UNIVERSITY OF EDUCATION, WINNEBA

MATHEMATICS TEACHERS' PERCEIVED KNOWLEDGE AND PRACTICE FOR TEACHING PROBLEM-SOLVING IN PUBLIC JUNIOR HIGH SCHOOLS IN BEREKUM WEST DISTRICT

COLLINS ASOMA

MASTER OF PHILOSOPHY

2020

UNIVERSITY OF EDUCATION, WINNEBA

MATHEMATICS TEACHERS' PERCEIVED KNOWLEDGE AND PRACTICE FOR TEACHING PROBLEM-SOLVING IN PUBLIC JUNIOR HIGH SCHOOLS IN BEREKUM WEST DISTRICT

COLLINS ASOMA
(8180030024)

A thesis in the Department of Basic Education,
Faculty of Educational Studies, submitted to the School of
Graduate Studies, in partial fulfilment
of the requirements for the award of degree of
Master of Philosophy
(Basic Education)
in the University of Education, Winneba

NOVEMBER, 2020

DECLARATION

Student's Declaration

I, Collins Asoma, declare that this thesis, with the exception of quotations and references contained in published works which have all been identified and duly acknowledged, is entirely my own original work, and it has not been submitted, either in part or whole, for another degree elsewhere

Signature: ....

Date: ……………………………….……..

Supervisors' Declaration

We hereby declare that the preparation and presentation of this work were supervised following the guidelines for supervision of thesis as laid down by the University of Education, Winneba.

MR. Clement Ayarebilla Ali (Principal Supervisor)

Signature: ....

Date: ……………………………….……..

Mr. Nixon Saba Adzifome (Co-Supervisor)

Signature: ....

Date: ……………………………….……..

DEDICATION

To my family.

ACKNOWLEDGEMENTS

I would like to thank my family, friends, and supervisors for their support throughout this process; thank you for all of your help, guidance, and wisdom.

I want to especially acknowledge my wife, Agyeiwaa Joyce, without whom none of this study would have been possible; you are amazing.

TABLE OF CONTENTS

Contents **Page**

LIST OF TABLES

LIST OF TABLES

Table **Page**

LIST OF FIGURES

LIST OF ACRONYMS

BWD:	Berekum West District
CRDD:	Curriculum Research and Development Division
GES:	Ghana Education Service
J.H.S:	Junior High School
NGO	Non-governmental agency
JIPA:	Japan International Cooperation Agency
MOE:	Ministry of Education
NaCCA:	National Council for Curriculum and Assessment
NCTM:	National Council of Teachers of Mathematics
TIMSS:	Trends in International Mathematics and Science Studies

ABSTRACT

This study explored public Junior High School Mathematics teachers' perceived knowledge and practice for teaching problem-solving, their problem-solving instructional strategies, how they engage pupils in problem-solving and measures to improve the teaching of problem-solving in the Berekum West District in the Bono Region of Ghana. The study employed Polya's problem-solving models to develop a conceptual framework for the teachers' perceived knowledge, practice of problem-solving, engagement of pupils in problem-solving and measures to improve the teaching of problem-solving. Four research questions were formulated to guide the study. The study employed a sequential explanatory research design and used the census technique to collect data from all the 80 Junior High School Mathematics teachers for the quantitative phase and trimmed the number down to only five teachers for the qualitative phase. The research main instruments used for the study were questionnaire (structured), interview guide and observation guide. Questionnaire data were analysed using descriptive statistics (frequency, mean & standard deviation). The qualitative data were analysed thematically. Thus, the interviews were subjected to interpretive thematic analysis and the observations were also subjected to content analysis. The results revealed that public Junior High School Mathematics teachers in the Berekum West District have a good perceived knowledge for teaching problem-solving in general. Besides, majority of the respondents agreed that Mathematics problem-solving task pupils to reason logically and critically. Also, the results suggested that even though public Mathematics teachers in the Berekum West District have a good perceived knowledge of problem-solving, they moderately used problem-solving instructional strategies. In terms of pupils' engagement in problem-solving, the results revealed a high mean score for manipulative materials than the other kinds of pupils' engagement. In conclusion, public Junior High School Mathematics teachers in the Berekum West District were inactive in engaging pupils in problem-solving. It was therefore recommended that District Directorate of Education should focus on regular teacher collectives where Mathematics teachers will meet to share ideas, solve problems, discuss ways of teaching Mathematics through problem-solving.

CHAPTER ONE

INTRODUCTION

1.0 Overview

This chapter presents the background to the study, statement of the problem, the purpose of the study, research objectives, research questions, significance of the study, the delimitations of the study as well as the organisation of the study.

1.1 Background to the Study

Petersen (2016) also views problem-solving as everyday tasks that challenge the solver to use their predictive and analytical skills. In this situation, the problem solver must use prediction and analysis to find a solution to the problem. Problem-solving is very important in the study of Mathematics. It offers opportunities for students to engage in meaningful Mathematics discourse, including analysing various representations of and justifications for their solutions. Problem-solving instructional practices force students to become active participants in the learning process and engage teachers to participate actively as learners in the classroom along with students.

In this context, Bay (2000) explains teaching via problem-solving as a method by which Mathematics teachers may provide more meaningful instruction. Advancing his argument, Bay (2000) further explains that teaching via problem-solving (teaching through a problem-solving approach) is teaching Mathematics content in a problem-solving environment. Students develop, extend and enrich their understanding by solving problems (Hieber & Wearne, 2003). Teaching through problem-solving prepares students for a life full of able-mindedness. It also helps students to develop confidence as problem solvers and become mathematical risk-takers (Tratton, & Midgett, 2001).

Besides, Van de Walle (2007) equally observes that the teaching of Mathematics through a problem-solving approach, helps teachers to engage students fully in essential mathematical learning. This implies that problem-solving permeates every mathematical task and as a generic skill, it involves independent thinking and critical analysis of issues which is important for life-long learning. It is therefore desirable that students develop mathematical problem-solving skills at the basic school as a sign of readiness for life-long learning and the job market.

According to Cai and Lester (2012), problem-solving can promote conceptual understanding of students, develop their capacity to reason and communicate mathematically and cultivate their interests and curiosities in Mathematics. That is to say, Problem-solving provides a good ground for students to develop and exercise their cognitive abilities, thereby enabling them to make effective decisions. Through problem-solving, students acquire skills that are needed in everyday activities. It offers students the training that they may need to prepare for a career, helps students get more out of life, increases their knowledge and understanding of the universe and helps them acquire skills that make their lives more interesting and enjoyable (Bryant, 2009).

The successful implementation of a problem-solving Mathematics curriculum depends on the teacher's conception and experiences. Moreover, current technology and scientific advancement being experienced worldwide require that Ghanaian learners are taught to go beyond low-level comprehension and mere memorization of facts and formulae if they are to become problem solvers of the future. Teachers, therefore, should be adequately equipped to be able to develop in their pupils or learners higher-level thinking skills in Mathematics.

The acquisition of problem-solving abilities and the deepening of the students' problem-solving skills depend on the teacher. Specifically, the teacher has to make a good choice, which includes, choosing appropriate problems that could engage students, selecting appropriate methods of teaching, providing an enabling environment for students to explore, taking appropriate decisions to avoid risks and sharing failures and successes in real practice. A teacher's role in developing problem-solving skills in Mathematics is drastically transformed from his or her traditional role as a source of knowledge and authority to a guide and facilitator.

As reported by Baki (2004) in the study of problem-solving experiences of pre-service Mathematics teachers, "students felt that the teacher was closer to them by giving up the role of authority or a distributor of pre-organized knowledge and assuming the role of a counsellor, bridging their previous experience with appropriate use of software" (p. 179). If education is going to achieve its aim of producing experts or effective leaders for the various field of the economy, or teachers must certainly know how to teach students to solve mathematical problems (Roberts, Sharma, Britton & New, 2009).

Skills in problem-solving are applied in all spheres of human endeavour. They are applied in commerce, industry or science. Because of this, problem-solving is recommended as a powerful instructional tool in Mathematics education (Roberts, Sharma, Britton & New, 2009). Teachers can equip students with problem-solving skills to enable them to solve real-life problems only if teachers' practices are tailored towards achieving the objectives of problem-solving delineated by the curriculum. A careful look at Junior High School Mathematics curriculum, sufficient provision has been made in the curriculum documents to guide teachers to use problem-solving in Mathematics lessons delivery (Anamuah-Mensah & Mereku, 2005). From the above, it is clear that Mathematics problem-solving provides a natural environment for learning and the primary mechanism by which humanity advances in knowledge.

Teachers' understanding and knowledge of problem-solving are crucial in the teaching and learning of Mathematics. Teachers need to be proficient in problem construction and teaching pedagogy that will support students' problem-solving proficiency and skills (Chapman, 2012). Teaching Mathematics problem-solving requires the teacher to be knowledgeable and understand problem-solving in order to inculcate problem-solving in Mathematics lessons.

Also, Mathematics teachers should understand that problem-solving is not only a process but also a way of thinking to teach for problem-solving proficiency. Mayer and Wittrock (2006) described problem-solving thinking in terms of inductive and deductive reasoning, critical thinking, creative thinking and decision making. Moreover, teachers have been required to use problem-solving strategies in teaching and learning of Mathematics (Anderson & White, 2004). Problem-solving skills are necessary for all areas of life, and classroom problem-solving activities can be a great way to get students prepped and ready to solve real problems in real-life scenarios. Teachers need to use a problem-solving approach to teaching Mathematics. Moreover, in the real world, students encounter problems that are complex, not well defined, and lack a clear solution and approach. They need to be able to identify and apply different strategies to solve these problems.

Andrews and Xenofontos, 2015 stated that just because students know the strategies do not mean they will engage in the appropriate strategies. Therefore, teachers need to provide opportunities where students can explicitly practice learning how, when, and why to use strategies effectively so that they can become self-directed learners (Andrews & Xenofontos, 2015).

Mathematics teachers must create a classroom environment in which students are problem solvers. This helps tie struggles to strategies so that the students will not only see value in working harder but in working smarter by trying new and different strategies and revising their process. In doing so, students will more successful in the next time. However, teaching using problem-solving approach is rarely seen in most Junior High School Mathematics classrooms in Ghana. The teaching approach observed in several classrooms of some teachers during Mathematics lessons is predominantly a conventional approach (a teacher-centred or teacher-directed instructional approach). Mereku, (2015) stated that the conventional lecture approach in the classroom is of limited effectiveness in both teaching and learning and in such a lecture, pupils assume a purely passive role and their concentration fade off gradually. This has accounted for the abysmal performance of Ghanaian pupils in Mathematics at both Primary and Junior High Schools (JHS). Pupils have weak problem-solving abilities and they are unable to comprehend the language of the text (Mereku & Anamuah-Mensah, 2005).

. A careful look at Junior High School Mathematics curriculum, sufficient provision has been made in the curriculum documents to guide teachers to use problem-solving in Mathematics lessons delivery but it is not known whether teacher make the same sense of it. Again, limited knowledge of teachers in problem-solving influences it uses in the Mathematics classroom.

It is against this background that this study was carried out to investigate Junior High School Mathematics teachers' perceived knowledge, problem-solving instructional strategies, how the teachers engage pupils in problem-solving and measures to be adopted by teachers to improve the teaching of problem-solving.

1.2 Statement of the Problem

The primary aim of Mathematics teaching and learning is to develop students' ability to solve a wide variety of complex Mathematics problems and also to apply Mathematics to real-world situations. The continuous downward trends of the performance of students in Mathematics, said by Mireku, (2015), suggests that teachers are failing to help students develop the ability to solve problems in Ghanaian basic schools. However, many efforts have been put in place to curb this problem by the government (Ministry of Education), Non-governmental agencies (NGOs) and other stakeholders to improve pupils' performance in Mathematics in their Basic Education Certificate Examination (BECE).

One way to ensure pupils' participation in Mathematics is their ability to use problem-solving. The Ghana Mathematics syllabus for Junior High School recommends the use of Mathematics in solving daily problems by applying the appropriate Mathematics problem-solving strategies (NaCCA, MOE, 2019). Research has shown that Mathematics teachers do not often use problem-solving as a teaching approach and that they fail to teach pupils how to solve problems (Mereku, 2015).

Pupils' inability to use appropriate problem-solving strategies to deal with new mathematical situations and the unpopularity of problem-solving in schools is a reflection of Mathematics teachers' perceived knowledge, instructional strategies, how they engage pupils' and measures to improve the teaching of pupils in problem-solving. The teaching approach observed in several classrooms in Berekum West district during JHS Mathematics lessons is mostly a traditional approach (a teacher-centred or teacher-directed instructional approach) (Hattori, 2008). A careful look at Junior High School Mathematics curriculum, sufficient provision has been made in the curriculum documents to guide teachers to use problem-solving in Mathematics lessons delivery but it is not known whether teacher make the same sense of it. Again, limited knowledge of teachers in problem-solving influences it uses in the Mathematics classroom.

It is therefore vital to explore public JHS Mathematics teachers' perceived knowledge of problem-solving, their problem-solving instructional strategies, how the teachers engage pupils in problem-solving and measures to be adopted to improve the teaching of problem-solving.

1.3 Purpose of the Study

The study aims at exploring Mathematics teachers' perceived knowledge and practice for teaching problem-solving in public junior high schools in Berekum West District.

1.4 Objectives of the Study

The research objectives of the study were:

1. To find out Mathematics teachers' perceived knowledge of problem-solving in public Junior High Schools in Berekum West District.
2. To explore the extent to which Mathematics teachers' use problem-solving instructional strategies in their teaching in public Junior High Schools in Berekum West District.
3. To explore Mathematics teachers' engagements with pupils' in problem-solving in public Junior High Schools in Berekum West District.
4. To determine the measures to improve the teaching of problem-solving in public Junior High Schools in Berekum West District.

1.5 Research Questions

The study was guided by the following research questions:

1. What is Mathematics teachers' perceived knowledge for teaching problem-solving in public junior High Schools in Berekum West District?
2. To what extent do Mathematics teachers' use problem-solving instructional strategies in their teaching in public Junior High Schools in Berekum West District?
3. How do Mathematics teachers' engage pupils' in problem-solving in public Junior High Schools in Berekum West District?
4. What measures can be adopted to improve the teaching of problem-solving in public Junior High Schools in Berekum West District?

1.6 Significance of the Study

The findings of this study may provide information for policymakers in education about teachers' problem-solving knowledge in the Mathematics classroom. Thus, future education policy formulation and direction can base decisions on the results of the study.

Again, the findings may guide curriculum developers in planning and designing an enriched Mathematics curriculum for Ghanaian Basic schools in problem-solving skills.

Also, the findings serve as the bases for organising professional development courses and in-service training programmes for teachers in teaching Mathematics through problem-solving. In this context, the study provides vital information for teacher education institutions for designing functional Mathematics programmes.

Moreover, it is envisaged that the findings may contribute to the existing knowledge of theory, practice and policy in the teaching and learning of problem-solving in a language of instruction.

1.7 Delimitation of the Study

According to Simon and Goes (2013), the delimitations of a study are those characteristics that arise from limitations in the scope of the study defining the boundaries of the study. In cognizance of the fact that this study will inure to the benefit of all basic schools in the country, it would have sufficed the researcher to conduct a nation-wide study. This notwithstanding, the study is restricted to Berekum West District.

Mathematics problem-solving has a wide coverage. However, the study considered only the following aspects; Mathematics teachers' perceived knowledge for teaching problem-solving, teachers' instructional strategies for teaching problem-solving, how the teachers' engage pupils' in problem-solving and measures that can be adopted to improve the teaching of problem-solving among public Junior High Schools in Berekum West District.

In any case, the outcome of the study could be generalized to other Districts in the country that has the same characteristics as Berekum West District.

1.8 Limitations

This study may be subject to methodological setbacks in as much as the use of questionnaire as a quantitative data collection tool is concerned. Respondents might not be true as the case may be for reasons best known to them. In that sense, it may in one way or the other have influenced the study findings due to the subjectivity on the part of the respondents. The search for literature was a challenge but then, an effort was made to review literature related to the studies within the African Sub-Region.

Finally, there may be difficulties in getting data from institutional authorities due to personal and ethical concerns, but then, protocols will be used to break-through such barriers.

1.9 Operational Definition of Terms

The purpose of this section is to provide explanations of some of the terms used in this study.

The following terms are used as explained below:

Achievement: Being used here to mean competence or ability or performance score.

A Problem-solving approach: Is a learner-centred teaching approach that engages learners actively in the learning process and encourages them to understand concepts and develop procedural skills meaningfully.

Facilitators: subject teachers who teach basic school learners.

Motivation: In this study, motivation is used to refer to the learner/student desire that influences his/her behaviour and persistence in all learning and academic activities.

Knowledge of the subject matter: Knowledge of subject matter is understanding concepts, algorithms, facts, procedures, and connections among concepts.

Knowledge of subject matter includes knowledge of purposes of instruction, justifications for learning a topic, important ideas for a topic, prerequisites, and typical school problems.

1.10 Organisation of the Study

This study was put into five chapters. The first chapter is the introductory chapter and has eight sub-headings made up of the background to the study, statement of the problem, the purpose of the study, objectives of the study, research questions, significance of the study, delimitation of the study, and organisation of the study. Chapter Two reviewed related literature on teachers' perceived knowledge for teaching problem-solving.

The third chapter dealt with the methodology of the study while Chapter Four was devoted to results and discussions. Chapter Five provided a chapter summary of the study, conclusions, and recommendations.

1.11 Summary Chapter

In this chapter, the researcher has discussed the background to the study, statement of the problem, the purpose of the study, research objectives, research questions, significance of the study, the delimitation of the study, limitations, operational definition of terms, as well as the organisation of the study. The next chapter presents a review literature to the study.

CHAPTER TWO

REVIEW OF RELATED LITERATURE

2.0 Overview

Hammond and Wellington (2013) state that a literature review involves researchers establishing an overview of what has already been studied in the field or area under investigation. It includes the perspectives of a selection of researchers and disciplines examined from a historical perspective to current thinking on the subject under investigation. This chapter presents the theoretical framework of the study, the conceptual framework of the study, the empirical review of the study, and the chapter summary.

