set's centre of balance, or a typical or representative value. Finally, when it came to the conceptual understanding of the arithmetic mean, it was discovered that the vast majority of participants from various departments were unable to explain the mean in a given statement.

5.2.3 Research question 3: What level of conceptual understanding with respect to the SOLO taxonomy do pre-service mathematics teachers have about the arithmetic mean?

According to the quantitative analysis of SOLO Taxonomy levels reached by students, 31.6 % (117) of students reached the Pre-structural level, while 23.5 % (87) reached the Uni-structural level. This leaves 55.1 % of students achieving the lowest levels of the SOLO taxonomy (204). This means that the majority of PSTs who advanced to these levels were either incompetent or could only perform basic tasks. In addition, 25.7 % of students (95%) achieved the Multi-structural level. I4.6 % (54) of the participants were able to reach the SOLO taxonomy's relational level. Finally, only 4.6% (17) reached the SOLO taxonomy's highest level, the Extended Abstract. This means that only about 5% of students were able to correctly answer all questions about the AM in the SUTAM. There was no significant difference between PST's Arithmetic mean thinking Levels in the three departments at $p > 0.05$, according to the findings of this research question. This revealed that students in the mathematics, chemistry, and physics departments all have the same Arithmetic mean level of thinking at the start of their university professions.

5.3 Implications for Practice

It has been proposed that the best way to learn statistical concepts like the arithmetic mean is to first develop their statistical sense, or representativeness in the case of the arithmetic mean, and then connect this conceptual understanding to the governing mathematical aspects (Jones et al., 2004). Many of the participants in the current study had a limited understanding of what constitutes the arithmetic mean. Their point of view was based on the arithmetic mean formula, not on a developed understanding of the arithmetic mean as representative of the data set. The participants' mathematical knowledge of the arithmetic mean was most likely developed outside of the statistical sense of representativeness. The relevance of this study's findings in terms of embedding basic concepts in students cannot be overstated. In every teaching of mathematics or other fields of science, it is the understanding of concepts that must be improved.

Almost all of the pre-service teachers in this study understood that average and mean are the same thing. They presumed that when the word "average" is used, it should refer to the mean. The general term average, on the other hand, can obscure any of the three measures of central tendency when it comes to statistical literacy. This finding implies that the terms of measures of central tendency given in the context of everyday life situations should be clear in the curriculum specifications. As a result, mathematics teachers must plan teaching and learning activities in accordance with these curriculum specifications in order to provide students with experiences with the concept of the average in everyday life. Through such exposures, the knowledge that the word average is a reflection of central tendency and that all three measures of central tendency; the mean, the median, and the mode contribute to the idea of average can be developed.

The level of students" responses in the SOLO taxonomy can help the teacher know how students solve mathematical problems. At each level of response based on SOLO, teachers can use to design models or learning strategies to determine the causes of misconceptions. Biggs and Collis (1982) believe that teacher intervention is crucial to students, achievement of Algebra learning. They also believe that if individuals operate at different levels of the SOLO taxonomy ineffective rote learning occurs. Therefore, teachers must have a good understanding of the taxonomy and become aware of the level of statistics, particularly the arithmetic mean. This would allow them to teach effectively the subject matter.

The findings of this study revealed that subject matter knowledge is more important than procedural knowledge in improving statistical literacy. Knowing how to calculate the mean isn't enough. As a result of this finding, school textbooks should include activities that go beyond calculation and procedures for calculating the arithmetic mean. The knowledge of SOLO taxonomy can also be used by textbook authors to create more difficult questions that assess mathematical thinking. This will make it easier for teachers to evaluate their students based on the various learning outcomes they anticipate. More SUTAM tests for students in elementary and secondary school could lead to significant improvements in their performance on NEA, TIMSS, and WASSCE exams.

5.4 Conclusion

The study examines the students' experiences with the concept of arithmetic mean, which they encounter frequently in their daily lives. Despite the fact that data handling is now part of the curriculum, there are still gaps in understanding basic concepts such as the arithmetic mean. The researcher found quantitative and qualitative differences in understanding in the sample of three departments, but not to the extent expected. The researcher was able to observe various types of difficulties students have with the arithmetic mean as a result of this study. The findings show that students are unfamiliar with the concepts of typical value, representative value, and fair share, and thus have no idea how to react when confronted with such data in their arithmetic mean calculations.