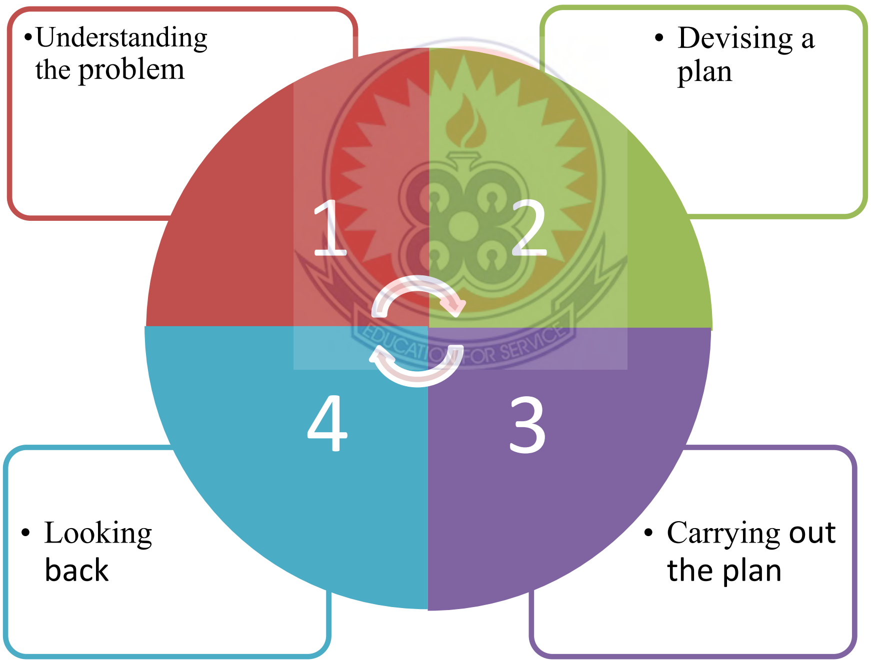
2.1 Theoretical Framework for the Study

The theoretical model was adopted from Polya's problem-solving approach. George Polya (1887-1985) was a Mathematical giant who made classical analysis into problem-solving in the twentieth century. According to Polya's Pillars of Mathematical Problem-solving, problem-solving can be viewed as an activity involving a variety of skills. Problem-solving is supposed to be a practical art: a lesson taught and learned. It is about the ability of the child to make use of previous knowledge and apply it in a new situation (Polya, 1954).

The child's ability to remember basic arithmetic skills, how and when to incorporate them into a new situation, and then the ability to do so are three distinct skills. The child may have all three skills that facilitate problem-solving, however, a child's inability to use one of the skills does not mean that he or she understands the problem. It may mean that the child's learning style has not been adequately addressed (Polya, 1954). Similarly, the fact that the child can perform the procedures in isolation does not mean that he or she has the knowledge to apply them or can interpret the numbers involved (Bley & Thornton, 2001). A good problem is one of modification that can be made for students with different skills, abilities, and learning styles. Therefore, According to Polya, teachers are therefore encouraged to use multiple solutions and multiple methods of solution in a classroom, which will promote problem-solving skills.

2.1.1 Polya's Problem-solving Approaches

In 1945, George Polya published the book *How To Solve* This book quickly became the most popular publication. It offered a four-step approach to problem-solving, as described in Figure 1.



Source: Polya (1945, p.5)

Figure 1: Polya's stages of problem-solving.

Based on the Figure 1, Polya (1945) identified the four steps of problem-solving: understanding the problem, designing a plan, implementing it, and looking back. Wilson et al (1993) and Bennett (2007) observed that Polya's problem-solving steps can be interpreted as dynamic and cyclical processes in which problem solvers must move from one step to the next as they solve the problem. For example, a person may consider designing their plan first before the other stages. This may help them to understand the problem or rather the implementation of the plan may lead to a better way to solve the problem. For example, Polya (1957) explained the steps as a non-sequential process:

> Dying to find the solution, we may repeatedly change our point of view, our way of looking at the problem. …Our conception of the problem is likely to be rather incomplete when we start the work; our outlook is different when we have made some progress; it is again different when we have almost obtained the solution. (p.5)

Although the steps are not sequential and a problem solver does not necessarily have to skip one step before adopting another, it is necessary to address the steps and their use:

Understanding the problem is the first and most important of Polya's problem-solving stages. This step requires identifying the unknown data and conditions of the problem. According to Polya (1985), there is no chance of solving a problem if one does not first understand the problem. Understanding the problem requires not only knowing what to do, but also knowing the key pieces of information that need to be gathered in some way to get the answer. Since it is often impossible to absorb all the important information about a problem at once, Polya advised that you should always read a problem several times, both at the beginning and during the course of the work.

During the resolution process, it may be necessary to revisit the original issue from time to time to ensure that it is on the right track. As Polya (1957) said, it makes no sense to answer a question that you do not understand. Polya added that a problem solver must be motivated to solve the problem, and teachers can motivate students by choosing a problem that is appropriate for the students' level of difficulty. The student must at least understand the problem before designing a plan of attack. However, many teachers do not teach their students this crucial step in Polya's problem-solving strategy, such as converting contextual information into conceptual understanding. As a result, students are often confused in their efforts to solve problems simply because they do not fully understand what they are being asked to do (Rudd, 2010). For students to be successful at the understanding stage, teachers need to teach students to ask complete and conceptual questions. Rudd outlined the following questions as guidelines that help the problem solver to understand a problem:

1. What are you asked to find or show?
2. What type of answer do you expect?
3. What units will be used in the answer?
4. Can you give an estimate?
5. What information is given? Do you understand all the terms and conditions?
6. Are there any assumptions that need to be made or special conditions to be met?
7. Is there enough information given? If not, what information is needed?
8. Is there any extra information given? If so, what information is not needed?
9. Can you restate the problem in your own words?
10. Can you act out the problem?
11. Can you draw a picture, a diagram, or an illustration?
12. Can you calculate specific numerical instances that illustrate the problem?

Polya's second stage, finding a strategy, tends to suggest that it is fairly simple to think of an appropriate strategy. However, there are certainly problems for which children may find it necessary to play with information before they can think of a strategy that might produce a solution. This exploratory step will also help students to better understand the problem and to become aware of some of the information they had overlooked after the first reading. The development of a plan is crucial in the solution-finding process. Planning is the foundation for the success of any activity. It has been said that "He who does not plan, plans to fail".

The plan is not meant to be the step-by-step procedural instructions that most students receive in the classroom. Rather, it is about choosing appropriate heuristic strategies to investigate a problem that is unfamiliar to them. Most students have not learned problem-solving strategies and are therefore unable to proceed at this stage of the problem-solving process. As a result, students' ability to solve an unfamiliar problem will be diminished. In fact, according to Polya (1957), the main success in solving a problem is to develop the idea of a plan. The plan to be devised may be slower or slower depending on who is solving the problem or on the sudden event. To design a plan, the problem can be compared to a problem that has already been solved or to a similarly solved problem. Teachers should use step-by-step procedures during lessons delivery.

Implementing the plan is the third stage in Polya's (1957) problem-solving, as mentioned earlier. In Polya's third step, the chosen strategy is used to solve the problem step by step, and if the solution cannot be found, the strategy is modified. The final step is to check the solution against the original problem to see whether or not the answer is reasonable and whether or not there is another way for the solution.

Achieving it means adopting the skills and knowledge that should be automatic in the problem solver. Skills and knowledge can be resources, such as content knowledge, algebraic skills or Mathematical techniques. Teachers should note that teaching problem-solving in Mathematics is about having a sufficiently, and strong foundation of Mathematical knowledge. Research has shown that there is a significant positive correlation between a student's level of prior knowledge and their ability to solve non-routine Mathematical problems (Lee & Chen, 2009). However, a student's ability to use his or her Mathematical concepts, theorems, formulae, and procedures to solve Mathematical problems may not be sufficient. Rather, it is the student's ability to apply his or her knowledge to solve a problem. The "plan'' used to solve a problem is often referred to as a problem-solving strategy. For some problems, you may start by using a strategy and then realize that the strategy does not match the information given or does not lead to the required solution. In this case, you must choose another strategy. In other cases, you may need to use a combination of strategies.

Several problem-solving strategies are described as follows (in no particular order). Polya (as cited in Lee & Chen, 2009) proposed the following strategies for developing a plan:

1. Use guess and check. When a problem calls for a numerical response, the student can make a random guess and verify it using the facts and information provided in the problem. If the answer is incorrect, the student can make and verify a new answer. Each subsequent guess should lead to a better understanding of the problem and a more appropriate guess. In some cases, the guessing and checking strategy can also be used for problems where the answer is non-numerical.
2. Draw a picture or diagram/use a graph or numerical line. A picture or graph can illustrate relationships between given facts and information that is not as easily visible in words or numbers.
3. Use manipulatives or models. When a problem requires moving or rearranging elements, a physical model can be used to illustrate the solution.

4. Make a list or table. A list or table can be useful for organizing information. It may be possible to make an ordered list or table of all possible solutions and then choose the solution that best fits the facts and information given in that list. In some cases, the answer to the problem is a list or table of all possible solutions.
5. Eliminate possibilities. When there is more than one possible solution to a problem, each possibility should be examined. Possible solutions that do not work are eliminated from the list of possible solutions until an appropriate response is determined.
6. Use cases. Some problems can be divided into cases. Each case can be examined separately.
7. Solve an equivalent problem. In some cases, it is easier to solve a related or equivalent problem than to solve a given problem.
8. Solve a simpler problem. It may be possible to formulate and solve a problem that is simpler than the given problem. The process used to solve the simpler problem may provide insight into the more complex given problem.
9. Look for a pattern. Patterns are useful in many problem-solving situations. This strategy will be especially useful for solving many real-world problems. "Models are a way for young students to recognize order and organize their world" (NCTM, 2000, p.91).
10. Choose the operation/write a formula or number sentence. Some problems are easily solved by applying a known formula or number sentence. The difficulty often lies in choosing the appropriate formula or operation.

11. Make prediction/use estimation. All the elements of a problem must be carefully considered when making a use prediction or estimate. This careful examination can provide useful insight into the solution to the problem.
12. Work the problem backwards. If the problem involves a sequence of steps that can be reversed, it may be useful to work on the problem in reverse. Young children may already have some experience working backwards. To solve many mazes and puzzles, it is sometimes easier to start at the end than at the beginning.
13. Use logical reasoning. Mathematics can and should make sense. Sometimes logical reasoning and careful thought are enough to solve a Mathematical problem.

Again, the implementation of the plan is very important in the problem-solving procedures. The problem solver must be patient. According to Polya (1957), patience is needed not only to execute the plan, but also to make the necessary adjustments to the plan, or even abandon it altogether and design a new one. The problem solver must critically analyse and design a plan. If an appropriate plan is used for the given problem, then the plan is appropriate for that given problem, then the plan can be implemented to solve the problem. However, in some cases, if the initial plan does not succeed, another plan must be developed. The initial strategy may need to be modified, or a new strategy may be chosen. Students must realize that not all problems will be solved at once. A failed attempt can be considered a learning experience. Teachers should encourage and engage students to use computers, calculators, or other manipulative tools when dealing with routine tasks.

The last stage in Polya's problem-solving is to look back. This step is not just about checking the answer, but much more than that. When a good problem solver works on a problem, he or she pays attention to his or her problem-solving process during and after he or she has solved the problem. This involves examining both the problem and the solution, looking for alternative methods of solution, considering extensions, connections and related problems, and thinking about his or her own solution process. According to Polya (1962), perhaps the best time to think about methods is when the reader has finished solving a problem. It is particularly difficult to motivate students to look back after solving a problem, but according to Polya (1957), Research has shown that students need these high-level problem-solving skills to recognize that there are multiple solutions and strategies for solving problems and to become experts in the field (Adiguzel & Akpinar, 2004). The following are some questions that may be useful in the process of looking back.

1. Is the answer reasonable?
2. Is there another method of solution that will allow the answer to be easily verified?
3. Does the answer match the data for the problem?
4. Does the answer meet all of the conditions or requirements of the problem?
5. Is there more than one answer?
6. Will the solution process be useful for solving similar or related problems?

Polya's problem resolution stages provide a fundamental framework for solving problems. The notion of heuristics is a distinctive feature of Polya's approach to problem-solving. In his book *"How to Solve It"* (1957), Polya has a section entitled "Short Dictionary of Heuristics" in which Polya identified strategies that apply to a wide range of problems. Polya explains heuristics as the study of methods of solution.

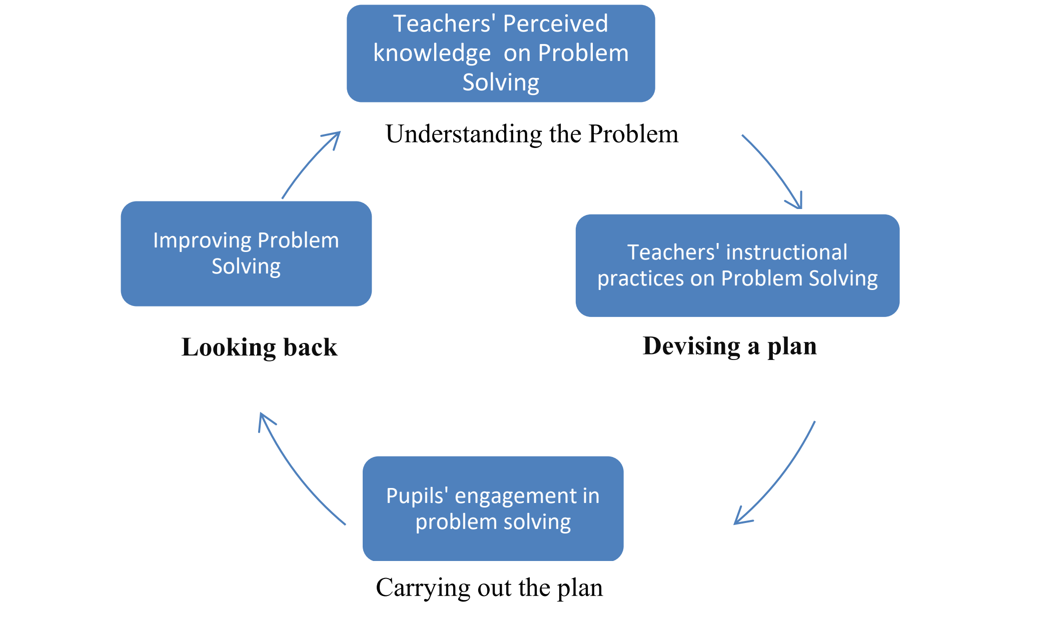
Research has also shown that some Mathematics teachers have considered heuristic strategies, or simply heuristics, as synonymous with problem-solving strategies, while others describe heuristics as being contained within a broader set of problem-solving strategies. Schoenfeld (1987) described heuristic strategies as follows:

> Heuristic strategies are rules of thumb for making progress on difficult problems. There are, for example, heuristic strategies for understanding a problem (focusing on the unknown, on the data, drawing a diagram, etc.), for devising a plan (exploiting related problems, analogous problems, working backwards, etc.), and for carrying out and checking a solution. (p.284).

Polya's (1945) four-step problem-solving process is similar to Goldman's (1989) four-phase model, which includes the following elements: 1, read and become familiar with the problem, 2, find the necessary information, 3, establish the problem with numbers and symbols and solve it; and 4, see if the solution makes sense (as cited by Kargas & Stephens, 2014). Polya's problem-solving steps are simple and do not require any prior knowledge, Mathematical skills, or choice of strategy. These are also very important factors that teachers and students need to be aware of. Lesh and Zawojewski (2007) also consider Poyla's four stages of problem-solving as a linear problem-solving procedure, whereas strategies are the different tools used by the problem solver to interpret, move forward and try different ways of solving the problem.

2.2 Conceptual Framework

The conceptual framework of the study draws on Polya's problem-solving strategies to help Mathematics teachers develop a teaching procedure for Mathematics classes, as shown in Figure 2.



Source: Adapted from Polya (1945)

Figure 2: Conceptual framework for teaching problem-solving.

The Conceptual framework was adopted from Polya's problem-solving approach. According to Polya's Pillars of Mathematical Problem-solving, problem-solving can be viewed as an activity involving a variety of skills. And problem-solving is supposed to be a practical art: a lesson taught and learned. It is about the ability of the child to make use of previous knowledge and apply it in a new situation.

2.2.1 Teachers' perceptions of problem-solving

Research shows that teachers have varied perceptions of problem-solving and that their classroom practices are influenced by these perceptions. Dollah (2006) stated that students' willingness to accept a difficult problem is considered an important part of problem-solving. In problem-solving, it does not necessarily matter whether a task is difficult or not, as long as the student accepts it as a challenge. Accepting a challenge here implies that the pupil is prepared to find appropriate methods to solve a problem. Saleh (2009), in a study on problem-solving methods of secondary school Mathematics teachers, indicated that teachers perceive Mathematics problem-solving as difficult word questions, which are challenging and normally related to everyday problems. It also involves manipulating numbers and symbols and requires the use of multiple skills and strategies.

Van de Walle (2007) described problem-solving as a main pedagogical strategy used to fully engage the student in an important Mathematics learning situation. It also goes beyond the domain of Mathematics to include activities of daily life in general. Anderson, Peter, and White (2004) described problem-solving as a process by which students explore non-routine issues. These explorations involve the use of a wide range of strategies to solve unfamiliar tasks, as well as the development of processes of analysis, reasoning, generalization, and abstraction. In exploration, students make mistakes and backtrack. Making mistakes and stepping back is a natural part of problem-solving. Henderson (2002) has supported this assertion by saying that some teachers have the view that making mistakes and having to backtrack is a natural part of problem-solving. He added that teaching Mathematics through problem-solving is not logical but involves trial and error.

Traiton and Midgett (2001) argue that problem-solving is a means by which students make sense of Mathematics and learn content, skills, and strategies, and that when students learn Mathematics through problem-solving, they better understand both content and pedagogy and make sense of the reasoning behind the solution process. According to Goldberg (2005), Mathematical problem-solving involves the ability to read the process and solve Mathematical situations. Problem-solving, according to the National Council for teachers of Mathematics (NCTM, 2000), involves getting involved in a task for which there is no immediate answer. The same concept has appeared in many studies at different times (Van De Walle, 2007).

2.2.4 Instructional strategies for teaching problem-solving

According to Biddlecomb and Carr (2011), strategies can be thought of as a grouping of activities, mental or physical techniques aimed at solving a problem. There are many strategies that students can use to find answers to problems. Students need to be aware of these strategies and use them.

In mathematics education, students should be allowed to discover for themselves diverse ways they can use to find a solution to the problem (Cotic & Zuljan, 2009). Problem-solving strategies should be varied and children should have many opportunities to try them out in concrete ways, either orally or in written work. Many children fail in mathematics because their mathematical vocabulary is insufficient to cope with the terminology of the problems. Emphasis should be placed on developing the necessary vocabulary in a consistent manner across all classes. Some strategies that can be taught to children are: the use of appropriate manipulatives/board models/patterns and equipment to solve a particular problem. These strategies are not intended to overload students' learning capacity. There are useful skills that students should receive from their teacher.

Quality mathematics education "should equip students with declarative and procedural knowledge and skills and enable them to become progressively independent" (Cotic & Zuljan, 2009). Students with these skills will have more knowledge about how to solve mathematical problems independently. Ross and Maynes (1983) identified instructional strategies for teaching problem-solving. Teaching strategies are discussed below:

2.2.5 Selection of an appropriate domain of problem-solving tasks

Taking into account the underlying cognitive structures and processes of students is a difficult and costly educational undertaking that requires elaborate analysis and planning. The demands placed on teachers by this approach are very high and many problem-solving areas simply do not warrant such exhaustive efforts. For example, there seems little point in making such special efforts to teach basic skills, such as numeracy, that almost all children learn. Moreover, there is little point in selecting puzzles of puzzling toys, made by a few adults, that have little to do with school learning, however useful these activities may be in developing and testing designs on how learning takes place. Thus, several criteria come to mind when selecting appropriate tasks.

Problem-solving tasks should be prominent in several classes and subjects. They should have direct application to real-life problems that students face during and after their school experience. Problems should be part of the explicit goals of education. Problems should be tasks that children have difficulty mastering and/or tasks that are an area that does not receive sufficient attention in the existing curriculum. Four types of problems have been identified that meet all of these criteria: (1) decision-making problems in which the student must choose the best course of action in a complex situation; (2) correlational problems in which the student attempts to find an association between two or more variables in circumstances where the values of the variables cannot be physically manipulated; (3) experimental problems in which the student attempts to establish a causal relationship between two or more variables by physically manipulating the variables; and (4) comparative problems in which the student establishes similarities and differences between two or more entities. These types of problems are found at all grade levels and in all subjects, in educational activities, and in specific contexts.

Constructing learning hierarchies

Ross and Maynes' (1983) instructional strategy is also a process of developing learning behaviour. A hierarchy of learning consists of a series of descriptions of cognitive behaviours that reduce the gap between experts and novices in problem-solving. The resulting learning hierarchies provide a blueprint for development, describing the specific changes needed to improve students' problem-solving performance. Each level of the hierarchy can be treated as a learning objective to be achieved through specific instruction.

Devising instructional strategies

The simplest teaching strategy is to give students the opportunity to practice component operations, ideally using less difficult examples of the type of problem to be solved. The second category of pedagogical strategies consists of confronting the pupil with a problem that cannot be dealt with by a simple executive scheme, so that he or she recognises the need for change. The third set of teaching strategies involving teacher modelling is also suggested. Providing students with simplified routines that mimic a mature executive scheme is a useful intermediate process (Bereiter & Scardamalia, 1982). Such simplified routines are analogous to the problem-solving strategies that students invent as their thinking matures (Resnick, 1976).

A good teaching routine is one that is easily demonstrated and can be easily adapted by students to solve problems that cannot be dealt with by less complex schemes. When modelling, the teacher should think aloud so that the advantages of the new solution and the disadvantages of the old one is obvious to the learner. The fourth category of teaching strategies is to help students become more aware of their thought processes. Bereiter and Scardamalia have suggested that effective executive routines are more likely to develop if teachers reveal normally hidden processes to students, for example by focusing their attention on particular mental operations. This is probably particularly important for lower-level operations that disappear in expert processes or are so compact in higher-order routines that they become invisible.

Similarly, requiring students to articulate goals and verbalize plans for solving a problem also improves performance, as demonstrated in an experiment by Pellegrino and Schadler (cited in Resnick, 1976). These strategies can contribute directly to increasing problem-solving skills and can have an indirect effect by improving communication between students and teachers.

One of the greatest practical difficulties in providing problem-solving instruction that addresses students' underlying cognitive structures and processes is that these mental activities are internal. Making these structures and processes visible and giving those labels enables teachers to diagnose more effectively what students are thinking. It also makes it much easier to describe to students what they are expected to do. A final teaching strategy in this category is to test reality. Expert problem-solving is characterised by effective verification and monitoring of partial solutions.

On the other hand, novices, especially young children, are unaware of the internal inconsistencies of their solutions and are unable to determine whether a solution is contrary to common sense (Brown, 1978). Most schools place greater emphasis on replicative learning (reproduction of knowledge) than on applicative learning (problem-solving) (Broudy & Spiro, 1977). Broudy and Spiro (1977) have suggested that it is wise for students to keep personal and academic patterns as separate as possible, as attempts to integrate the two sets can lead to interference and may reduce students' performance in tasks that require the exact reproduction of knowledge.

Although reality tests disrupt the functional mental segregation of academic achievement in most programme contexts, they are an important step in the development of problem-solving ability and require a special effort on the part of teachers to overcome student resistance. Therefore, it is desirable to introduce real-life examples and tasks into school problem-solving in order to foster the tendency to check solutions against common sense and the belief that problem-solving is a useful activity. The use of real-life examples also has the effect of reducing the demands on processing capacity because the situations are familiar; this reduction frees up processing space for the acquisition of new operations.

Designing teacher training

Ross and Maynes (1983) explained that the initial training of teachers involved in their projects focused on the benefits to students of learning problem-solving strategies and on the principles of teaching as described above. The emphasis on procedural rather than theoretical knowledge was based on the belief that it is not necessary to understand the constructs of a technology in order to use it. In organising teacher training, teachers were expected to be taught problem-solving in a similar way to the methods used for students; that is, the lowest levels were generated by the teachers and progressively revised collectively with the advice of the programme manager. Teachers should be provided with very detailed lesson plans and sets of materials for students. Ross and Maynes found in their project that slavish adherence to these lesson plans was not necessary; it was assumed that teachers would make adaptations to meet the perceived demands of their classroom situations.

2.2.6 Pupils' engagement in problem-solving

Pupils' engagement is seen as the interest and enthusiasm that students show for school and classroom activities (Olson & Peterson, 2015). Pupils' engagement in this sense is a complex term because it includes attendance, attention, and participation in the classroom, and thus the child's psychological and behavioural attitudes (Sinclair, Lehr & Anderson, 2003). Christenson, Reschly and Wylie (2012) also believe that student engagement is the link between home, school, peers, and the child's community, based on the child's interest. Thus, students who are primarily engaged in classroom activities pay close attention, contribute to class discussion, prepare notes and ask questions when necessary.

In a Mathematics class focused on problem-solving, students are expected to exhibit these characteristics and behaviour. However, students who are not involved in problem-solving in Mathematics are passive, annoyed and even angry because of their presence in class. Fredricks, (2004) has proposed that student involvement in the classroom is very important for learning and depends on successful teacher-student interaction in the classroom.

Pupils' engagement has been categorized into three dimensions: cognitive, behavioural, and effective engagement (Fredrick, 2004). First, cognitive engagement refers to the child's willingness to demonstrate and make efforts to understand and perform difficult tasks. Thus, it involves the child in the learning process. Fredricks, (2004) also found that cognitive engagement is complex and encompasses investment in learning, self-regulation and strategy. Cognitive engagement can be thought of as students using deep strategies such as integration (active engagement) and students relying on memorization (superficial engagement) (Kong et al., 2003). Pupils who focus more on task mastery goals are more active in cognitive engagement than pupils who are oriented towards social recognition.

Second, behavioural engagement is a continuum of participation development (ie. respect for school and classroom procedures, taking initiative in the classroom, involvement in school activities and, finally, participation in school governance). Behavioural engagement, however, is classified into three categories (Fredricks et al., 2004). First, behavioural engagement refers to the positive conduct of students, which consists of following rules and adhering to classroom norms. Second, it involves participation in learning. It involves reflecting on behaviours of persistence, concentration, attention, asking questions and contributing to class discussion. Thirdly, it also deals with school-related activities. Kong, Wong, and Lam (2003) summarized the three identified dimensions of behavioural engagement: attention, diligence, and time spent learning Mathematics outside of classroom instruction.

The final dimension of pupils' engagement is emotional or affective engagement. Emotional engagement involves a sense of belonging and acceptance of the goals of schooling (Kong et. al., 2003). It encompasses the beliefs, attitudes and emotions experienced by students (Fredrick et al., 2004). Other researchers focus on aspects of emotional engagement such as anxiety, interest, boredom, achievement orientation, frustration, sense of belonging and being valued in school (Fredricks, 2004).

2.2.7 Ways of promoting problem-solving in Mathematics instructions

Reflection is essential to a teacher's professional growth. This implies that teachers should have the opportunity to reflect on their teaching, if possible, with others about teaching Mathematics through resolution. Teachers need training and experience in problem development and knowledge of Mathematical content. An effective way to help teachers develop these skills is to create teacher collectives. Teacher collectives are "groups of teachers, either in the same school or in neighbouring regions, who meet regularly to solve problems, share ideas, discuss pedagogy, plan lessons, and reflect collaboratively on teaching and learning" (Institute for Advanced Study/Park City Mathematics Institute International Seminar, 2006, p.11).

Peer observation and team teaching are also effective ways of developing teachers' knowledge and skills in problem-solving (Institute for Advanced Study/Park City Mathematics Institute International Seminar, 2006). Ali, et. al., (2010) recommended a comprehensive training programme; a seminar and workshops for primary school Mathematics teachers to enable them to use a problem-solving method in the classroom. They also suggested organising training sessions for student teachers to be trained in problem-solving and proposed to transform Mathematics textbooks into a form of problem-based learning. Taplin (2010) wrote that problem-solving has been recommended as the organisational tool around which the Mathematics curriculum should be built.

Furthermore, the NCTM (2000) strongly argues that problem-solving should be the driving force behind the Mathematics curriculum. Since many teachers rely on textbooks as the main source of information for instruction, the presence of problem-solving questions in Mathematics textbooks will encourage them to teach Mathematics using problem-solving approaches.

Murat and Memnum (2008) have argued that the establishment of a constructivist social learning environment and the content of teaching have erased negative attitudes and problem-solving beliefs of students. From a constructivist perspective, meaning is understood as the product of individuals (here, teachers) "establishing relationships, reflecting on their actions, and modelling and constructing explanations" (Fosnot, 2013). The beliefs of contemporary theorists and researchers have shifted from the isolated mastery of concepts by students to the belief that true learning is about interaction, development and growth (Fosnot, 2013). In the cognitive sciences, we are told that students learn through a progressive structuring and restructuring of the experience of knowledge and that deep conceptual learning is about structural changes in cognition and that without exchange with the environment, disorder would result (p. 279).

According to Lambdin (2003), "in order to be able to solve problems, one must have a deep and conceptual understanding of the Mathematics involved; otherwise one will only be able to solve routine problems" (p. 7). Teachers believe that students need to have the basic knowledge to solve not only routine problems, but also complex higher-order thinking problems. To prepare students for the future and the problems they will encounter, students should learn Mathematics by solving problems. Beliefs such as "a Mathematical problem has only one way to solve, only one correct answer, and students can never solve an unusual problem correctly" are removed from students' minds. The provision of valuable resources and more time are important steps in promoting the teaching of problem-solving; problem-solving in the Mathematics curriculum can only be valued if it is included in the assessment of high stakes. In addition, teachers need readily available examples of useful non-routine problems, especially in textbooks (Anderson, 2009).

2.3 Empirical Review on Problem-Solving

The empirical review discuss the concept of problem and problem-solving, teachers' knowledge on problem-solving, problem-solving approaches to teaching mathematics, benefits of teaching mathematics problem solving, challenges that militate the successful teaching of problem-solving

2.3.1 The concept of problem and problem-solving

A problem can be described as a situation where something needs to be found or shown and the way to find or show it is not immediately obvious (Grouws, 2003). In other words, the situation may seem unfamiliar to the researcher and may not have a clear path to follow (Grouws, 2003). However, a problem can be defined as a situation for which there is no ready-made solution (Henderson & Pingry, as cited in Karuku, 2013). Polya has also indicated that having a problem means consciously seeking appropriate action to achieve a clearly designed, but not immediately achievable, goal. Thus, what may be a problem for one individual may not be a problem for another (as cited in Grouws, 2003).

Lester (2013) proposes that whether a situation is a problem for an individual is determined by the individual's reaction to it. Lester argues that in order for a situation to be a problem for an individual, the person must first be "aware of the situation" and "be interested in solving it", and then the person must be "unable to directly proceed to a solution", so they must "make a deliberate attempt to find it" (Lester, 2013). Furthermore, a problem for a particular individual today may not be a problem for him or her tomorrow (Henderson & Pingry, cited in Karuku, 2013).

Problem-solving is considered to be one of the fundamental vital functions of the brain's natural intelligence (Wang & Chiew, 2010). The daily experience of the rational human being is to make decisions related to certain problems that require a solution, even if the problem is minor or crucial. Problem-solving in the Mathematical context can be thought of as the process by which students encounter a problem or question for which they do not have an immediate apparent solution or an algorithm that they can apply directly to obtain an answer (Chimni, Sankar & Tripathi, 2016).

According to Anderson and White (2004), problem-solving is the process by which students explore non-routine issues. It involves the use of a variety of strategies to solve unfamiliar tasks through analysis, reasoning, generalization and abstraction. Petersen (2016) also views problem-solving as everyday tasks that challenge the solver to use their predictive and analytical skills. In this situation, the problem solver must use prediction and analysis to find a solution to the problem.

Problem-solving, as published by the National Council of Teachers of Mathematics (NCTM) in its Principles and Standards for Mathematics in Schools, is seen as a task for which there is no immediate answer (NCTM, 2000). In this case, the answer to the problem is not known in advance. Because people solve problems every day, they achieve something without knowing in advance how to do it. To solve a problem, we need to coordinate our knowledge, experience, intuition and diverse analytical skills (Lester, 2013) with our Mathematical reasoning (Willoughby, as cited in Banks & Kenz, 2011). Problem-solving can generally be considered to be the most important cognitive activity in all life (Jonassen, 2000).

According to Sepeng (2014) problem-solving is considered crucial in the teaching and learning of Mathematics. In addition, Kuzle (2013) sees problem-solving as a process oriented in this way; learners play an active role in generating ideas to solve the problem. The ability to generate ideas to solve the problem should emphasise the ability of the problem solver to understand that problem-solving requires higher order critical thinking as a solution is, by definition, not immediately observable.

2.3.2 Teachers' knowledge of problem-solving

Dvorak (2004) defined knowledge as a belief in what we know. McQueen (1999) clarified that knowledge is the experience, understanding and comprehension of a phenomenon or the context of a problem that governs our behaviour in order to obtain a required response. Knowledge has been conceptualized by Singh (2008) as codified information that includes intuition, interpretation, context, experience and wisdom, among others. In psychology, knowledge is divided into two categories, declarative knowledge and procedural knowledge (Stemberg, 2002).

Declarative knowledge knows something that can be stated as a true statement and procedural knowledge is the ability or aptitude to do something (Stemberg). Over the decades, much effort and attention has been devoted to the knowledge that teachers should have about problem-solving and teaching through problem-solving. Ball, Thames, and Phelps (2008) proposed that significant attention has been given to the training of Mathematics teachers as the basis for the acquisition of knowledge in problem-solving.

Procedural knowledge does not reflect knowledge about problem-solving, but rather as an important aspect of the knowledge acquired. Ball et al. also suggested that Mathematical ability does not fully account for the knowledge and skills required for effective problem-solving. Thus, teachers need a specific type of knowledge that is not required in other professional contexts. For example, the knowledge needed to teach effective problem-solving should be more than general problem-solving skills.

The National Council of Teachers of Mathematics [NCTM] (2000) has defined teaching problem-solving as engaging learners in a problem for which the solution is as yet unknown. The act of finding a solution to a difficult task is a form of cognitive processing in which teachers engage pupils when they are faced with a difficult problem and do not have an absolute solution (Mayer & Wittroek, 2006). For successful problem-solving, Schoenfeld (as cited in Ball et al., 2000) argued that Mathematics teachers must be well equipped and skilfully use appropriate resources (Mathematical concepts and procedures), heuristic strategies (specific and general heuristics), metacognitive control (monitoring and supervision of the entire problem-solving process), and appropriate beliefs (perspective, motivation, and self-confidence).

2.2.2 The importance of metacognition as an aspect of teachers' knowledge

Another important issue that teachers should pay attention to in problem-solving is the role of metacognition. As Polya (1957, 1962, & 1985) insisted, the success and failure of problem-solving depends on metacognitive behaviour teachers. These metacognitive behaviours include: understanding what the problem requires, choosing a particular strategy to solve the problem, evaluating whether the strategy leads to a solution, and examining whether the answer makes sense. One might think that Polya values metacognition in Mathematical problem-solving, even though he has never used the term. According to Silver (1982), "If we take a metacognitive perspective, we can consider many of Polya's (1957) heuristic suggestions as metacognitive prompts" (p. 21). In Polya's (1962) retrospective look, the problem solver can ask many useful questions such as "What was the decisive point? What was the main difficulty? What could I have done better? I failed to see this point: which item of knowledge, which attitude of mind should I have had to see it?" (p. xii)

Metacognition includes not only knowing one's own cognitive abilities, but also regulating one's behaviour in response to what one knows (Lester, 2013). This concept is known as self-regulation, which is closely related to control.

Metacognitive behaviour in problem-solving was a topic of discussion in the 1980s (Campione, Brown and Connell, cited in Donaldson, 2011), but research on metacognition no longer occupies a prominent place in Mathematics education. Metacognition, however, remains a tacit part of the problem-solving discussion (Donaldson, 2011). For example, the literature on Mathematics instruction is replete with terms such as tracking and reflection, and ideas such as self-assessment and knowledge of one's own cognition. The NCTM (2000) considers the development of students' metacognitive abilities as an important part of classroom instruction: "Students should be encouraged to monitor and evaluate themselves. Good problem solvers realize what they know and don't know and what they are good and not so good at" (p. 260).

The following quote shows the multiple dimensions of metacognition that the NCTM (2000) values in Mathematics education. These dimensions include reflection, metacognitive inquiry, and monitoring.

> Reflective skills (called metacognition) are much more likely to develop in a classroom environment that supports them. Teachers play an important role in helping to enable the development of these reflective habits of mind by asking questions such as "Before we go on, are we sure we understand this?" "What are our options?" "Do we have a plan?" "Are we making progress or should we reconsider what we are doing?" "Why do we think this is true?" Such questions help students get in the habit of checking their understanding as they go along. As teachers maintain an environment in which the development of understanding is consistently monitored through reflection, students are more likely to learn to take responsibility for reflecting on their work and make the adjustments necessary when solving problems (pp. 54-55).

Note the emphasis on the teacher's role in promoting metacognition. Teachers are responsible for creating classroom environments in which they encourage metacognitive behaviour and provide opportunities for students to reflect on their work. Teachers promote metacognition by modelling metacognitive behaviour, such as thinking aloud and asking metacognitive questions (Donaldson, 2011).

2.2.3 Guidelines for teaching problem-solving

Foshay and Kirkley (1998) identified the following as the basis for problem-solving instruction in the PLATO system. The following is a summary of their principles: First, the context of problem-solving must either alternate between declarative and procedural knowledge or integrate both. In any problem or situation, there must be both declarative and procedural knowledge, and each must have appropriate pedagogical importance. When teaching declarative knowledge, emphasis should be placed on mental models appropriate for solving the problem ahead, explaining the structures of knowledge and asking learners to predict what is going to happen or explain why something has happened. They also suggested the use of direct (deductive) teaching strategies for declarative knowledge and well-structured problem-solving and the use of inductive teaching strategies to encourage synthesis of mental models and for moderate and poorly structured problem-solving.