Despite its fundamental nature, the results show that the students polled are unfamiliar with some of the arithmetic mean's main properties. What's more, there were no statistically significant differences in arithmetic mean performance between the trained teachers and the direct applicants. The trained teacher was obviously expected to perform at a higher level because they had studied at a higher level and supposedly benefited from greater maturity and experience. Some of them had even previously taught these concepts to others. In this case, the findings contradict those of Watson and Moritz (2000), who claim that as students‟ progress through the educational system, their level of understanding increases.

Students, in my opinion, have problems because they are unfamiliar with the conceptual aspects of the arithmetic mean. They look at the calculus algorithm and how to use the right representations, but they don't go over the most important aspects. Because most students are familiar with the calculus algorithm, they can obtain the arithmetic mean for a given set of data if that is what is asked of them, as discovered in this study. However, this procedural knowledge is unrelated to conceptual aspects, corroborating Mokros & Russell's (1995) conclusion that some students have a poor conceptual understanding of the arithmetic mean because they think of it as a pure algorithm. Students know how to use computational algorithms to calculate statistical means, but they don't use them to explain real-life situations.

Students are in the process of teaching and learning rarely faced with the following situations:

- Open-ended tasks in a mathematical and everyday context, like: What happens with arithmetic mean if one number is changed or added?
- Reverse questions in mathematical and everyday context: The arithmetic mean of two numbers below 15 is 10. What are these two numbers?
- Questions fostering reasoning and decision making, including explanation: The mean temperature at a beach is 20°C. Is the place a good choice for the summer holidays if you want to swim on a warm beach every day?

We can improve students" performance with the development of metacognitive strategies by asking questions to clarify exactly what students are trying to do or say, and we can make students more aware of the mental processes they use.

5.5 Recommendations

From the findings and conclusion above, it is recommended that;

- Lecturers and tutors in Ghanaian teacher education institutions must ensure that teacher trainees are taught the concepts of the arithmetic mean before teaching them how to compute these concepts.
- Students should be well-versed in the properties and definitions of the arithmetic mean. Real-life or problem-solving scenarios must be included or presented during teaching sessions so that students learn how to apply their newly acquired knowledge to solve everyday problems and in a variety of situations.
- Mathematics teachers must try to develop their SUTAM (Part B) to assess the thinking levels of students in every topic taught. This will enable them to plan

an appropriate intervention for each student. It will also enable the teachers to make an informed decision on how to help students improve their mathematical knowledge.

- It has been observed that teachers' statistical knowledge is frequently similar to that of their students (Groth & Bergner, 2006; Jacobbe & Horton, 2010; Leavy & O‟Loughlin 2006). As a result, teachers must employ various technologies, such as the internet, to gather information, as well as seek out various assessment tools that assess mathematical thinking, in order to have a diverse set of materials to use in their teaching and learning activities.
- Curriculum developers, textbook authors, and policymakers should examine the SOLO taxonomy for insights into how to improve learner achievement in Mathematics in general, and arithmetic mean in particular. This is due to the fact that the taxonomy has been found to help learners in Ghana improve their conceptual understanding of arithmetic mean and statistical thinking levels.

5.6 Suggestions for Further Research

The educational implication of the findings of this study calls for further research in Ghana. The following are suggested for further research:

- This study only involved 370 pre-service mathematics, chemistry and physics teachers. The subjects were drawn from the pre-service teachers who enrolled in the 4-year Bachelor of Science Education (BSc. Ed.) program at UEW. Therefore, it is recommended that the present study be extended to other pre-service mathematics teachers enrolled in a similar program in other public universities, and teachers‟ training institutes to verify and elaborate on the findings of the present study.

- This study focused on pre-service teachers‟ conceptual understanding of the arithmetic mean. Therefore, it is recommended that the study be extended to in-service mathematics teachers and school students at various levels. This may contribute toward a wider knowledge base of the teachers‟ and students‟ subject matter knowledge of measures of central tendency involved in the enhancement of statistical literacy.
- This study examined the levels of pre-service teachers‟ conceptual understanding of the arithmetic mean. The present study did not examine pre-service mathematics teachers‟ beliefs about the arithmetic mean. Thus, it is recommended that further research examine pre-service teachers‟ beliefs about the arithmetic mean as well as their beliefs about teaching and learning the arithmetic mean.
- Future studies might investigate the extent to which mathematics teachers use textbooks when teaching the concept of arithmetic mean, what representation forms they use and what solution strategies they apply when solving arithmetic mean problems. For this purpose, whether students learn the concept of arithmetic mean deeply can be revealed. The results might enable us to see the effect of textbooks on the performances of students.

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APPENDICES

Appendix A Letter for Consent

Dear Student,

I am an MPhil Mathematics Education student of the University of Education, Winneba. I am conducting a research study to enable me write my thesis. You are my chosen participants for this study. Your background in senior high core and elective mathematics is enough as a prerequisite knowledge for this test. The answers are for educational purposes and are in no way meant for individual or personal assessment. Your answers will be treated as strictly confidential, therefore, the result from this test is not going to be part of your assessment, feel free to answer all questions as frankly as possible. Your participation is very important; however, you have the right to decline to participate in the study.

If you agree to be part of this study, kindly give your consent by filling the consent form. Thank you for your consideration.

I, ________________________________, give my consent to be part of this study. I understand that all information including my student"s identification number will be kept confidential. I understand that these activities will not disrupt my program and results of the test will not form part of my assessment.

Signed: …………………………………….

Date: ………………………………………

Appendix B Introductory Letter

UNIVERSITY OF EDUCATION, WINNEBA
FACULTY OF SCIENCE EDUCATION
DEPARTMENT OF MATHEMATICS EDUCATION
P. O. Box 25, Winneba, Ghana   math@uew.edu.gh
+233 (020) 2041076

Date: *February* 10, 2021

TO WHOM IT MAY CONCERN

Dear Sir/Madam

LETTER OF INTRODUCTION

I write to introduce to you the bearer of this letter, **Godfred Kofi Osei,** a postgraduate student at the University of Education, Winneba. He is reading for a Master of Philosophy degree in Mathematics Education and as part of the requirements of the program, he is undertaking research titled – *Pre-service teachers' conceptual understanding of the mean.*

He needs to gather data to be analysed for the said research and he has chosen to do so in your institution. I would be grateful if he is given the needed assistance to carry out the exercise.

Thank you.

Yours faithfully,

……………………….

Dr. Joseph Issah Nyala
Senior Lecturer,
Dept. of Mathematics Education
University of Education, Winneba.
Email: jinyala1957@gmail.com
Tel: +233 246004930

www.uew.edu.gh

Appendix C Test for Students

UNIVERSITY OF EDUCATION, WINNEBA
FACULTY OF SCIENCE EDUCATION
DEPARTMENT OF MATHEMATICS EDUCATION

B. Sc. (MATHEMATICS, CHEMISTRY AND PHYSICS EDUCATION EDUCATION)

STUDY TEST FOR LEVEL 100 STUDENTS

PART A: PERSONAL DATA

INDEX NO. (LAST FIVE DIGITS ONLY): ………………

INSTRUCTIONS: Answer all questions. Tick as appropriate, and where possible provide short answers.

1. Gender: Male [ ] Female [ ]

2. Age: Below 20 [ ] 20 – 25 [ ]
26 – 30 [ ] 31 – 35 [ ]
36 – 40 [ ] Above 40 [ ]

3. What program did you study at the S. H. S level? ……………..……………………

4. In which year did you complete S. H. S? ………………………...…………

5. In which region did you attend S. H. S? ……………………………………..

6. Did you study Elective Mathematics in S. H. S.? Yes [ ] No [ ]

7. Have you studied any advanced mathematics after S. H. S? Yes [ ] No [ ]

8. Are you a trained teacher? Yes [ ] No [ ]

9. Have you taught mathematics before? Yes [ ]

No [ ]

10. If your answer to 9 is yes, then at what level of education did you teach?

Pre – school Level [ ] Primary Level [ ]

J. H. S. Level [ ] S. H. S. Level [ ]

11. If your answer to 9 is yes, how many years of mathematics teaching experience do you have? …………….

PART B: TEST ITEMS

Your score in this will not be part of your continuous assessment. It is strictly for research purposes. So, feel free to answer all questions as truthful as you can.