Second, they stressed that Mathematical problems should be moderate and clearly structured. This means that teachers need to teach problem-solving skills in the context in which they will be used. For example, using current problems in explanations, practice and assessments, with simulations, games and scenario-based projects. In other words, problem-solving should not be taught as an independent, abstract and decontextualized skill. On the other hand, when teaching moderately structured problem-solving, teachers should encourage learners to use their declarative knowledge (context) to invent a strategy appropriate to the context and the problem. This allows learners to use many appropriate strategies to arrive at a solution and to compare their effectiveness and efficiency.

In addition, they suggested that for each problem, teachers need to help learners understand the problem (define the goal) and then break it down into intermediate objectives. In addition, asking frequent questions about the strategies adopted by the learners encourages them to reflect on their problem-solving strategies as well as the ability to grasp the generalization part of the skill, across many similar problems in a different context.

Finally, teachers need to use context, problems and teaching styles that enhance learners' interest, motivation, confidence, perseverance and self-awareness and reduce anxiety. They added that the planned lesson should be refined from a novice level to an understanding of knowledge at an expert level. According to Bransford, Brown and Cocking (2000), a general strategy can be summarized in five steps. They go on to explain that these steps involve using previous information and translating it into a new problem.

Bransford, Brown, and Cocking (2000) developed the following guidelines, based on studies that compare experts and novices in problem solving and use recorded differences to help students progress through the problem-solving process:

Comprehend the problem. In this step you must accurately assess the situation, identify and understand the problem. This allows the problem solver to decide what information is important, what information can be ignored, and what additional information may be needed.

Represent the problem in formal terms. This step allows the problem solver to simplify a complex problem into its essential parts or units. This makes it easier to find the solution. The main objective of this guideline is to determine the relationships between the unknown and the known. This can be achieved by simplifying the problem situation by visualizing the events described in the problem and then describing it using a sketch or diagram. Then, reaffirm what you intend to find by identifying the desired unknown and naming specific variables.

Plan a solution. This step involves the ability to describe how the problem will be solved and whether it will provide a reasonable solution before implementing the plan. In some cases, logical steps can be expressed mathematically in a practical way, while others involve selecting an equation that specifies how the variables are related.

Execute the plan. The fourth stage of this guideline involves executing the plan to get the solution. This is the process whereby the problem solver inserts all of the known quantities into the solution to determine a value.

Evaluate and interpret the solution. The final guideline deals with how well the solution resolves the original problem given. This involves checking the work to find out if it is properly stated, reasonable and that it answers the question asked. To do this, the problem solver needs to explain or restate the solution in terms that relate to the original problem.

2.3.3 Problem-solving approach in Mathematics education

Problem-solving approaches to teaching Mathematics have been identified in the problem-solving literature. These include teaching for problem-solving, teaching about problem-solving, and teaching through problem-solving (Shroeder & Lester, 1989; Siernon & Booker, 1990; Anderson, 2009; Foong, 2002). Each of these approaches has implications for the types of activities and strategies that can be presented to students in Mathematics classrooms. All three approaches involve the use of problem-solving strategies and heuristics.

However, problem-solving education treats problem-solving as a process of enquiry, while education for and about problem-solving treats problem-solving as an object of enquiry. It has been argued that all three approaches have their place in Mathematics education, although problem-solving education is considered the most appropriate. Schroeder and Lester (1989) also pointed out that all three approaches have value but that teacher should be aware of the shortcomings of teaching for and about problem-solving approaches if used in isolation. They argued that when teaching for problem-solving, a problem can be reduced to applications of recently learned concepts and does not necessarily require extensive Mathematical thinking on the part of students.

They also indicated that teaching problem-solving can lead to problem-solving being treated as another subject in the curriculum. Finally, they recommended that problem-based teaching is most likely to promote understanding.

Teaching for problem-solving

Teaching for problem-solving involves students learning Mathematical content so that they can apply it to solve problems related to that content area (Anderson, 2009: Foong, 2002). Lester (2013) has presented it as the approach of an end. Indeed, the acquisition of Mathematical knowledge is focused on solving both routine and non-routine problems. Routine problems are practical and contain at least one arithmetic operation or ratio (Altun, cited by Kayan, 2007).

On the other hand, non-routine problems mainly concern the development of learners' Mathematical thinking skills and the stimulation of their Mathematical creativity (Polya, 1966). In this approach, teachers provide students with the skills and knowledge to solve Mathematical problems. The problems are usually related to the Mathematical content that has just been studied, and students are offered a variety of applications in which this Mathematics can be used (Anderson, 2009). In problem-solving instruction, the focus is on learning Mathematics with the primary goal of applying it to problem-solving in a wide range of situations after learning a particular topic (Foong, 2002). This approach is often associated with closed problems in terms of clearly formulated tasks where the only correct answer can always be determined in a fixed way from the necessary data given in the problem situation. These closed problems include content-specific routine problems, multi-stage problems as well as non-routine heuristic problems (Foong, 2002). When teaching problem-solving, learners should be allowed to apply their concepts and understanding to solve both routine and non-routine problems. In addition, teachers teaching problem-solving should focus on the learner's ability to transfer what they have learned from one aspect to another (Schroeder & Lester, as cited in Webb & Sepeng, 2012).

Teaching about problem-solving

Teaching about problem-solving includes advice on the problem-solving process and instruction on a variety of problem-solving strategies. It often includes the recommendations of Polya's (2004) problem-solving strategies. For example, in order for a teacher to teach problem-solving, he or she must first choose a problem-solving model and then systematically follow that model. For example, Polya suggested the following steps: understand the problem, design a plan, execute the plan and then look back. The goal of a teacher who teaches problem-solving is to follow the four steps that the learner can use to solve a problem.

According to Lester (2013), when teaching problem-solving, the teacher shows how to solve certain problems and teaches learners to focus on the most important procedures and strategies for solving those problems. When teaching problem-solving, students learn to use a variety of problem-solving strategies or heuristics, such as making a list, drawing a diagram, staging it and solving a similar problem by guessing and checking (Anderson, 2009). It is emphasized that the approach does not promote learners' original thinking skills because they must choose from a variety of solutions or strategies that will apply to a particular problem.

In Mathematics studies, the most popular model for problem-solving is that of Polya (1945). This model involved four steps: understanding the problem, designing a plan, executing the plan and looking back. According to Polya, in order to solve a problem, these four steps must be followed. In this approach, learners are expected to design their strategies or heuristics, (such as making a list, drawing, modelling, conjecture, guessing and checking) to solve the problem (as cited in Anderson, 2005). These problem-solving strategies are supposed to help learners choose the right path that seems to lead to a better outcome (Anderson, 2005).

However, the use of problem-solving strategies does not guarantee that a solution will be found if it exists, but rather strategies increase the likelihood that the solution can be found (Frensch, 2014). Thought studies have much to say about teaching problem-solving as the most preferred and widespread approach in textbooks, but it also has a limitation. According to Schroeder and Lester, problem-solving is seen as a subject to be added to the curriculum, as an isolated unit of Mathematics, not as a context in which Mathematics is learned and applied (as cited in Webb & Sepeng, 2012).

Teaching through Problem-Solving

In this approach, problems are used as a vehicle for learning Mathematics (Anderson, 2009; Foong, 2002). Problem-solving instruction focuses more on student understanding. In this approach, an attempt is made to make sense of the Mathematical procedures needed to solve a problem as a problem; students make a commitment to do Mathematics (Foong, 2002). This approach makes problem-solving a means rather than an end. Problem-solving instruction begins with a problem;

teachers pose problems to challenge students' knowledge, requiring the student to organize his or her understanding to solve the problem (Anderson, 2009). Students learn and understand important aspects of the concept or idea by exploring the problem situation (Cai & Lester, 2012).

These are often more open-ended problems that allow for several correct answers and multiple approaches to solutions. In problem-solving instruction, problems are not only the focus of organization and the stimulus for student learning, but also serve as a vehicle for mathematical exploration (Cai & Lester, 2012). Pupils play a very active role in their learning by exploring situations with the help of their teacher and "inventing" their solution strategies. Pupils' exploration of the problem is an essential component of problem-solving instruction.

In problem-solving education, learning takes place during the problem-solving process. When students solve problems, they use every approach they can think of, build on all the knowledge they have acquired and justify their ideas in a way they find convincing (Cai & Lester 2012). The learning environment of problem-solving instruction provides a natural framework for students to present various solutions to their group or class and learn Mathematics through social interaction, i.e., negotiation and the search for a common understanding (Cai & Lester 2012). These activities help students clarify their ideas and gain different perspectives on the concept or idea they are learning. Problem-solving instruction, as noted by Chapman (2012), is more process- and strategy-focused than product-focused.

According to Baki, (2004) and Chapman, (2012), problem-solving instruction involves creating an environment where students can discuss their views on a problem and explain their methods of enquiry and generalisation to their classmates. Van De Walle (2007), for his part, explained that problem-solving instruction requires students to read a problem carefully, analyse it for all the information they have, and then examine their Mathematical knowledge to see if they can find a strategy that will help them find a solution. This process forces the reorganization of existing ideas and the emergence of new ideas as students work on problems with the help of a teacher who acts as a facilitator by asking questions that will help students review their knowledge and build new connections.

The role of the teacher in this approach is essentially non-judgmental rather than authoritarian. Instead of being the sole source of knowledge and solutions, the teacher creates a classroom climate and culture that encourages and facilitates student initiative and stimulates interactive and collaborative problem-solving. Norton, McRobbie and Cooper (2002) therefore see Mathematics instruction through problem-solving as an approach in which teachers see themselves as guides, listeners and observers rather than as authorities and disseminators of knowledge and information. However, Mathematics education through problem-solving is a relatively new idea in the history of problem-solving in the Mathematics curriculum (Cai & Lester, 2012). As problem-solving instruction is a new concept, it has not been the subject of much research.

2.4 Benefits of Problem-Solving in Mathematics Education

To learn in the 21st century, students need to be critical thinkers and problem solvers. However, problem-solving has become the predominant method of teaching Mathematics in this modern society because of its many advantages. Indeed, many jobs in our modern society require employees to use their knowledge of technology and their problem-solving skills. This new approach to Mathematics education is based on the belief that the main reason to study Mathematics is to learn how to solve everyday problems (Posamentier, Smith & Stepelman, 2006).

Moreover, the problem-solving process is not only about solving problems but also about building new Mathematical knowledge. The National Council of Teachers of Mathematics (NCTM, 2000) has stated that students can acquire new Mathematical knowledge by teaching those problem-solving strategies: to solve problems that arise in Mathematics and other contexts, to apply and adapt a variety of appropriate strategies to solve a problem, and to follow and reflect on the Mathematical problem-solving process. This places the teacher at the centre of the problem-solving process and its application. It also provides a basis that teachers can use to help their learners achieve higher levels of Mathematical thinking and learning. In addition, research has shown that problem-solving is a goal of mental development and a teaching method in Mathematics education (Jonassen, 2004). The reason for this emphasis is that problem-solving is not only about meeting everyday challenges, but also about the future of society and the improvement of the workforce.

Problem-solving helps learners to see Mathematics as a dynamic discipline, in which they have the opportunity to organise their ideas, engage in Mathematical discussion and defend their conjectures (Manuel, 1998). In thinking about their solution, students use a variety of Mathematical skills, develop a deeper understanding of the structure of Mathematics and acquire a disposition to generalization that also helps them to develop patterns of thinking, habitual persistence and curiosity, and confidence in unfamiliar situations that serve them well outside the Mathematics classroom (NCTM, 2000). In the face of the new and unfamiliar situation, they solve all the difficulties of such a frequently posed situation as the essence of problem-solving (Jonassen, 2004). Thus, problem-solving is not just about remembering well learned facts or procedures, but rather a good problem solver must demonstrate a certain degree of creativity and originality (Polya, as cited in Jonassen, 2010).

2.5 Challenges of Teaching Mathematics Problem-Solving

Although the use of problem-solving in Mathematics education has many advantages, its successful implementation also faces some obstacles. The challenges of teaching problem-solving in Mathematics addressed in the research literature can be grouped into three broad categories. These are problems related to the teacher, the students and the curriculum.

2.5.1 Challenges related to teachers

Buschman (2004) has argued that teaching through problem-solving for learners to understand poses many challenges for the teacher. One of the challenges faced by many teachers is that their previous professional experience does not prepare them to teach problem-solving strategies (Buschman). In addition, teacher educators are not prepared to teach in a way that incorporates their new ideas and knowledge (Artzt, Armour-Thomas & Curcio, 2009). This is because most primary school Mathematics teachers are trained as general educators and often do not have the Mathematics training required to teach using problem-solving approaches.

As generalist teachers, they may not have sufficient knowledge to anticipate anything other than limited curricular goals or teaching styles and may therefore be handicapped in achieving a problem-solving orientation (Andrews & Xenofontos 2014). The situation in Ghana is no different from Andrews & Xenofontos observation regarding teacher preparation. This may lead to teachers not being equipped both in terms of content knowledge and pedagogical content knowledge to teach Mathematics using problem-solving strategies. Xenofontos & Andrews added that the use of problem-solving approaches requires both thorough preparation and the development of ways to maintain at least a minimum of control in the classroom and, perhaps more importantly, the ability to anticipate the objectives of Mathematics teaching in the light of such an orientation.

In a study on the problem of language in problem-solving in Mathematics in basic school, Mereku (2015) observed that problem-solving is unpopular in basic schools in Ghana because many teachers do not know how to introduce it into the classroom; they cannot solve the problems themselves, and they cannot explain why students find problem-solving so difficult to learn. The author further pointed out that teachers find it difficult to teach problem-solving. Mclntosh, Jarrett and Peixotto (2000) have stated that teaching Mathematics through problem-solving is difficult for teachers because they have insufficient knowledge of the subject matter, insufficient knowledge of instructional content and personal problems. In addition, they do not have the Mathematical expertise to understand the different approaches that students might use to solve a problem and to identify promising approaches to problem-solving.

However, they often provide a strong justification for not including problem-solving activities in their Mathematics instruction: it is too time-consuming, too demanding, and not measured and tested in public examinations. The authors also observed that teachers are generally expected to cover broad areas of Mathematical content, yet problem-solving takes too much time to teach. As a result, many teachers tend not to feel ready to use a problem-solving approach to teaching Mathematics. In addition, teachers often find it difficult to see their students struggling with frustration in problem-solving situations, in terms of when to give advice and intervene.

Mathematical knowledge for teaching is an essential ingredient for effective teaching (Ball & Bass, 2000). In addition, some teachers lack the knowledge, skills, and expertise to teach Mathematics through problem-solving (Anderson, 2009). Lack of Mathematical knowledge for teaching reduces teachers' confidence in teaching Mathematics through problem-solving. These teachers rely on traditional methods where students memorize rules at the expense of teaching the construction of meaningful knowledge through problem-solving. In addition, teachers' lack of ownership of the Mathematics content of the curriculum does not encourage teachers to practice problem-solving (Anderson, Sullivan & White, 2004).

Mathematics teachers seem to have more confidence in the teaching methods they have experienced in their school life (Saleh, 2009) and indeed, teachers who have not experienced a problem-solving method in their professional training tend not to emphasize problem-solving approaches in teaching. Taplin (2010) has stated that although problem-solving is emphasized in the Mathematics curriculum around the world, teachers still do not know how best to teach problem-solving skills. In fact, the difficulties associated with teaching student achievement mean that many students avoid teaching problem-solving because they are uncomfortable with their problem-solving skills (Ellison, 2009).

2.5.2 Challenges related to students

Henderson (2002), in his study of faculty conceptions of teaching and learning problem-solving in the introduction to computational physics, described students' knowledge/skills in problem-solving as poor. At the same time, a study conducted by Adesoji (2008) shows that students with a high level of proficiency have a better understanding of problem-solving. Thus, it is relatively easy to teach these students Mathematics using problem-solving. However, those with low proficiency could also develop problem-solving skills if they were exposed to strategies for teaching problem-solving (Adesoji, 2008). In a research study, Saleh (2009) confirmed that students' knowledge base is a key factor in teaching Mathematics through problem-solving and concluded that problem-solving is not good for students with low ability.

The inability of students to read and understand poses yet another problem for teachers when teaching Mathematics using problem-solving strategies. Fletcher and Santoli (2003) reported that Mathematics vocabulary is generally not taught in schools and that if students do not read good textbooks, they have no place to understand Mathematical terms. Therefore, an emphasis on vocabulary instruction in Mathematics curricula is crucial if students are to learn Mathematics through full problem-solving.

2.5.3 Challenges related to the school curricular

Anderson, Sullivan, and White (2004), in their study of the influence of perceived constraints on teachers' beliefs and practices in problem-solving, identified textbooks and assessment regimes used in schools, as well as the timing of Mathematics classes, as barriers to teaching Mathematics through problem-solving. In addition, the conservative teaching methods of other teachers in the school as well as parents' requirements to prepare their wards for contests are other factors identified as barriers to the implementation of problem-solving instructions in the Mathematics classroom.

Mclntosh, et al (2000) in a review of the literature on problem-based Mathematics education found that many textbooks do not provide a sufficient number of non-common problems from which teachers can choose. This affects teachers' use of problem-solving methods for teaching Mathematics, as they rely mainly on textbooks as a source of information. Ali, Hukamdad, Akhter and Khan (2010), in a study to investigate the effects of the use of problem-solving methods on student achievement in elementary Mathematics education, found that traditional textbooks do not meet the criteria for a problem-solving approach. This phenomenon of textbooks not presenting sufficient problem-solving questions has the potential to prevent teachers from teaching Mathematics using problem-solving approaches.

Zanzali (2003), in a study to document the constraints that teachers face in implementing curriculum aspirations, identified the influence of the examination on the content and manner in which Mathematics should be taught to students as a barrier for teachers to use problem-solving in teaching. Given that the mark of a good teacher is to help students pass their exam and that the trend of exams parallels teaching through problem-solving, its implementation in the classroom has become an issue of concern for teachers.

The results of Saleh's (2009) study indicate that limited time for Mathematics lessons and problem-solving methods that are not necessary to answer examination questions are some of the reasons that hinder the teaching of Mathematics through problem-solving. Teachers participating in the study felt that teaching Mathematics through problem-solving takes time.

As Anderson (2009) points out, a school's culture can sometimes be a barrier to the implementation of new educational innovations. Curricula, Mathematics textbooks, homogeneous classrooms, assessment practices, staff attitudes, and time are some of the constraints the school may face in implementing problem-solving in Mathematics. The school culture may hinder teachers' planning and approaches because of prescribed curriculum practices as well as the traditional beliefs of other staff members. In school, there is so much competing for time. These include the compulsory curriculum, external evaluation procedures, and curriculum workload.

However, teachers feel that teaching problem-solving takes a lot of time, and if time is not sufficient, it is better to use storytelling as a way of teaching Mathematics.

Teachers expressed a lack of resource materials available for teaching problem-solving (Foong, Yap, & Koay, 1996). Research has emphasized the importance of teaching through problem-solving and teaching problem-solving skills to students, but the pressure on teachers to raise their students' test scores forces them to stick to textbook routines for teaching Mathematics instead of using problem-solving approaches (Tratton & Midgett, 2001).