1. During the last five (5) days the temperature in degrees in Dubai were recorded as:

Day 1 = 28, Day 2 = 29, Day 3 = 30, Day 4 = 32, Day 5 = 36.

Do you think there is one single number that can represent the temperature in degrees during the last 5 days?

Yes: [ ] No: [ ]

Explain your answer:

…………………………………………………………………………………

…………………………………………………………………………………...

…………………………………………………………………………………

………..........................

2. Friends decided to share the cookies they brought to their party. Each one brought a different number of cookies, but Dela brought the biggest number (6

cookies). When they were given the shared cookies, each one received 8 cookies. Do you think this could happen?

Yes: [ ] No: [ ]

Explain your answer:

……………………………………………………………………………………

……………………………………………………………………………………...

……………………………………………………………………………………

………….....................

The table below represents the distribution of ages of a group of students taking a course. Use it to answer questions 3 and 4

Age (x)	5	6	7	8	9
Freq. (N)	2	3	3	2	1

3. A student said the arithmetic mean of the data is 10. Without calculating, can this be true?

Yes: [ ] No: [ ]

Give reason(s) for your answer?

....………………………………...…………………………………………

……..……………………………...…………………………………..........

…..………………………………...…………....................................

....

4. Calculate the arithmetic mean for the distribution.

....……………………………...……………………………………………

……..………………………………...……………………………..................

…..…………………………………………...…………...

....

5. The weights of a class of 100 sociology students were measured, and the arithmetic mean was found to be 69.5kg. What does it mean to say that the arithmetic mean of all the weights is 69.5kg?

....

....

....

6. What is the purpose of finding the arithmetic mean of a data set?

....

……………………………………………………………………………

……………………………………………………………………………

7. The ages of 10 chemistry students were recorded as 23, 24, 19, 27, 22, 20, 19, 30, 21, and 31. Compute the mean of this data.

………………………………………………………………………………

………………………………………………………………………………

………………………………………………………………………………

………………………………………………………………………………

……………………………….

8. A student has gotten the following grades on his tests: 87, 95, 76, and 88. He wants an 85 or better overall. What is the minimum grade he must get on the fifth test in order to achieve that average?

………………………………………………………………………………

………………………………………………………………………………

………………………………………………………………………………

……………………………………………………………………………………

……………………………….

9. Can the arithmetic mean of a data be zero?

Yes: [ ] No: [ ]

When can that be?

………………………………………………………………………………………

………………………………………………………………………………………

………………………………………………………………………………………

……………………….

10. There are ten people in an elevator, four women and six men. The average weight of the women is 120 pounds, and the average weight of the men is 180 pounds. What is the average of the weights of the ten people in the elevator?

……………………………………………………………………………………

……………………………………………………………………………………

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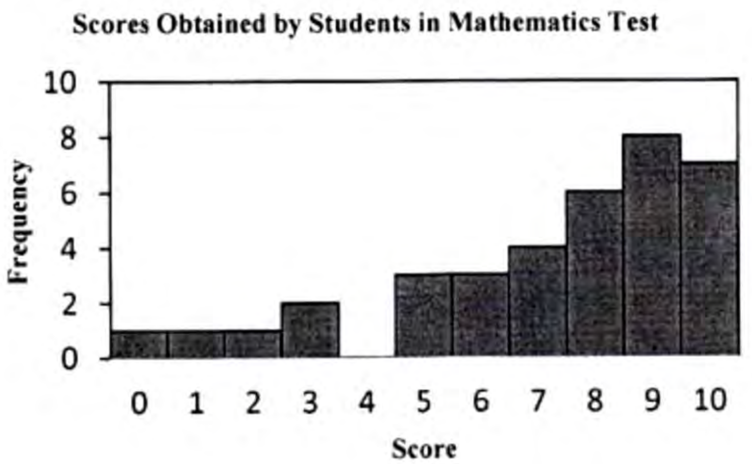
……………………………………………………………………………………

11. The scores obtained by students in a mathematics test are shown in the representation below:



Find the mean score obtained by the students.

……………………………………………………………………………………

……………………………………………………………………………………

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……………………………………………………………………………………

12. Leticia is doing research on prices of snacks in order to buy the cheapest and save her allowance. She has written down prices from three different places and has found out that the mean price of the snacks is Gh¢ 3.00.

Write T (True) or F (False) for the possible values that Leticia has found:

A. 1, 1, 1, [ ] B. 1, 3, 5, [ ] C. 3, 3, ,3 [ ]

D. 1, 3, 6, [ ] E. 9, 9, 9, [ ] F. 1, 2, 6, [ ]

Appendix D Answers to Research Instruments

(Part B of SUTAM)

1. Yes

    a. A single number can represent the temperature in the last five (5) days if the temperature of each day is added together and then divided by the number of days.

    b. By finding the arithmetic mean

2. No

    Since the biggest number of cookies brought to the party was 6, and it was shared equally between them, the highest each one can received will be less than 6.

3. No

    a. The arithmetic mean of a set of data is always between the highest and the lowest value of a set of data

    b. 10 is greater than the highest value in the data set which is 9.

4. $Mean(\bar{x}) = \frac{\sum fx}{n}$ **M1**

    $\bar{x} = \frac{74}{11}$ **M2**

    $\bar{x} = 6.73$ **A1**

5. A fair share of their weight is 69.5kg or 69.5 kg is the representative or the typical weight of all the weights.

6. a. to get a representative value for the data set

    b. to get the balancing point (Centre) of the data set.

7. $Mean(\bar{x}) = \frac{\sum x}{n}$

$\bar{x} = \frac{236}{10}$ **M1**

$\bar{x} = 23.6$ **A1**

8. Let the minimum grade be x

$\bar{x} = \frac{\sum x}{n}$ **M1**

$85 = \frac{346+x}{5}$ **M1**

$425 = 346 + x$ **M1**

$x = 79$

The minimum grade he needs in order to achieve his average is 79 **A1**

9. Yes

   a. When all data values are zero (0)

   b. When data values comprise both positive and negative numbers

10. Weight of the six men $= 180 \times 6 = 1080$ **M$\frac{1}{2}$**

Weight of the four women $= 120 \times 4 = 480$ **M$\frac{1}{2}$**

Total weight $= 1080 + 480 = 1560$ **M$\frac{1}{2}$**

$\bar{x} = \frac{total\ weight}{number\ of\ people\ in\ the\ elevator}$ **M$\frac{1}{2}$**

$\bar{x} = \frac{1560}{10}$ **M1**

$\bar{x} = 156\ pounds$ **A1**

11. $\bar{x} = \frac{\sum fx}{n}$

$\bar{x} = \frac{0(1)+1(1)+2(1)+3(2)+4(0)+5(3)+6(3)+7(4)+8(6)+9(8)+10(7)}{1+1+1+2+0+3+3+4+6+8+7}$ **M3**

$\bar{x} = \frac{260}{36}$ **M1**

$\bar{x} = 7.22$ **M2**

12.

A. False

B. True

C. True

D. False

E. False

F. True

Appendix E Sample of Students' Responds

UNIVERSITY OF EDUCATION, WINNEBA
FACULTY OF SCIENCE EDUCATION
DEPARTMENT OF MATHEMATICS EDUCATION

B. Sc. (MATHEMATICS EDUCATION)
MATD 113 (PROBABILITY AND STATISTICS I)
FEB. 19, 2021

STUDY TEST FOR LEVEL 100 STUDENTS

PART A: PERSONAL DATA

INDEX NO. (LAST FIVE DIGITS ONLY): 10515

INSTRUCTIONS: Answer all questions. Tick as appropriate, and where possible provide short answers.