Anderson (2009) also found that students are sometimes so attentive to certain established procedures for solving Mathematical problems. These students resist teacher initiatives or consider adopting problem-solving approaches in Mathematics instruction. They prefer to have Mathematics explained to them rather than be guided by the teacher to explore and build their understanding. He also found that diversity in classrooms, students understanding of language and their attitudes and beliefs towards Mathematics are potential factors that militate against the implementation of problem-solving curriculum in Mathematics. Some students would benefit from teaching strategies for problem-solving. As in the parable, if you teach a man to fish, he can fish all his life. Similarly, if you teach a student to solve a problem, he can do it for the rest of his life.

Pupils should have the opportunity to make connections between real world applications and Mathematics. Pupils enjoy Mathematics more when there are connections with their favourite basketball player or musician. These are concrete cases where pupils learn to apply their skills to their daily lives. Pupils should be able to apply what they learn at school to become productive citizens of society. Hiebert and Wearne (2003) have written that tasks such as problem-solving can promote students' conceptual understanding, their ability to reason Mathematically, their ability to communicate Mathematically and to capture their interest. Capturing a student's interest is necessary for learning. If the problems apply to real life, it may be easier for the student to understand the problem. Giving students relevant problems gives meaning to Mathematical concepts.

The National Research Council (1989) found that it is not the memorization of important Mathematical skills, but the assurance that Mathematical tools can be found and used in problem-solving. Students gain this confidence through the process of creating, constructing and discovering Mathematics. When it is part of the students' daily routine, they are more competent and able to develop, carry out and execute their plan.

Helping students to better solve problems is not only a fundamental part of learning Mathematics, but also in all areas and at all levels. This competence is a continuous process that students should develop at their own pace. The hope of problem-solving instruction is that students will continue to use problem-solving skills throughout their lives.

Hahkioniemi, Leppaaho and Francisco (2012) studied the problem-solving processes of ninth-grade students when working on an open problem using dynamic geometry software. The aim of the researchers was to conceptualize the problem-solving processes of students when they engage in open problem-solving using technology under the guidance of a teacher. Data was collected by video recording a 45-minute lesson with two video cameras and capturing student computer screens. According to Hahkioniemi et al. (2012), several models describe the process of Mathematical problem-solving (e.g., Polya, 1945; Schoenfeld, 1985).

However, the use of technology, classroom conditions and the nature of the problem can affect the problem-solving process. They reported that, for example, the use of technology enhances the exploration of the problem. Research has shown that in actual implementations of problem-solving in the classroom, there are time constraints and the use of open-ended problems emphasizes posing problems and creating different ideas for solutions (Nohda, 2000).

As part of the study, future teachers were trained as Mathematics survey directors. For example, trainee teachers practised guiding students in hypothetical teaching situations (e.g., Hahkioniemi & Leppaaho, 2012). Next, each trainee teacher implemented an investigative Mathematics lesson in grades 7-12. In analysing the problem-solving processes of pairs of students, the study by Hahkioniemi et al. (2012) highlighted the following four phases: Framing the problem, Exploring the solution, Conjecting and Justifying or investigating the conjecture. The researchers noted that the lesson succeeded in engaging students in rich and creative Mathematical thinking and that students were active and enthusiastic.

According to Hahkioniemi et al (2012), this was the case even though it was the first student lesson where GeoGebra was used so intensively, the open problems were new to them and the lesson was taught by a trainee teacher under ordinary classroom conditions in 45 minutes. Thus, it also shows that early experiences of open problem-solving enriched by technology can be very positive. In the empirical part of the study by Kolar, Mastnak and Cadez (2012) conducted among primary school student teachers, the aim was to explore their skills in inductive reasoning. In the early school years, inductive reasoning is often used as a strategy for teaching basic Mathematical concepts, as well as for solving problem situations (Kolar et al., 2012).

In the same research, emphasis was placed on the use of inductive reasoning to solve a Mathematical problem.

The study then analysed the relationship between depth and problem-solving strategy and found that not all strategies were equally effective in seeking generalisations about problems. Inductive reasoning is used as a strategy in the teaching of basic Mathematical concepts, as well as in problem-solving situations. Researchers believe that in Mathematics, only teachers who have problem-solving skills can create and manage classroom situations that contribute to the development of these skills in children. The empirical study was based on the descriptive, occasional, non-experimental method of educational research (Hartas, 2010). This method, according to Kolar et al. (2012), allowed them to explore problem-solving strategies related to generalization among primary school students. The study found that the majority of students generally perceive the given situation as a problem; however, their abilities to delve deeper into the problem are quite different.

Based on the stages of inductive reasoning according to Canadas and Castro (2009), it can be inferred that students' responses focused on the following three stages: observation of specific cases, model finding and prediction, and generalization. They found it important to establish that the stage that an individual student reaches is largely influenced by his or her choice of strategy. When we develop Mathematical thinking in the early years of school, we deal mainly with situations in which children have to reason inductively.

In the action research study, Lopez (2008) examined the influence of Mathematical strategies on the Mathematical ability of college students. The study took place in an urban school in Orlando, Florida, with 12 eighth graders assigned to the researcher's intensive Mathematics class. The objective of this action research was to observe the students' Mathematical abilities and to study whether teaching Mathematical problem-solving strategies in Mathematics will improve students' Mathematical thinking and their ability to understand and solve word problems. During the first five weeks of the study, students' Mathematics journals revealed that students barely understood the problem-solving task at hand and used the problem-solving strategies ineffectively. Twenty-five percent (3/12) of the students scored 4 on the holistic scale on the pre-test. Thirty-three (4/12) scored a 3 on the holistic scale and fifty percent (6/12) scored a 2 on the holistic scale. In general, eighty-three percent of students scored at or below average in their Mathematics journal.

As part of remedial intervention, more direct instruction has been provided to help students use problem-solving strategies effectively. Pupils who showed difficulties in their Mathematical, strategic and communication skills were given in-depth instruction on the objectives and purpose of the maths journals. Students were instructed to clarify their thinking process and to present their thoughts and ideas more clearly. After the intervention, the students' results showed an increase in the correct use of reading and Mathematical strategies. At the end of the study, all students were at or above average on the holistic scale. This increase revealed that students were performing in Mathematics, strategy and communicating their knowledge effectively. Students gave clear insights and understanding, gave organized thinking, new ideas and constructed Mathematics for themselves.

2.6 Summary of Review

This chapter presented the theoretical and conceptual framework of the study and reviewed other documents relevant to the study. The study explored the teacher perceived knowledge needed for Mathematics education and focused on problem-solving in Mathematics education. In order for teachers to use problem-solving as a strategy for teaching Mathematics, they must have the problem-solving strategies. Polya's model therefore formed a coherent framework for this research study.

Three problematic approaches to the teaching of Mathematics are identified in the research literature. These approaches are: teaching for problem-solving, teaching about problem-solving, and teaching through problem-solving (Scheoder & Lester, 1989; Siernon & Booker, 1990; Anderson, 2009; Foog, 2002). The problem-solving literature has also revealed that knowledge of problem-solving strategies and heuristics by teachers and students will enable them to practice each of the problem-solving strategies in the Mathematics classroom (Polya, 1985; Verschaffel, et al, 2002).

Teachers' perceptions of problem-solving vary from teacher to teacher. Some teachers have conceptualized problem-solving as solving difficult word problems involving real-life situations (Saleh, 2009), while others believe that problem-solving is a process of accepting and solving difficult Mathematical problems (Dollah, 2006).

Teachers should have the opportunity to reflect on their teaching, if possible, with others, regarding the teaching of Mathematics through problem-solving. Teachers are particularly in need of training and experience in problem development and knowledge of Mathematical content. An effective way of helping practising teachers to develop these skills is to create teacher collectives. Teacher collectives are "groups of teachers, either in the same school or in neighbouring regions, who meet regularly to solve problems, share ideas, discuss pedagogy, plan lessons, and reflect collaboratively on teaching and learning" (Institute for Advanced Study/Park City Mathematics Institute International Seminar, 2006, p. 11). Peer observation and team teaching are also effective ways of developing teachers' knowledge and problem-solving skills (IAS/PCMIIS, 2006).

Ali, et al (2010) recommended an extensive training programme, seminars and workshops for primary school Mathematics teachers to enable them to use a problem-solving method in the classroom. They also suggested organising training sessions for student teachers to be trained in problem-solving and proposed to transform Mathematics textbooks into a form of problem-based learning. Since many teachers depend on textbooks as the main source of information for teaching, the presence of problem-solving questions in Mathematics textbooks will encourage them to teach Mathematics using problem-solving methods.

Murat and Memnum (2008) have argued that the establishment of a constructivist social learning environment and the content of teaching have erased negative attitudes and problem-solving beliefs of students. Beliefs such as that a math problem has only one way to solve, only one correct answer, and that ordinary student can never correctly solve an unusual problem are eliminated from students' minds. Problem-solving involves solving open-ended problems that have no apparent method of solution (Hiebert, 2003; Lambdin, 2003; Van De Walle, 2007).

The challenges of teaching problem-solving in Mathematics are related to the teacher, the student and the curriculum. Teachers' beliefs about problem-solving, their knowledge of the content of Mathematics and problem-solving, and their knowledge of the instructional content of problem-solving affect their classroom problem-solving practices in Mathematics instruction (Anderson, Sullivan, & White, 2004; McIntosh, Jarrett, & Peixotto, 2000; Ball & Bass, 2000; Saleh, 2006; Ellison, 2009; Xenofontos & Andrews, 2014; Mereku, 2015). In addition, issues of inadequate problem-solving in Mathematics textbooks, school timetable constraints, and the conservative teaching methods of other teachers in schools are seen as some of the challenges that the curriculum has posed to teachers in their attempt to use problem-solving approaches in Mathematics instruction (Ali et al, 2010).

Studies have shown that problem-solving is more difficult for students with low ability (Adesoji, 2008; Saleh, 2009). In addition, when students believe they should be taught Mathematics, they tend to resist teachers who teach through problem-solving (Anderson, 2009).

CHAPTER THREE

METHODOLOGY

3.0 Overview

This chapter addresses the procedure of carrying out the research and consists of the philosophical underpinning, research design, research setting, population, sample and sampling techniques and research instruments. It also deals with the pilot study, data collection, data analysis procedure, and some ethical considerations.

3.1 Philosophical Underpinning of the Study

A philosophical underpinning of research is how someone sees the world through his or her sunglasses. A study by Tashakkori and Teddie (1998) indicated that there are three approaches to research. These include; qualitative, quantitative and mixed methods. However, each one of these approaches is characterized by a paradigm that the research design and methods of data analysis are derived from (Creswell & Creswell, 2017). Paradigm is a basic set of assumptions that guide researcher methodology (Creswell & Creswell, 2017). The quantitative approach is viewed from the post-positivist perspective that employs the use of inquiry (experimentation, survey, etc.), and uses numeric data for its measurement and analysis (Adeleke, 2017). The quantitative methods deal with the use of numbers and variables that can be muted in a systematic process of describing a phenomenon.

Qualitative, on the other hand, is described as constructivist or transformative-emancipatory paradigms (Adeleke, 2017). This approach employs designs such as case study, narrative and usually in the form of textual data. Thus, researchers using this approach study things in their natural settings and give its interpretations (Lincoln, 2000).

The final method that has to do with mixed methods uses both quantitative and qualitative methods in its data analysis to address research questions. There is a range of philosophical approaches that underpin mixed methods research (Mesel, 2013). Therefore, Creswell and Plano-Clark (2014) outlined three (3) different approaches commonly used in mixed methods. These are pragmatism, transformative, critical realism.

This study adopted the pragmatist approach of philosophy. The pragmatic approach is where the researcher uses "what works" to seek answers to the study (Creswell & Plano-Clark, 2014). According to the pragmatist, research questions are very crucial to both the subjectivist and objectivist to reveal the true nature of the study (Creswell, & Plano-Clark, 2014). The pragmatist mixed-method approach was adopted based on the following reasons; it gives a greater validity through corroboration between both quantitative and qualitative data. This gives a clear and comprehensive overview of the phenomenon under study, to be able to answer different research questions and to be able to use the qualitative phase to develop the quantitative phase (Bryman, 2016).

Also, it is intended to collect both quantitative and qualitative data to give a vivid description of the problem under study.

Finally, this philosophical approach enabled the researcher to develop a thorough understanding of teachers' perceived knowledge, instructional strategies, how they engage pupils' and measures to be adopted to improve the teaching of problem-solving.

Both qualitative and quantitative approaches were employed in the study. The use of a quantitative approach was necessary to determine the extent that enable the researcher to obtain data that could be analysed statistically to explore Mathematics teacher's perceived knowledge for teaching problem-solving. Fetters et al. (2013) noted, using a mixture of both qualitative and quantitative approaches enables researchers to obtain and utilized much more comprehensive data that include numbers, statistics, words, and narrative. By using both the qualitative and quantitative approaches in this study, the researcher was able to provide a much more detailed analysis to achieve the objectives of the study.

3.2 Research Design

Burns and Grove (2010) defined research design as a blueprint that guides the researcher when conducting the study utilizing maximum control over the variables to validate the findings from the study. Simply put, a research design is a framework that is used to answer the research question and validate the findings.

In general, there are several established research designs that the researcher could choose from: comparative design, cross-sectional design, longitudinal design, case study design or the traditional experimental design Creswell (2017). Amedahe (2002) has noted that in every research study, the choice of a particular research design must be appropriate to the subject under investigation and that the various designs in research have specific advantages and disadvantages.

This study adopted both quantitative and qualitative methods because it is to gain a deeper understanding of the problem under study. Morrison (2012) outline that adopting both the quantitative and qualitative methods (two traditional approaches) give an in-depth understanding of a phenomenon. According to Creswell (2017), explanatory sequential mixed methods is one in which the researcher first conducts quantitative research, analyzes the results and then builds on the results to explain them in more detail with qualitative research. Sequential explanatory design has two main distinctive phases. Thus, the quantitative phase followed by the qualitative phases.

One of the strengths of using the sequential explanatory research design is that the two phases (quantitative and qualitative) make it straightforward to implement because the researcher conducts the two methods separately and collects one data at a time, the final report from the study provides a clear delineation for readers.

Therefore, to explore teacher's perceived knowledge, instructional strategies, pupils' engagement and ways to improve the teaching of problem-solving this study adopted the sequential explanatory mixed-method research design. In this study, the researcher first collected the numeric data (quantitative) and analyzed it. Then, the context is also collected to help to explain the numeric data (quantitative) in the first result. The rationale for this approach is that the quantitative data and their subsequent analysis provide a general understanding of the research problem. The qualitative data and their analysis refine and explain those statistical results by exploring participants' views in more depth (Creswell, 2009). the findings from the qualitative data are to enrich the findings from the quantitative data (Mason, 2006) and to generate new knowledge (Creswell, 2009).

3.3 Research Setting

This study was conducted in the Berekum West District in the Bono Region of Ghana. Berekum West is located in the North-Western corner of the region. The Berekum West District Assembly is one of the 12 administrative districts of the Bono Region. It was established by a Legislative Instrument L.I. 2337. Jinijini serves as the administrative capital of the District. The population of the Berekum West District in 2010 was 50,749 based on a selection of twenty communities and out of these figures, 25,324 (49.9%) were males and 25,425 being females representing (50.1). The current projected population of the District for 2018 is 79,656 (Ghana Statistical Service, 2010). The Akan ethnic group forms the majority amongst the dialect groups in the District.

Agriculture is the dominant economic activity in terms of employment and income. The major crops cultivated in the District are plantain, cocoyam, cassava, vegetables, yam, maize and some exotic crops such as cashew, cocoa, citrus, palm kernel, and mango. Some of the major trees found within the District are Wawa, Odum, Sapele, Teak, and Mahogany, etc. There are a total number of 104 schools both privately and publicly owned in the District. Out of this number, 35 are pre-schools (KG), 39 Primary Schools, 45 Junior High Schools and 2 Senior High Schools. There is no Vocational/Technical School in the District. The District has five (5) circuits, which consist of both rural and urban communities. The urban segment of this District is among the few privileged places in the Region where access to basic social amenities is guaranteed. The availability of these amenities has attracted many rural dwellers. This has increased a number of students/pupils in the turban and rural communities in the District. This has also resulted in a high teacher-student ratio in the classroom.

3.4 Population

Kusi (2012) defines a population as a group of individuals or people with the same characteristics and in whom the researcher is interested. Banerjee and Chaudhury (2010) defined a study population as all the individuals or objects/elements that have specific characteristics that are of interest to a researcher for a particular investigation. The population of this study consisted of all Mathematics teachers in the Berekum West District. The large group to which the researcher wish to generalize the results of the study becomes the targeted population

(Sasso, & Ross, 2019). The target population consisted of public basic school teachers in Berekum West District.

Precisely, the accessible population for this consisted of all public Junior High School Mathematics teachers in the Berekum West District. Most importantly, JHS Mathematics teachers were selected for the study because it is at that level that the students are prepared for their external examination (Basic Education Certificate Examination) and it is believed that if students are well equipped with problem-solving, it would positively affect their performance in Mathematics. Table 1 presents the distribution of public JHS Mathematics teachers in the Berekum West District.

Table 1: Distribution of Public Junior High School Mathematics Teachers in the Berekum West District

S /N	Name of Circuit	No. of Schools	Total No. of Mathematics Teachers
1	Jinijini North	8	15
2	Jinijini South	8	15
3	Nsapor	8	15
4	Jamdede	10	17
5	Fetentaa	11	18
Total	**5**	**45**	**80**

Source: Statistics Office, GES, Berekum West District, 2019/2020

In all, there are eight (80) public JHS Mathematics teachers in the Berekum West District.

3.5 Sample and Sampling Techniques

Sampling is the process of selecting a portion of the population to represent the entire population (Alhassan, 2006). Berekum West District was selected conveniently as the researcher in the area. This made it easier to obtain the data needed for the study. The study adopted the census technique for the quantitative phase. The census was used because the researcher wanted to seek the perceived knowledge and skills of all the Mathematics teachers in the District.

Again, the convenient sampling technique was used to select the schools for the qualitative phase based on the researcher's discretion, availability of time and resources. The technique was used to select five teachers, one female and four male teachers for both the interview section and classroom observation.

3.6 Data Sources

To be able to achieve the stated objectives, the study relied on both primary and secondary sources of data. The primary data were collected from teachers. The instruments that were employed in collecting the primary data included a questionnaire, interview, and observation. The kind of primary data collected included data on the demographic characteristics of respondents. Secondary data was obtained from the District Education Office and headteachers in the public junior high schools in Berekum West District.

Another set of secondary information was collected from the internet, published data, survey reports, books, journals, students' theses, as well as other unpublished documents.

3.7 Methods of Data Collection

The data collection process of the study involves the development and administration of data collection instruments. Acquiring an in-depth understanding of public JHS Mathematics teachers' perceived knowledge and practices requires collecting a wide variety of information using three instruments. The purpose of the study was to explore public JHS Mathematics teachers' perceived knowledge, instructional strategies, pupils' engagement and measures to improve problem-solving in teaching Mathematics. Three research instruments namely; questionnaire, interview guide, and observation checklists were used for the data collection (See Appendix A, B, and C).

According to Creswell (2009), using more than one instrument for data collection allows for triangulation. Triangulation is the process whereby the researchers look for convergence among multiple and different sources of information to validate a phenomenon. In short, triangulation helps to increase the credibility of a study (Creswell, & Miller, 2000). The interview guide was also unstructured questions that were administered to the Mathematics teachers in all public junior high schools in the Berekum West District. Observation checklist was also used to collect the qualitative data.

3.7.1 Questionnaire

This study needs data to describe the attributes of Mathematics teachers in public Junior High School, their perceived knowledge for teaching problem-solving, their instructional strategies and pupil's engagement of problem-solving. The questionnaire for data collection was developed by the researcher based on the research questions of the study. Thomas (2003) suggests using a questionnaire in collecting large data because (i) large quantity of data can be collected in a relatively short period; (ii) a wide variety of information can be obtained from participants, particularly if the questions are multiple-choice; and (iii) data can be collected from participants in distant places and in the absence of the researcher.

Regardless of the strength of a questionnaire, it has a low response rate and also response bias is more likely to occur (Creswell & Creswell, 2017). The questionnaire was chosen because it requires little time for respondents to complete. It also allows for broad geographical sampling and it can be used to cover a large sample as well (Leung et al., 2019). Salant and Dillman (1994) indicated some reasons to follow when designing a questionnaire for data collection. These include; a well-designed questionnaire demands adequate time in planning, it should be attractive to the respondents and it should be easy to be handled by respondents.

This study used the structured questionnaire to seek teachers' perceived knowledge, instructional strategies and how teachers engage in problem-solving. This provided a numeric data from the participants. The questionnaire (see Appendix A, B, C and D) was in four sections: Section A, Section B, Section C and Section D. Section A consisted of items that sought respondents' background information (excluding names and other personal identities). The section provided information such as gender, age, professional qualification and years of teaching Mathematics.

Section B consisted of twenty (20) statements that were derived from literature on problem-solving. It was a closed-ended item type. It was measured with 4-Likert scale (strongly disagree, disagree, agree and strongly agree). Respondents were expected to tick a statement according to the scale of their decision. Section C contained a statement on teachers' instructional strategies for teaching problem-solving. The section focused on the nature of Mathematics problems as well as the techniques teachers used to teach Mathematical problem-solving. It was eleven (11) statements measured on the 4-Likert scale.

Finally, Section D deals with how teachers engage pupils in problem-solving. The statements were derived from the three dimensions of the pupils' engagement. These include cognitive engagement, behavioural engagement and affective engagement. It was also nine statements measured on a 4-Likert scale.

3.7.2 Interview guide

Due to the need for more detailed data to support those obtained in the questionnaires, the researcher used interviews to create effective means of obtaining such detailed and sincere qualitative information from individual participants (Thomas, 2003). According to Mitchell and Jolley (2012), an interview is a survey in which the researcher orally asks participants questions to describe a phenomenon. They provide elaborate responses and a forum for sincere participation in a study. They opined that there are three main types of interviews namely; structured, semi-structured and unstructured interviews. Mitchell and Jolley explained that the structured interview is a type in which all respondents are asked a standard list of questions in a standard order. A semi-structured interview, like the structured interview, is constructed around a core of standard questions. However, the interviewer may expand on any question to explore a given response in greater depth.

Finally, Mitchell and Jolley explained that with the unstructured interview, the interviewers have objectives that they believe can be best met without an imposed structure. The interviewer is free to ask what he/she wants, how he/she wants to, and the respondent is free to answer how he/she pleases.

An unstructured interview guide was used to collect qualitative data from Berekum West JHS Mathematics teachers on their perceived knowledge, instructional strategies, pupils' engagement and measures to be adopted to improve the teaching of problem-solving. The guide was designed based on the issues emerging out of the results of the quantitative data. The interview questions were centred on the following themes; teachers' perceived knowledge of problem-solving, problem-solving instructional strategies, pupils' engagement and measures to improve the teaching of problem-solving (see Appendix B).

The interview was conducted on a one-on-one basis in the school setting and stored on a tape recorder. This enabled the participants to express their views and concerns freely and explicitly. The detailed nature of face-to-face interview helped the researcher to obtain from interviewees, such information they were unable to give using a questionnaire; thereby helping to complement the quantitative data obtained by the questionnaire.

3.7.3 Observation checklist

According to Amedahe (2002), observation is a method of data collection that employs vision as its main means of data collection. In observational studies, researchers collect data on the current status of subjects by watching them, listening and recording what they observe rather than asking questions about them. Observation can also be used to collect explanatory data about what is happening in a situation or a set-in perspective data obtained by questionnaire or interviews (Robson & McCartan, 2016). The observation checklist provides the researcher to develop a holistic understanding of a phenomenon under study that is as objective and accurate as possible given the limitations of the method (DeWalt & DeWalt, 2002).

Observation also allows the researcher to follow up on results emanating from the questionnaire and interview instruments. It is the interest of the researcher to explore teachers' perceived knowledge, instructional strategies, pupil's engagement and measures to improve problem-solving, therefore the need to employ an observation checklist among the research instrument (see Appendix C). The observation section provided the opportunity for the researcher to follow up on the result emanated from the interview guide and the questionnaire to ascertain what happens in the classroom.

The observation checklist took the form of semi-structured typed. It first looked out for general information about the lessons. Such information includes the following; topic/sub-topic, lessons, duration, class attendance, the physical appearance of the class as well as classroom activities. It also consisted of fifteen (15) structured statement designed to observe pupils engagement of problem-solving. These statements were rated on a three (3) point Likert scale; (1 = Not at all, 2 = Some evidence, 3 = Clear evidence). These rating keys help the researcher to describe each of the lessons observed.

3.8 Pilot Testing of Instruments

Wilkinson and Birmingham (2003) assert that it is common to construct a questionnaire with ambiguous layouts and mistakes in items. Similarly, Awanta and Asiedu-Addo (2008) caution that, it is possible to design a questionnaire that is reliable but invalid, due to inconsistencies in responses and failure to measure exactly what the scales are intended to measure. Because of this, the instruments in this study were pilot tested to minimize mistakes and errors to increase reliability and validity. Piloting research instruments is a procedure in which a researcher tries the instruments on a small number of individuals and makes necessary changes to improve the instruments, based on feedback from those involved in the trial (Creswell, 2014).

The instrument was piloted in the Berekum Municipality of the Bono region of Ghana. Berekum Municipality was chosen for the pilot study because the selected population for the work has comparable characteristics as those in Berekum West District. Also, the school environment in terms of infrastructure, teaching and learning materials were similar to those selected for the main study. Eighty (80) teachers were conveniently sampled for the pilot testing. This consisted of public JHS Mathematics teachers in the municipality. The pilot test helped to assess the strengths and/or weaknesses of the research instruments.

Also, it enabled the researcher to modify and change some of the statements that looked inappropriate and difficult to understand. This helped to reduce ambiguity and misinterpretation. According to Awanta and Asiedu-Addo (2008), piloting a test enables the researcher to modify items that are difficult to understand, ambiguous and incorporate new categories that could be relevant to the study. Two days were used to distribute the questionnaire to the teachers. A teacher used a maximum of ten (10) minutes to complete the questionnaire (e.g., Problem-solving involves tasks that challenges pupils' ability).

After the analysis, the test items for Section B were reduced from 20 to 15. This was because some of the items were similar in meaning to the others and some others were not measuring the objective of the section.

Again, Section C was reduced from 11 to 8 as Section D was also reduced from 10 to 7 (e.g., I pose an open-ended problem for pupils to solve was the same as teacher introduces lesson by posting open-ended problems to pupils to solve). Some test items were put together and restructured to make them meaningful. Other essential corrections concerning the numbering of the items and the format of the items were all made to give a vivid understanding of the questionnaire.

3.8.1 Reliability of the instruments

The Cronbach's Alpha statistics were used to calculate for the internal consistency for the questionnaire. This was done using the Statistical Package for Social Science (SPSS) version 22.

Dillman (2000) asserted that piloting a test of a research instrument helps to ensure the internal validity and reliability of the data. The questionnaire was piloted among eighty (80) JHS Mathematics teachers in the Berekum Municipality. Berekum Municipality was used for the pilot study because they are similar to the area of the study. The reliability coefficient of the questionnaire was calculated using Cronbach's Alpha and coefficient of 0.75 was recorded. Creswell (2014) opted that a Cronbach alpha coefficient of 0.70 is considered reliable and good indicative of internal consistency. The reliability coefficient guided the researcher to identify and correct some items that were wrongly formulated. Those that give some unintended results, as well as those, were similar in meaning to other items.

3.8.2 Validity of instruments

The validity of the instruments was established through the face and content validity. According to Awanta and Asiedu-Addo (2008) validity of an instrument is established when the items measure what they are intended to measure. The face validity was determined by the help of some Master of Philosophy past students of the Department of Basic Education and another College of Education Mathematics tutors. The instruments were given to lecturers for their comments and suggestions. The purpose of this was to assess each item's content, accuracy and format.

3.9 Data Collection Procedure

According to Creswell (2017), the site where research takes place and gaining permission before entering a site is very paramount in research. The researcher visited some schools with an official letter of introduction from the University of Education, Winneba, and seeking permission from the heads of schools to carry out the study. The researcher then sought permission from the school heads to organize the basic mathematics teachers for the study. The researcher familiarized himself with teachers and explained to them how the questionnaires should be responded to as well as how the interview will be conducted

3.9.1 Administration of the questionnaire

The questionnaire was administered to all JHS Mathematics teachers in the Berekum West District. The researcher used four (4) days to administer the questionnaire to all public JHS Mathematics teachers in Berekum West District. To ensure a high return rate, the researcher arranged with the teachers on the distribution and collection of the questionnaire. Eighty (80) questionnaires were administered and all were retrieved. The participants were assured that there was no wrong or correct answer to the items and their identity was highly confidential.

Also, they were free to make any corrections concerning their decisions they feel to be inappropriate. The arrangement was made on the date of completing the questionnaire so that the participants could respond to them at their own free and leisure time within the shortest possible time.

3.9.2 Conduction of interviews

The study further uses a semi-structured interview to collect qualitative data to help explain some of the items in the questionnaire to a greater depth. The researcher used a semi-structured interview guide to gain an in-depth understanding of the problem understudied. It was a one-on-one interview. Five (5) public JHS Mathematics teachers were used for the interview section. These comprise four male teachers and one female teacher. The interviewees were part of the sample that was used for the administering of the questionnaire. The proceeding of the interview was audio-taped and transcribed subsequently.

3.9.3 Conduction of classroom observation

The researcher visited five different schools and observed one lesson from those schools to gather observational data. The videotape recording was not only to augment the use of the observation schedule but also to provide information on other aspects of the lesson that were not captured in the schedule. A maximum of 70 minutes lesson was observed and the observation check list was ticked. Additional notes were taken during the lesson to take care of relevant issues not covered by the observation schedule, such as the topic, sitting arrangement other relevant materials and equipment (teaching/learning materials) used in the lesson.

Notes were also taken on the nature of classroom activities and the involvement of the students in the class activities. The videotape recording of each observed lesson was transcribed verbatim on the same day the lesson was observed. This was to ensure the credibility and accuracy of the transcription since some of the observed episodes and other relevant issues would be fresh on the researchers' mind.

3.10 Data Analysis Procedure

Questionnaire data were analysed using descriptive statistics (frequency, mean & standard deviation) and Qualitative data were analysed thematically. The quantitative data was first analysed and then followed by the qualitative data. Both analyses gave a clear picture of the problem understudied.

3.10.1 Quantitative data

The returns from the questionnaires were cleaned, coded and entered into SPSS version 22. SPSS version 22 was used for the analysis because it is user-friendly and popularly used for most of the quantitative data analysis. The researcher did most of the analysis to check for accuracy and accountability. Frequencies and percentages were employed in analysing the demographic data. Means and standard deviations were employed in analysing the research questions (1, 2 and 3).

3.10.2 Qualitative data

The primary method of analysis for the qualitative (interview guide) in this study was thematic analysis. Thematic analysis is a method of identifying, analysing and reporting themes or patterns within data set (Braun & Clarke, 2006). Reicher and Taylor (2005) noted that with qualitative analysis the researcher needs to be clear and explicit about what they are doing and what they say they are doing matches up with what they do.

Content analysis was also used to analyse qualitative data. The content analysis is a common approach to qualitative data analysis. This analytical tool is defined as a process of coding and identifying themes or patterns. There are three types of content analysis: conventional, summative and directed. In the conventional content analysis, the coding categories are derived from the data. In the summative content analysis, the process involves counting and comparisons. The directed approach, on the other hand, starts with a theory as a guide to the analytical process (Hsieh & Shannon, 2005).

A summative approach to content analysis was used to analyse the observed data in this study. Summative content analysis usually begins with identifying and quantifying certain words or content. The quantification is not initially an attempt to infer meaning but to explore teachers' instructional strategies of problem-solving. The summative process then goes on to where the codes are interpreted to discover underlying meaning. In this way, the counting process allows for interpretation of the associated context (Hsieh & Shannon, 2005).

3.11 Ethical Considerations

Kusi (2012) opines that in educational research, ethics are the issues that are related to how the researchers conduct themselves or their practices and the consequences of these on the participants in the research. Similarly, Cohen, Manion, and Morrison (2012) suggested two concerns to watch for in ethical considerations; first, how the research has been conducted concerning the research subject (matters such as informed consent, confidentiality, and persons involved). Secondly, acknowledgement of the contribution of all the people who have been involved in the research and as well as open recognition of individuals whose research influenced this present study.

Having discussed the methodological aspects of the research, the researcher contemplated on ethical issues of the study. This became evident since teachers' knowledge; instructional strategies, pupil's engagement and measure to be adopted to improve the teaching of problem-solving describe their moral, cultural, and social behaviours (Creswell 2017). Asking teachers to reveal their knowledge and practices to an unknown person, therefore, raises important ethical considerations. The ethical issues the researcher considered were: access, confidentiality, maintaining the anonymity of respondents, and data security.

3.11.1 Access

To try out a successful study in research, it is paramount to seek permission before entering in for data collection (Creswell, 2009). Upon this, the researcher obtained an official letter (see appendices) from the Department of Basic Education, outlining the need and purpose of the study and asking participants to give their maximum co-operation. The letter was first sent to the directorate of Berekum West, Ghana Education Service to gain access to the various schools and also as a form of documentation. The Education Director of Berekum West gave a permission letter for access and allowing the participants to participate in the study (see appendices). A copy of the directorate letter was then sent to the head teachers at JHS where the research was carried out.

3.11.2 Confidentiality

The participants were assured that all their information gathered would be treated as confidential data. Thus, the data was used for the stated purposes and no other person would have access to the gathered data. The participants were been informed that their names and other personal details of theirs would be omitted (Patton, 2002; Liamputtong, & Ezzy, 2005). The participants were assured that if the anonymity is being threatened, all other records would be destroyed. At the end of the process, all documents will be shredded and tapes will be erased (Patton, 2002; Liamputtong, & Ezzy, 2005).

3.11.3 Anonymity

One of the important ethical considerations the researcher considered was "maintaining the anonymity of respondents". Providing anonymity of information collected from research participants means that either the project does not collect identifying information of individual subjects (e.g., name, address, an Email address; etc.), or the project cannot link individual responses with participants identities. In this study, the researcher did not seek any information that was likely to reveal the identity of the respondents. This was done to protect the identity of research respondents. Personal anonymity may be central to gaining reliable information and that the issue of anonymity was dealt with when one respondent asked whether they had to give their names on the questionnaire.

3.11.4 Trustworthiness

The trustworthiness of this study is enhanced by including participants differing viewpoints, giving more credibility to the findings. The trustworthiness is again enhanced by the exact description of the procedure, by motivated participants and by the important quotations from the interviews (Hanson et al., 2015). Lincoln and Guba (2007) stated that the concept of trustworthiness is very important because it is necessary to estimate the accuracy of a qualitative study or a mixed-method study.

Also, by way of ensuring credibility the researcher followed this procedure:

1. The interviews were conducted using languages that were understood by both the researcher and participants to avoid misunderstanding between the researcher and the interviewees.
2. The observation took place in a quiet and serene environment void of distortions.
3. The supervisor for this study's regular inspections by giving constructive criticisms helped the researcher to check for flaws and problems in the study.

Participation of the participants in the interviews was strictly voluntary and their privacy and confidentiality were strongly maintained at all time

3.12 Chapter Summary

This chapter discussed the methodological procedure that was followed in the study. Issues relating to population, sampling procedure, instrumentation, data collection, and analysis as well as the ethical principles were discussed. Descriptive statistics, as well as content and thematic analysis help, answered the research questions. The next chapter presents the analysis of data collected and the discussion of findings.

CHAPTER FOUR

RESULTS AND DISCUSSION

4.0 Introduction

This chapter presents the results and the discussion of findings obtained from the analysis. The chapter commences with a discussion on the response rate and it is followed by the presentation of the background information of the respondents. Thereafter, the analysis of the research questions is done, and finally, the discussion of the results is presented.

4.1 Response Rate

A total of eighty (80) questionnaires were distributed to the respondents. However, seventy-five (75) questionnaires were retrieved from the respondents, representing a response rate of 93.75%. Five (5) questionnaires were not used in the analysis because they contained a lot of missing data that could distort the findings of the study. Nevertheless, the response rate obtained in this study was considered acceptable based on the suggestion of Saunders, Lewis and Thornhill (2009) that a response rate of 30% to 40% is adequate in surveys.

4.2 Demographic Characteristics of the Respondents

The demographic characteristics of the respondents were examined under the following: sex, age, academic qualification, years of teaching Mathematics and rank. As indicated in Table 3, more male teachers (n=61, 81.3%) than female teachers (n=14, 18.7%) were involved in the study. Concerning age, the findings showed that most of the teachers who participated in the study were 20-30 years (n=32, 42.7%) than those who fell between 31-40 years (n=31, 41.3%), 41-50 years (n=10, 13.3%) as well as those who were 51 years and above (n=2, 2.7%). The distribution of the respondents on academic qualification revealed that one respondent, representing 1.3% had teachers' certificate "A", 42 respondents representing 56.0% were diploma/HND holders, 30 respondents representing 40.0% had bachelor's/post-diploma degree and the remaining two respondents, representing 2.7% had master's degree. The composition of the respondents by work experience showed that most of the respondents had 1-5 years of work experience (n=37, 49.3%) as compared to those with 11-15 years (n=15, 20.0%), 6-10 years (n=14, 18.7%), 16-20 years (n=6, 8.0%) and 21 years and above (n=3, 4.0%).