1. Gender: Male [✓] Female [ ]
2. Age: Below 20 [ ] 20 – 25 [ ]
   26 – 30 [✓] 31 – 35 [ ]
   36 – 40 [ ] Above 40 [ ]
3. What program did you study at the S. H. S level? General Arts.
4. In which year did you complete S. H. S? 2013
5. In which region did you attend S. H. S? Central
6. Did you study Elective Mathematics in S. H. S.? Yes [✓] No [ ]
7. Have you studied any advanced mathematics after S. H. S? Yes [ ] No [✓]
8. Are you a trained teacher? Yes [✓] No [ ]
9. Have you taught mathematics before? Yes [✓] No [ ]
10. If your answer to 10 is yes, then at what level of education did you teach?
    Pre – school Level [ ] Primary Level [✓]
    J. H. S. Level [ ] S. H. S. Level [ ]
11. If your answer to 10 is yes, how many years of mathematics teaching experience do you have? four

10/20

PART B: TEST ITEMS

Your score in this will not be part of your continuous assessment. It is strictly for research purposes. So, feel free to answer all questions as truthful as you can.

1. During the last five (5) days the temperature in degrees in Dubai were recorded as:
   Day 1 = 28, Day 2 = 29, Day 3 = 30, Day 4 = 32, Day 5 = 36.
   Do you think there is a one single number that can represent the temperature in degrees during the last 5 days?

   Yes: [ ] No: [✓]

   Explain your answer: Because the Temperature in degrees and should be measure in degree.

2. Friends decided to share the cookies they brought to their party. Each one brought a different number of cookies, but Dela brought the biggest number (6 cookies). When they were given the shared cookies, each one received 8 cookies. Do you think this could happen?

   Yes: [ ] No: [✓]

   Explain your answer: Because it Dela who brought the biggest number (6 cookies) adding to that of his friends mean each will get less than 8 cookies.

The table below represents the distribution of ages of a group of students taking a course. Use it to answer questions 3 and 4

Age (x)	5	6	7	8	9
Freq. (N)	2	3	3	2	1

3. A student said the arithmetic mean of the data is 10. Without calculating, can this be true?

   Yes: [ ] No: [✓]

   Give reason(s) for your answer?

   Because the ~~A~~ arithmetic mean will give you less than 10. The arithmetic mean is given by $\Sigma fx / \Sigma f$ = 74/11 = 6.73.

4. Calculate the arithmetic mean for the distribution.

$\frac{\Sigma fx}{\Sigma f} = \frac{74}{11} = 6.73$ M3A1

5. The weights of a class of 100 sociology students were measured, and the arithmetic mean was found to be 69.5kg. What does it mean to say that the arithmetic mean of all the weights is 69.5kg?

The arithmetic mean of all the weights is 69.5kg is given us the general average weights measured of 100 sociology student.

6. What is the purpose of finding the arithmetic mean of a data set?

To give you the average of ~~data~~ number of particular data involved.

7. The ages of 10 chemistry students were recorded as 23, 24, 19, 27, 22, 20, 19, 30, 21, and 31. Compute the mean of this data.

$\bar{x} = \frac{23+24+19+27+22+20+19+30+21+31}{10} = \frac{236}{10} = 23.6$ M1A1

$\bar{x} = 23.6$

8. A student has gotten the following grades on his tests: 87, 95, 76, and 88. He wants an 85 or better overall. What is the minimum grade he must get on the fifth test in order to achieve that average?

$\frac{87+95+76+88+x}{5} = 85 \Rightarrow \frac{346+x}{5} = 85$ M3

$346+x = 425 \Rightarrow x = 425-346 \Rightarrow x = 79$ A1

9. Can the arithmetic mean of a data be zero?

Yes: [✓] No: [ ]

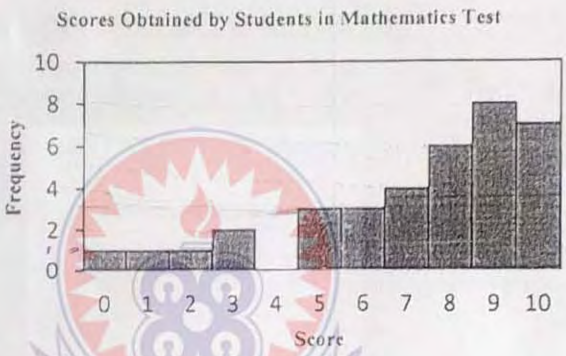
When can that be?

when all the samples are zero.