Table 3: Demographic Characteristics of the Respondents

		Frequency	Percent
Sex	Male	61	81.3
	Female	14	18.7
	Total	**75**	**100.0**
Age	20-30	32	42.7
	31-40	31	41.3
	41-50	10	13.3
	51 and above	2	2.7
	Total	**75**	**100.0**
Academic Qualification	Cert A	1	1.3
	Diploma/HND	42	56.0
	Bachelor's Degree/Post Dip	30	40.0
	Master's Degree (MPhil., Med, MA)	2	2.7
	Total	**75**	**100.0**
Years of teaching Mathematics	1-5 year(s)	37	49.3
	6-10 years	14	18.7
	11-15 years	15	20.0
	16-20 years	6	8.0
	21 years and above	3	4.0
	Total	**75**	**100.0**
Teachers' Rank	Superintendent	4	5.3
	Senior Superintendent	34	45.3
	Principal Superintendent	19	25.3
	Assistant Director and above	18	24.1
	Total	**75**	**100.0**

Source: Fieldwork Data, 2020

In terms of their rank, the information showed that most of the teachers were at the Senior Superintendent rank (n=34, 45.3%), 19 teachers, representing 25.3% were at the rank of Principal Superintendent, 18 teachers, representing 24.1% were at the Assistant Director and above ranks, while the remaining four teachers, representing 5.3% were at Superintendent rank.

4.3 Presentation of Findings

This section of the study presents the findings of the study and they are presented based on the research questions.

Research Question 1: What is Mathematics teachers' perceived knowledge for teaching problem-solving in public junior High Schools in Berekum West District?

Research Question 1 investigated JHS Mathematics teachers level of knowledge of Problem-solving. In view of the 4-point Likert scale used, which makes the average/fair score to be 2.5 ([1+2+3+4] ÷4), in this study, the determination of the level of JHS Mathematics teachers knowledge on problem-solving was done using mean and standard deviation such as that mean <2.50 indicated low perceived knowledge, 2.50 ≤ mean < 3.50 indicated fair perceived knowledge, and mean ≥ 3.50 indicated good perceived knowledge. The general level of perceived knowledge of Mathematics teachers on problem-solving is shown in Table 4.

Table 4: Descriptive Statistics on Mathematics Teachers' Perceived Knowledge on Problem-Solving

Statements on Perceived Knowledge	Min.	Max.	Mean	Std. Dev.	Perceived Knowledge
1. Mathematics problems should task pupils to reason logically and critically	3	4	3.68	0.47	Good Knowledge
2. Mathematics problems should have connection with pupils' real-life situation	2	4	3.61	0.54	Good Knowledge
3. Mathematics problem should challenge pupils to apply their daily skills in solving it	2	4	3.60	0.52	Good Knowledge
4. Mathematics lessons should guide pupils to self-develop strategies in solving problems	3	4	3.53	0.50	Good Knowledge
5. Problem-solving involves tasks that challenge pupils' ability	2	4	3.39	0.59	Fair Knowledge
6. Mathematics problem should require pupils to conjecture their strategies in solving it	1	4	3.20	0.79	Fair Knowledge
Overall Perceived Knowledge			**3.50**	**0.57**	**Good Knowledge**

Source: Fieldwork Data, 2020

Review of Table 4 has revealed that Mathematics teachers perceived the problem-solving to involve tasking learners to reason logically and critically (M=3.68, SD=0.47) than problems having a connection with learners' real-life situation (M=3.61, SD=0.54), Mathematics problems that challenge the learner to apply their daily skills in solving it (M=3.60, SD=0.52). Again, they perceived problem-solving to be Mathematics lessons that should guide learners to develop strategies in solving problems (M=3.53, SD=0.50), than Mathematical problems involving tasks that challenge the learners' ability (M=3.39, SD=0.59), and Mathematical problems that require learners to conjecture their strategies in solving it (M=3.20, SD=0.79) components. Overall, Mathematics teachers' perception of the general knowledge yielded a mean of 3.50 (SD=0.57). However, the results indicated that Mathematics teachers' perceived all the components to be an embodiment of problem-solving and that it could be said that Mathematics teachers had good perceived knowledge in problem-solving.

Research Question 2: Which Instructional Strategies do Mathematics teachers' employ for teaching problem-solving in public Junior High Schools?

The second research question dwelled on the instructional strategies that were employed by Mathematics teachers in teaching problem-solving. In ascertaining the extent of usage of the problem-solving instructional techniques, mean and standard deviation were used such that mean <2.50 indicated rarely used, $2.50 \leq$ mean < 3.50 indicated moderately used, and mean ≥ 3.50 indicated frequently used. The results on the instructional techniques and the general level of usage are presented in Table 5.

Table 5: Descriptive Statistics on Instructional Strategies of Mathematics Teachers Employ in Teaching Problem-Solving

Problem-solving Instructional Techniques	Min.	Max.	Mean	Std. Dev.	Extent of Usage
1. Task base Instruction	2	4	3.53	0.55	Frequently used
2. Cooperative Learning Technique	1	4	3.52	0.68	Frequently used
3. Project Work Technique	2	4	3.43	0.70	Moderately used
4. Guided Discovery Technique	1	4	3.31	0.94	Moderately used
5. Group Work Technique	1	4	3.31	0.70	Moderately used
6. Assignment Technique	2	4	3.29	0.59	Moderately used
7. Self-Instruction Technique	1	4	3.16	0.75	Moderately used
8. Inquiry Learning Technique	1	4	3.07	0.98	Moderately used
9. Computer Assisted Learning Technique	1	4	2.47	0.91	Rarely used
Overall Problem-Solving Instructions			**3.23**	**0.75**	**Moderately Used**

Source: Fieldwork Data, 2020

The information in Table 5 disclosed that there were several instructional techniques that Mathematics teachers employ in teaching Problem-Solving in Mathematics instruction. Particularly, the findings showed that task base instruction (M=3.53, SD=0.55) and cooperative learning technique (M=3.52, SD=0.68) which were indicated to be frequently used as compared to project work (M=3.43, SD=0.67), guided discovery technique (M=3.31, SD=0.94), group work (M=3.31, SD=0.70), assignment technique (M=3.29, SD=0.59), self-instruction (M=3.16, SD=0.75), and inquiry learning technique (M=3.16, SD=0.75) were all moderately used, while computer-assisted learning technique (M=2.47, SD=0.91) was rarely used. Generally, the findings of the study revealed that Mathematics teachers moderately used all the problem-solving instructional techniques outlined in the study.

Research Question 3: How do public junior high school Mathematics teachers engage pupils in problem-solving?

Research Question 3 sought to investigate the perception of Mathematics teachers on the extent they engage the learners in their instructional process in relation to teacher serving as a facilitator, use of collaborative learning, use of manipulative materials, active engagement of learners, knowledge application, and building lessons on relevant previous knowledge. In this study, mean and standard deviation were calculated to determine the perceived extent of the engagement of learners in problem-solving in Mathematic instruction such that mean < 2.50 indicated rarely engage, $2.50 \leq$ mean < 3.50 indicated occasionally engage, and mean ≥ 3.50 indicated always engage. The general perception of Mathematics teachers on the extent of engagement in problem-solving in their instructional process is presented in Table 6.

Table 6: Descriptive Statistics on Pupils Engagement in Teaching Problem-Solving

Kinds of Engagements	Min.	Max.	Mean	Std. Dev.	Levels of Engagement
Use of Manipulates	2	4	3.72	0.51	Always Engage
Active Engagement of Learners	1	4	3.45	0.79	Occasionally Engage
Teacher as a Facilitator	1	4	3.19	0.82	Occasionally Engage
Building Lessons on Learners RPK	1	4	3.15	0.75	Occasionally Engage
Motivation	1	4	3.15	0.83	Occasionally Engage
Application of Knowledge	1	4	2.92	0.82	Occasionally Engage
Assessment	1	4	2.60	1.00	Occasionally Engage
Curiosity	1	4	2.60	0.90	Occasionally Engage
Perseverance	1	4	2.19	0.85	Rarely Engage
Total Level of Engagements			**3.00**	**0.80**	**Occasionally Engage**

Source: Fieldwork Data, 2020

It is disclosed from the findings in Table 6 that there were many levels of engagement of learners in the problem-solving instructional process. Indeed, the findings revealed that the Mathematics teachers' indicated to always engage learners with the use of manipulative (M=3.72, SD=0.51), whereas active engagement of learners (M=3.45, SD=0.79), teacher as a facilitator (M=3.19, SD=0.82), building lessons on learners RPK (M=3.15, SD=0.75), motivation (M=3.15, SD=0.83), application of knowledge (M=2.92, SD=0.82), assessment (M=2.60, SD=1.00), and curiosity (M=2.60, SD=0.90) were occasionally used while perseverance (M=2.19, SD=0.85), was rarely used. Generally, it was concluded that Mathematics teachers occasionally (M=3.00, SD=0.80) engage learners in all the kinds of engagement outlined in the study by way of problem-solving instructional technique.

4.4 Qualitative Results from the Interview Guide

Five teachers were interviewed to capture all the research questions. The five teachers were selected from the seventy-five (75) teachers who responded to the questionnaire. The abbreviation TR followed by a number in the write connotes a teacher. As a follow up on Research Question one ***(what is the JHS Mathematics teachers' perceived knowledge of problem-solving?)***, all the five teaches were interviewed on the following questions:

Question: In your own opinion, what do you think problem-solving is All about?

This question was meant to solicit the teacher teachers' view on their understanding of what they know about problem-solving. From the responses, it was clear that almost all the teachers have a fair perceived knowledge about problem-solving. The following are some of the responses some of them gave in response to the question.

> *"Ah, problem-solving is a way of helping pupils to solve the problem that they encounter in their daily life"* (TR 1, Interviewed data, 2020).

> *"Ah Mathematics problem-solving has to do with the efforts that one makes to find a solution to a problem that probably you might ahh ahh have not met before"* (TR 2, Interviewed data, 2020).

> *"Hmmmmm…Problem-solving is all about giving Mathematics problem to children to think and reason very well before getting an answer" (TR 3, Interviewed data, 2020).*

> *"From the experience I have, problem-solving has to do with word problem where children are required to think and sometimes work in a group to provide a solution to an unsolved problem"* (TR 4, Interviewed data, 2020).

> *"Problem-solving is a process of finding a solution to a given problem"* (TR5, Interviewed data, 2020).

The next question the researcher asked was to find out when and how the teachers' came to realise problem-solving as a necessary focus in teaching Mathematics. The responses from the teachers' indicated that most of them heard of problem-solving at their training colleges and some workshops they attended. However, none of them had attended any workshop on problem-solving for the past few years. This also means that teachers have not been well introduced to problem-solving. The following are some of the views expressed by the teachers:

> *"I got to know problem-solving as a focus in teaching Mathematics through some books I have read. I got to know that problem-solving is one of the best approaches to evaluating students' application of Mathematics. So as a teacher I try to relate my lessons to students' daily life"* (TR 1 Interviewed data, 2020).

> *"I got to know problem-solving as being one of the ways of testing children understanding. I learnt that way back at college about 10 years ago and I sometimes use it in my lessons"* (TR 2 Interviewed data, 2020).

> *"I recognized problem-solving as a focus on teaching Mathematics right from training college. We were taught and when I became a teacher I sometimes try to also use problem-solving in my lessons*" (TR 3 Interviewed data, 2020).

> *"I heard of problem-solving from a workshop I attended some years ago. I got to know that problem-solving challenges children thinking so when I want to children to think deeply, I use problem-solving"* (TR 4 Interviewed data, 2020).

> *"I heard problem-solving from college and also from a workshop. It gives an in-depth understanding of both conceptual and procedural knowledge to pupils"* (TR5 Interview data, 2020).

From these excerpts, the researcher deduced that most of the JHS Mathematics teachers have heard about problem-solving as in brief and they have shallow perceived knowledge about what it is all about. For example, the definitions they gave were narrow.

Concerning Research Question two (**To what extents do public JHS Mathematics teachers apply problem-solving instructional strategies in their teaching?)**, the interview was to solicit teachers' views on the extent to which the teachers apply problem-solving instructional strategies in their teaching. The following questions were asked:

Do you use instructional strategies for teaching problem-solving?

> *"I love to give students group work assignments and also I sometimes encourage students to guess their answers before solving it"* (TR 1 Interviewed data, 2020).

> *"I discuss the problem with them using drawings and diagrams to illustrate or charts and as well as questions to model the problem. Then I lead students to solve the problem by asking leading questions. Finally, I help them to work backwards to be sure that the answer is correct"* (TR 2 Interviewed data, 2020).

> *"I have various ways of adopting problem-solving. For example, I use normal exercise being in the form of puzzles so that students can reason more and write few"* (TR 3 Interviewed data, 2020).

> *"I fact, I use a variety of strategies some of which are making a list, making a chart or a table, drawing a diagram, making a model, simplifying the problem, looking for a pattern working backwards. Sometimes, I even use a formula or equation or act out the problem situation, using guess and check"* (TR 4 Interviewed data, 2020).

> *"Through shared responsibilities between teacher and student (democratic classroom). Ah, for example, peer teaching, group discussions, group presentations and so on"* (TR5 Interview data, 2020.

The next question was asked to find out from the teachers some of the challenges of teaching Mathematics through problem-solving. The following excerpts are some of the views expressed by the respondents:

> *"Students prefer to be told Mathematics rather than to be guided by the teacher to explore and construct their understanding"* (TR 1 Interviewed data, 2020).

> *"Teaching through problem-solving requires a lot of time and if time is not sufficient, it is better to teach Mathematics by telling"* (TR 2 Interviewed data, 2020).

> *"Teachers have inadequate subject matter knowledge, pedagogical content knowledge and personal problems*" (TR 3 Interviewed data, 2020).

> *"Some teachers lack the requisite knowledge, skills and expertise for teaching Mathematics through problem-solving"* (TR 4 Interviewed data, 2020).

> *"Ah identifying the correct procedure in solving a specific problem is a challenge of teaching Mathematics through problem-solving" (*TR5 Interviewed data, 2020).

These excerpts conclude that most of the teachers do not incorporate problem-solving in their instructional lessons. Their responses show that they do not know the principles and guidelines of problem-solving. None of them was able to give a guiding principle of problem-solving as well as describing how she/he incorporates it in her/his lesson. The next research question also sought to find out how pupils were engaged in problem-solving. Some of the questions the researcher asked are as follows:

Question: Do you incorporate problem-solving into your Mathematics lessons?

> *"Well in my lessons, I provide the opportunity for students to reason logically and criticised their answer and I do this most of the time"* (TR 1 Interviewed data, 2020).

> *"Not often as and when what I am teaching calls for it. For example, I write Mathematics word problem form for the children to provide the solution"* (TR 2 Interviewed data, 2020).

> *"I like giving children projects as well as homework where they can get help from the house"* (TR 3 Interviewed data, 2020).

> *"By converting word problem into mathematical expressions, equations and so on"* (TR4 Interview data, 2020).

The next question was asked to find out from teachers how they engage pupils in Mathematics problem-solving. The following excerpts are some of the views expressed by the respondents:

> *"I give them questions I believe they can do to and they also perform"* (TR 1, Interviewed data, 2020).

> *"I engage pupils in problem-solving by putting them in groups so that they can work together like share ideas and things to solve the problem"* (TR 2, Interviewed data, 2020).

> *"You know children dislike Mathematics and at times difficult questions might scare them from even coming to school and you know our system too…., So I give them questions I believe they can do it and perform"* (TR 3, Interviewed data, 2020).

> *"Pupils in Mathematics problem-solving, hmm…. well, I give them questions and I encourage them to follow the procedures I used to solve it. Even if it is difficult for them, I encourage them to reason well and at times some students will get the answer"* (TR 4, Interviewed data, 2020).

> *"By allowing pupils to attempt problem-solving questions on their own"* (TR 5, Interviewed data, 2020).

Pupils' engagement from research can be grouped into teacher serving as a facilitator, use of collaborative learning, use of manipulative materials, active engagement of learners, knowledge application, and building lessons on relevant previous knowledge. The excerpts revealed that the teachers' attention was on the use of manipulatives materials than the others. This further concludes from the quantitative data that only manipulatives engagement was effectively making the rest ineffective.

The final research question also sought to find out measures to improve the teaching of problem-solving? Some of the questions the researcher asked are as follows:

Do you think there are advantages of teaching Mathematics throng problem-solving?

> *"Ah a lot of benefits, they can be independent when they face a real-life problem because already, they have been given the skills to manage the difficult problem so it helps the students a lot. If you teach problem-solving, it encourages the students to solve problems in real-life situations"* (TR 1, Interviewed data, 2020).

> *"I think that using a problem-solving approach in teaching Mathematics is interesting and enjoyable because students understanding the lessons better. After going through a lesson, students can be given a problem to help them revise what has been taught"* (TR 2, Interviewed data, 2020).

> *"If a student is confronted with an external problem at home, he/she uses the problem-solving techniques taught in school to address that situation"* (TR 3, Interviewed data, 2020).

> *"I must say students gain a lot of understanding when they are taught through a problem-solving approach. As students solve the problem in one topic, other topics came in, therefore problem-solving helps students to practice other topics. When it comes to exams, for example, the experience they gain through the approach helps them to understand the questions. They are also able to answer the questions accordingly. So, the benefits are……that they learn a lot, they acquire knowledge"* (TR 4, Interviewed data, 2020).

> *"It gives practical understanding to pupils as they solve a problem related to their everyday life"* (TR 5, Interviewed data, 2020).

The next question was asked to find out from teachers the disadvantages of teaching Mathematics through problem-solving. The following excerpts are some of the views expressed by the respondents:

> *"If you look at the way examination are set, students are likely not able to answer most of the examination questions well when they are taught through problem-solving. Problem-solving does not meet the demands of examination and students always learn to have at the back of their minds passing examinations"* (TR 1, Interviewed data, 2020).

> *"Hmm…teachers are not comfortable because he/she does not know much about problem-solving so he/she has little knowledge for the subject to teach"* (TR 2, Interviewed data, 2020).

> *"Ah…the large number of students in the class also makes it difficult to use a problem-solving approach in teaching because it is not easy going round all the students individually explaining to them or making sure they follow the correct path in solving a problem"* (TR 3, Interviewed data, 2020).

> *"Problem-solving is very time-consuming"* (TR 4, Interviewed data, 2020).

> *"Ah…. if the right technique is not used, it tends to confuse pupils"* (TR5, Interviewed data, 2020).

The final question was asked to find out from teachers in their opinion, how can teachers be encouraged to teach Mathematics through problem-solving. The following excerpts are some of the views expressed by the respondents:

> *"Organising in-service training for teachers to acquire knowledge in problem-solving will encourage teachers to teach Mathematics through problem-solving"* (TR 1, Interviewed data, 2020).
>
> *"Most teachers need to be trained to know how to use problem-solving because is like we know of problem-solving only when it comes to a word problem question. If teachers are trained to know how to use it, I think they will be encouraged to use it"* (TR 2, Interviewed data, 2020).
>
> *"I think if textbooks provide a lot of problem-solving questions it can help"* (TR 3, Interviewed, data, 2020).
>
> *"Ghana education service should organise in-service training courses for teachers especially on Mathematics problem-solving because we were brought out by teachers who did not have interest in Mathematics so they made Mathematics to look like it was a very difficult subject"* (TR 4, Interviewed, data, 2020).
>
> *"By organising seminars and workshops for teachers on problem-solving techniques"* (TR 5, Interviewed data, 2020).