10. There are ten people in an elevator, four women and six men. The average weight of the women is 120 pounds, and the average weight of the men is 180 pounds. What is the average of the weights of the ten people in the elevator?

$\frac{120+180}{2} = \frac{300 \text{ pounds}}{2} = 150$ pounds. MoAO

11. The scores obtained by students in a mathematics test are shown in the representation below:



Find the mean score obtained by the students.

Score (1+1+1+3+3+

MoAO

12. Leticia is doing research on prices of snacks in order to buy the cheapest and save her allowance. She has written down prices from three different places and has found out that the mean price of the snacks is Gh¢ 3.00.

Write T (True) or F (False) for the possible values that Leticia has found:

A. 1, 1, 1, [F] B. 1, 3, 5, [T] C. 3, 3, ,3 [T]

D. 1, 3, 6, [F] E. 9, 9, 9, [F] F. 1, 2, 6, [T]

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UNIVERSITY OF EDUCATION, WINNEBA
FACULTY OF SCIENCE EDUCATION
DEPARTMENT OF MATHEMATICS EDUCATION

B. Sc. (MATHEMATICS EDUCATION)
MATD 113 (PROBABILITY AND STATISTICS I)
FEB. 19, 2021

STUDY TEST FOR LEVEL 100 STUDENTS

PART A: PERSONAL DATA

INDEX NO. (LAST FIVE DIGITS ONLY): 27234.....

INSTRUCTIONS: Answer all questions. Tick as appropriate, and where possible provide short answers.

1. Gender: Male [ ] Female [✓]
2. Age: Below 20 [ ] 20 – 25 [✓]
   26 – 30 [ ] 31 – 35 [ ]
   36 – 40 [ ] Above 40 [ ]
3. What program did you study at the S. H. S level? ...Science..................
4. In which year did you complete S. H. S? ......2018..............................
5. In which region did you attend S. H. S? ..Ashanti Region...................
6. Did you study Elective Mathematics in S. H. S.? Yes [✓] No [ ]
7. Have you studied any advanced mathematics after S. H. S? Yes [ ] No [✓]
8. Are you a trained teacher? Yes [ ] No [✓]
9. Have you taught mathematics before? Yes [ ] No [✓]
10. If your answer to 9 is yes, then at what level of education did you teach?
    Pre – school Level [ ] Primary Level [ ]
    J. H. S. Level [ ] S. H. S. Level [ ]
11. If your answer to 9 is yes, how many years of mathematics teaching experience do you have? ....

PART B: **TEST ITEMS** 6/20

Your score in this will not be part of your continuous assessment. It is strictly for research purposes. So, feel free to answer all questions as truthful as you can.

1. During the last five (5) days the temperature in degrees in Dubai were recorded as:
   Day 1 = 28, Day 2 = 29, Day 3 = 30, Day 4 = 32, Day 5 = 36.
   Do you think there is a one single number that can represent the temperature in degrees during the last 5 days?

   Yes: [✓] No: [ ]

   Explain your answer: Because there is no huge difference between the temperatures

2. Friends decided to share the cookies they brought to their party. Each one brought a different number of cookies, but Dela brought the biggest number (6 cookies). When they were given the shared cookies, each one received 8 cookies. Do you think this could happen?

   Yes: [ ] No: [✓]

   Explain your answer: Because Dela bought the biggest number which was 6, so there is no way that anyone could received even 6 cookies much more 8 ~~each~~ cookies

The table below represents the distribution of ages of a group of students taking a course. Use it to answer questions 3 and 4

Age (x)	5	6	7	8	9
Freq. (N)	2	3	3	2	1

3. A student said the arithmetic mean of the data is 10. Without calculating, can this be true?

   Yes: [ ] No: [✓]

   Give reason(s) for your answer?

   To find the arithmetic mean of a data, you sum all the frequency and divide it by the number of frequency. And by doing that the arithmetic mean ~~will~~ will not ~~be~~ ~~more~~

4. Calculate the arithmetic mean for the distribution.

~~mean = $\frac{\Sigma f}{n}$ = $\frac{2+3+3+2+1}{5}$ = $\frac{11}{5}$~~

M0A0

mean = $\Sigma f/n$ = $\frac{5+5+6+6+6+7+7+7+8+8+9}{14}$ = ~~22~~ = 5.45

5. The weights of a class of 100 sociology students were measured, and the arithmetic mean was found to be 69.5kg. What does it mean to say that the arithmetic mean of all the weights is 69.5kg?

It means that the average weights of the 100 sociology student is 69.5kg

6. What is the purpose of finding the arithmetic mean of a data set?

to ~~the~~ find the average of the data set.

7. The ages of 10 chemistry students were recorded as 23, 24, 19, 27, 22, 20, 19, 30, 21, and 31. Compute the mean of this data.

Mean = $\frac{\Sigma f}{n}$ = $\frac{23+24+19+27+22+20+19+30+21+31}{10}$

= 236/10 = 23.6 = 24 M1 A1

8. A student has gotten the following grades on his tests: 87, 95, 76, and 88. He wants an 85 or better overall. What is the minimum grade he must get on the fifth test in order to achieve that average?

$\frac{87+95+76+88+x}{5} = 85$ = 346 + x = 425

M3

x = 79

∴ The student should get 79 in order to get 85 or better overall A1

9. Can the arithmetic mean of a data be zero?

Yes: [ ] No: [✓]

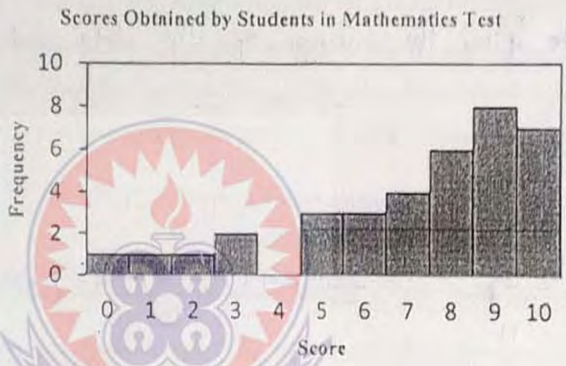
When can that be?

The arithmetic mean of a data cannot be zero.

10. There are ten people in an elevator, four women and six men. The average weight of the women is 120 pounds, and the average weight of the men is 180 pounds. What is the average of the weights of the ten people in the elevator?

Average weight of the women = 120 pounds
Average weight of the men = 180 pounds
therefore the average weight of the ten people = 120 + 180 =
300 pounds. MoAO

11. The scores obtained by students in a mathematics test are shown in the representation below:



Find the mean score obtained by the students.

Mean = $\frac{\Sigma f}{n}$ = $\frac{1+1+1+2+0+3+3+4+6+8+7}{11}$ Mean = $\frac{\Sigma f}{n}$

= $\frac{39}{11}$ = 3.55 Mean = $\frac{0+2+1+6+8+10}{6}$ MoAO

Mean = 5.166

12. Leticia is doing research on prices of snacks in order to buy the cheapest and save her allowance. She has written down prices from three different places and has found out that the mean price of the snacks is Gh¢ 3.00.

Write T (True) or F (False) for the possible values that Leticia has found:

A. 1, 1, 1, [F] B. 1, 3, 5, [T] C. 3, 3, ,3 [T]

D. 1, 3, 6, [F] E. 9, 9, 9, [F] F. 1, 2, 6, [T]

Appendix F Raw Scores

Table F1 : Raw Scores

Scores	N	N%	C.F	C.F %
0	49	13.2	49	13.2
1	14	3.8	63	17.0
2	20	5.4	83	22.4
3	14	3.8	97	26.2
4	6	1.6	103	27.8
5	26	7.0	129	34.9
6	34	9.2	163	44.1
7	1	.3	164	44.3
8	16	4.3	180	48.6
9	2	.5	182	49.2
10	51	13.8	233	63.0
11	14	3.8	247	66.8
12	27	7.3	274	74.1
13	4	1.1	278	75.1
14	10	2.7	288	77.8
15	19	5.1	307	83.0
16	18	4.9	325	87.8
17	8	2.2	333	90.0
18	16	4.3	349	94.3
19	4	1.1	353	95.4
20	17	4.6	370	100.0
Total	**370**	**100.0**	**370**	