From the above responses, organizing in-service training courses for teachers on problem-solving to develop teachers' perceived knowledge, providing incentives packages for teachers and providing teaching and learning materials can be encouraged Mathematics teachers to teach Mathematics through problem-solving in the Mathematics classroom.

4.5 Qualitative Results from the Observation Checklist

Junior High School Mathematics Teacher 1 (Female)

TR 1 had a Bachelor's degree in Mathematics and a Diploma in Education holder and had taught for eleven years. He is between 41-50 years of age. She is the Mathematics teacher for JHS 1, JHS 2 and JHS 3, with a total of 105 pupils in all the three classes. During the observation, she taught JHS 3 class. The classroom was to some extent spacious and had a dual-type desk.

The classroom arrangement inhibited small group formation but promoted pupils working in pairs. The class was sparsely equipped with teaching and learning resources as at the time of the research. There were no charts on the classroom walls and TR 1 mostly relied on the chalkboard to communicate and demonstrate a concept. During the lesson, her Pupils' were mostly engaged as a whole group in the same activities at the same time. The pupils were also mostly listening to a presentation from TR 1, which included extensive procedural instruction, demonstration and lecture. The process was however interspersed with some informal contributions from the pupils like answering oral questions. The lesson that was observed was on the topic of "shape and space".

Junior High School Mathematics Teacher 2 (Male)

TR 2 holds Diploma and B. Ed in Basic Education from the University of Education, Winneba. He falls within the 31–40-year group and has been teaching Mathematics at the Junior High level for nine years. He handled only JHS 2 and JHS 3 and is the form master for JHS 3. TR 2 has participated in several in-service training programmes that were organised by both his old and new district. On the day of the observation, TR 2 taught JHS 2 class. The classroom was spacious enough to accommodate the 33 pupils (19 boy and 14 girls) who were present that day. The pupils were seated in pairs on dual-type dusk in a traditional classroom seating arrangement.

The seating arrangement inhibited the quick formation of a small group for discussion and, therefore, the class activities were structured as a whole group, pair as well as the individual. The entire pupils were also engaged in the same activity at the same time. TR 2 mostly structured his lessons such that he was the main presenter while his pupils listened. The lesson observed was on Measurement of Area (Area of a Rectangle).

Junior High School Mathematics Teacher 3 (Male)

TR 3 was a Diploma in Basic Education holder. He had been in the teaching service for about 5 years and has participated in a few in-service training programmes. He was within the 20–30-year age range and was the only teacher who teaches Mathematics in the school. As at the time of the observation, he taught JHS 1 class which is made up of 42 (22 boys and 20 girls). TR 3 on the day of the observation was teaching Statistics and Probability (Sub-topic: Frequency). The classroom had no enough space as it was overcrowded. This inhibited the free flow of engagement between the teacher and the class and among the pupils.

As a result, TR 3 mostly structured her classroom activities to include the whole class and individuals. The pupils were mostly engaged in the same activity at the same time. Her lesson delivery mode was mainly lecture and demonstration. The classroom wall had a chart showing some basic geometric shapes on the wall.

Junior High School Mathematics Teacher 4 (Male)

TR 4 also holds B. Ed. Mathematics Education certificate from the University of Cape Coast. He is a graduate teacher and has been teaching for about twelve (12) years at the JHS. He is a JHS 2 form master. On the day of Observation, he taught the area of a circle. The class enrolment was 26 (15 boys and 11 girls). The classroom was spacious enough to contain all the pupils who were present that day. There were both dual and monotype desks as well as charts.

The class size and the space in the room could easily facilitate interactions among pupils. Major ways by which pupils' activities were structured was through the whole class engagement and the entire class was engaged in the same activity at the same time. The major role which the pupils played during the lesson was mostly giving chorus answers, clapping and copying examples from the chalkboard.

Junior High School Mathematics Teacher 5 (Male)

TR 5 had a Master's degree in Mathematics and a Diploma in Education holder and had taught for fifteen years. He is between 41-50 years of age. He is the Mathematics teacher for JHS 1, JHS 2 and JHS 3, with a total of 115 pupils in all the three classes. During the observation, he taught JHS 2 class. The classroom was not spacious and had a dual-type desk. The classroom arrangement inhibited small group formation but promoted pupils working in pairs. The class was sparsely equipped with teaching and learning resources as at the time of the research. There were charts on the classroom walls but TR 5 mostly relied on the chalkboard to communicate and demonstrate a concept. During the lesson, his pupils' were mostly engaged as a whole group in the same activities at the same time. The pupils were also mostly listening to a presentation from TR 5, which included extensive procedural instruction, demonstration and lecture. The lesson that was observed was on the topic of "Linear Equation".

Table 7 shows teachers scored after the lesson delivery.

Table 7: Problem-Solving Instructional Strategies by JHS Mathematics Teachers

SN	Instructional Practices	TR1	TR2	TR3	TR4	TR5
1	Teacher task pupils on problems that are within their ability and explain the problem for pupils understanding	2	2	3	2	2
2	Teacher pose open-ended problems and provide adequate time for pupils to solve	1	2	1	1	1
3	The teacher promptly reacts to pupils' responses and allow them to explain their procedures	1	1	1	1	1
4	Teacher frame problem so that the pupils can fully understand it	1	1	1	2	1
5	Teacher allow pupils to confer, consult and cooperate with friends when solving Mathematics problems	1	2	2	1	2
6	Teacher prefer giving routine procedures and much attention to pupils as an easy way of passing a test	3	3	3	3	3
7	Teacher easily taught new concept which is familiar to pupils and give up on problems which the pupils cannot solve immediately	2	3	3	2	3
8	Teacher allow pupils to device various ways of getting a solution and persistently make sure their answers are correct	1	1	1	1	1
9	Teachers devote much time for pupils to solve a Mathematics problem and gets excited when solving challenging problems	3	3	3	2	3
10	Teacher provides concrete material for students	3	3	3	2	3
11	Teacher allows students to work in groups	1	1	1	2	1
12	The teacher serves as a facilitator, a guide by allowing students to construct their knowledge during problem-solving lessons.	1	1	1	1	1
13	Teacher encourage students to pose their problem	1	2	2	1	2
14	Teacher makes pupils happy when they can solve difficult Mathematics problems	3	3	3	3	3

The table presents the overall ratings of all the participants during the lesson observation. The rating ranged from 1 (no evidence) to 3 (clear evidence). From the table, all the teachers were showed some evidence of Statement 1, except TR 3, who showed clear evidence 3 on Statement one (teacher tasks pupils on problems that are within their ability and explain the problem for pupils' understanding). The remaining three teachers showed some evidence of that statement. This means that only TR 3 has an in-depth knowledge of problem-solving. To Statement two, expert TR 2 who was rated 2, the remaining had 1. This indicates that TR 2 posed open-ended questions and also provided enough time for pupils to solve. All the teachers were rated 1 on Statement three (teacher promptly react to pupils responses and allow them to explain their procedures). All the teachers showed no evidence of statement 4, except TR4, who showed some evidence 2 on Statement four (teacher frame problem so that the pupils can fully understand it).

Three teachers showed some evidence 2 on statement 5 (teacher allow pupils to confer, consult and cooperate with friends when solving Mathematics problems) whilst two teachers showed no evidence 1 on Statement five. Of all the five lessons observed, the teachers showed clear evidence on Statement six (Teacher prefer giving routine procedures and much attention to pupils as an easy way of passing a test). For example, the pupils prefer giving them solved examples than unsolved examples. On Statement seven, teacher one and two showed some evidence 2 whilst the rest showed clear evidence 3. On Statement eight, all the teachers observed did not show any evidence on Statement eight (Teacher allow pupils to device various ways of getting a solution and persistently make sure their answers are correct). This is because all the pupils were using the teachers' procedures and they did not show any sign of new development in their solution. Also, on Statements nine and ten (10) (Teachers devote much time for pupils to solve a Mathematics problem and gets excited when solving challenging problems and Teacher makes pupils happy when they can solve difficult Mathematics problems), only TR 4 showed some evidence while the rest showed clear evidence on statements nine and ten (10). Statement nine and ten (10) were also based on affective engagement.

It can be deduced from the foregoing results that all the teachers except TR 4 paid much attention to manipulatives engagement of the pupils based on the instructions. On Statement 11 only teacher TR 4 showed some evidence 2 whilst the rest showed no evidence 1 (teacher allows students to work in groups). Of all the five lessons observed, the teachers showed no evidence on Statement 12 (the teacher serves as a facilitator, a guide by allowing students to construct their knowledge during problem-solving lessons. two teachers showed no evidence 1 whilst the rest showed some evidence 2 on Statement 13. All the teachers showed clear evidence 3 on Statement 14 (teacher makes pupils happy when they can solve difficult Mathematics problems).

4.6 Discussion of Findings

The purpose of this study was to explore public JHS Mathematics teachers' perceived knowledge of problem-solving, their practices of problem-solving instructional strategies, how pupils are engaged in problem-solving and measure to be adopted to improve the teaching of problem-solving. It is a belief that teachers' perceived knowledge of problem-solving may influence their practices as well as engaging pupils in problem-solving. The results of the quantitative data have been presented followed by the results of the qualitative.

Research Question 1

What is Mathematics teachers' perceived knowledge for teaching problem-solving in public junior High Schools in Berekum West District?

The questionnaire solicited views from the teachers to measure their perceived knowledge of problem-solving. The results from the questionnaire revealed that Berekum West JHS Mathematics teachers have good knowledge (M =3.50, SD =0.57) of problem-solving. Earlier studies conducted by Xenofontos & Andrews (2014), Mereku (2015), McIntosh, *et al* (2000), attested that teacher lack expert knowledge in the area of problem-solving therefore, they do not teach problem-solving in Mathematics. However, the current findings of this study are contrary to the conclusion drawn by Xenofontos & Andrews (2014), Mereku (2015), McIntosh, *et al* (2000).

The result of this current study supports the finding of Van de Walle (2007). Thus, Mathematics tasks should promote learner conceptual understanding, foster their ability to reason and criticize Mathematics problems.

Also, Stacy stated that successful mathematical problem-solving depends upon deep mathematical knowledge and personal attributes such as persistence, organisation and confidence (as cited in Van de Walle, 2007). These attributes of the teacher guide him or her to give tasks that challenges pupils' intellectual ability and reasons. This was evident during the interview and observation sections. Most of the respondents thought of problem-solving as involving pupils in tasks that challenge pupils to reason and critically work out for solutions. The observation guide also proved that the teachers' focus was giving Mathematics problem that allows pupils to reason and think critically.

Research Question 2

The second research question dwelled on the instructional strategies that were employed by Berekum West Mathematics teachers in teaching problem-solving. In this study, teachers' applications of problem-solving instructional strategies were assessed in nine main constructs namely, Task base Instruction, Cooperative Learning Technique, Project Work Technique, Guided Discovery Technique, Group Work Technique, Assignment Technique, Self-Instruction Technique, Inquiry Learning Technique and Computer Assisted Learning Technique. The result from the quantitative data revealed that JHS Mathematics teachers of Berekum West District "Moderately Used" problem-solving instructional strategies in their teaching (see Table 5).

The findings of the study confirm Matlala (2015) suggestion that Mathematics teachers find it difficult to teach through a problem-solving approach. Matlala further explained that the teachers still teach using the traditional approach namely: telling and showing as well as stepping in to show learners how to solve a mathematical problem.

Besides, Buschman (2004) opted that teaching through problem-solving is a challenge to many teachers. One of the challenges that were acknowledged by Artzt et al. (2008) was teacher educators' inability to prepare students to teach in a manner consistent with new ideas about learning and the nature of Mathematics. The qualitative data from the interview and observation also proved that public JHS Mathematics teachers in the Berekum West District "Moderately Used" problem-solving instructional strategies in their teaching. The interviewees were unable to describe how they use problem-solving instructional strategies in their teaching. They suggested that most of their classroom instructions were not always problem-based.

Also, the lessons the researcher observed revealed that the teachers moderately centred their instructions on problem-solving.

Research Question 3

Research question three sought to investigate how the public JHS Mathematics teachers engage pupils in problem-solving in relation to teacher serving as a facilitator, use of collaborative learning, use of manipulative materials, active engagement of learners, knowledge application, and building lessons on relevant previous knowledge.

The result from the Quantitative data revealed that JHS Mathematics teachers occasionally engage pupils in problem-solving (see Table 6). The result further indicated that among the engagement, only manipulatives engagement had a higher mean score of 3.72. This confirms Florence (2012) first conclusion that Mathematics manipulatives can help engage students for a longer period by helping them stay focused on particular tasks. Florence believes that lecture-based teaching can often seem boring but that manipulatives allow students to be actively involved in learning.

Generally, it was concluded that Mathematics teachers occasionally engage learners in all the kinds of engagement outlined in the study by way of problem-solving instructional technique.

Evident from the interview as well as the observational guide also confirmed that the teachers were focused on the use of manipulative.

Research Question 4

The final research question was also to find out measures to be adopted to improve the teaching of problem-solving in public Junior High Schools in Berekum West District? The interview solicited views from the teachers on how to improve the teaching of problem-solving. Most of the respondents thought of organising professional development courses for teachers are essential for effective teaching and learning of Mathematics. As such, the teachers in this study suggested that organizing in-service to sensitise and train Mathematics teachers on problem-solving will equip them with content knowledge, pedagogical content knowledge and knowledge of the curricular materials (Shulman, 1986) to enable them to teach Mathematics through problem-solving.

The study also confirms an earlier study by Ali, *et al*. (2010) recommended organizing an extensive training programme; seminars and workshops for Mathematics teachers in elementary schools to enable them to employ a problem-solving method in the classroom. They further suggested organizing training sessions for untrained teachers to be trained in problem-solving.

Again, the teachers also suggested that placing sufficient emphasis on problem-solving in the public examination will encourage teachers to teach Mathematics through problem-solving. Parents and students' expectations in competitive public examinations most often dictate to teachers the approach to use in teaching.

Consequently, Mathematics teachers teach for examinations instead of conceptual understanding. As stated by Anderson (2009), providing valuable resources and more time are important steps, in promoting the teaching of problem-solving, it is also possible that problem-solving in Mathematics curriculum will only become valued when it is included in high-stakes assessment. An equal important suggestion given by the teachers in this study was the school timetable slots for Mathematics lessons need to be increased so that teachers can allow students to explore, discuss and to construct their meaning and understanding rather than telling them Mathematics.

4.7 Chapter Summary

This chapter discussed the results and the discussion of findings obtained from the analysis. The chapter presented discussion on the response rate, followed by the presentation of the background information of the respondents, the analysis of the research questions and finally, the discussion of the findings.

CHAPTER FIVE

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

5.0 Overview

This chapter presents the summary of the findings; conclusions drawn as well as recommendations based on the findings of this study

5.1 Summary of the Findings

The purpose of the study was to explore public Junior High School Mathematics teachers' perceived knowledge of problem-solving, their problem-solving instructional strategies, how they engage pupils in problem-solving and measures to improve the teaching of problem-solving in Berekum West District of Bono Region of Ghana. The study adopted Polya problem-solving strategies. The target population for the study was all public JHS Mathematics teachers in the Bono Region of Ghana. The accessible population consisted of all the eighty (80) JHS Mathematics teachers in the Berekum West District. The study adopted three research instruments, namely: questionnaire (structured), interview (semi-structured) and observational Guide. The data collected (quantitative) were analysed using central tendency measures. The qualitative data from the interview were transcribed and analysed whilst the lessons observed were recorded and later described. The study was guided by the following research questions:

The study was guided by the following research questions:

1. What is Mathematics teachers' perceived knowledge for teaching problem-solving in public junior High Schools in Berekum West District?
2. To what extent do Mathematics teachers' use problem-solving instructional strategies in their teaching in public Junior High Schools in Berekum West District?
3. How do Mathematics teachers' engage pupils' in problem-solving in public Junior High Schools in Berekum West District?
4. What measures can be adopted to improve the teaching of problem-solving in public Junior High Schools in Berekum West District?

5.2 Major Findings of the Study

1. The study revealed that, though public JHS Mathematics teachers have heard of problem-solving in their Colleges of Education or some workshop they had attended, they had a good perceived knowledge of problem-solving.
2. Again, public JHS Mathematics teachers moderately used problem-solving instructional strategies in their teaching. It was deduced from the data that though their practices are influenced predominantly by child-centred method, it was not focused on problem-based approach.
3. Even though public JHS Mathematics teachers have heard about problem-solving, the study revealed that they occasionally engage pupils'. The use of manipulatives materials had higher concentration. This gives the impression that the teachers were interested in manipulatives engagement than the other materials.

4. Organising an extensive training programme; seminars and workshops for public JHS Mathematics teachers will improve the teaching of problem-solving.

5.3 Conclusions

The purpose of this study was to explore public JHS Mathematics teachers' perceived knowledge of problem-solving, their problem-solving instructional strategies, how they engage pupils in problem-solving and measures to be adopted to improve the teaching of problem-solving in Berekum West District. This study brought to light some level of perceived knowledge of public JHS Mathematics teachers in the District have on promoting learning in the 21st century. The findings revealed that JHS Mathematics teachers have a good perceived knowledge of problem-solving.

The study further revealed that with this level of JHS Mathematics teachers' perceived knowledge, they moderately used problem-solving instructional strategies in their teaching. Evident from the qualitative data (semi-structured interview and observation) proved that the public JHS Mathematics teachers were not expertise in the application of problem-solving in their teaching. When it comes to pupils' engagement, the study revealed that the public JHS Mathematics teachers occasionally engage pupils in problem-solving.

Several recommendations have been made to help curb this situation. However, it must be noted that the use of a problem-solving approach by teachers is a gradual process and cannot be achieved overnight. It may take a gradual approach to convince teachers that their present, traditional methods are less relevant and effective in relation to the needs of modern societies. To convince the majority of teachers of such a view, opportunities should be given to them to enhance their perceived knowledge of problem-solving, problem-solving instructional strategies and involvement of pupils in problem-solving. Teachers also need to challenge and critically reflect on their teaching methods more frequently. It is believed that if attention is given to problem-solving and the use of problem-solving instructional strategies, it will positively influence teaching and learning of Mathematics.

5.4 Recommendations

From the findings, the following recommendations were made:

1. District Directorate of Education should put in place a scheme that will address the professional development needs of Mathematics teachers on teaching Mathematics through problem-solving.
2. District Directorate of Education should focus on regular teacher collectives where Mathematics teachers will meet to share ideas, solve problems, discuss ways of teaching Mathematics through problem-solving. Teacher collectives are groups of teachers, either in the same school or nearby areas; those meet regularly to solve problems, share ideas, discuss pedagogy, plan lessons and reflect collaboratively about teaching and learning (IAS/PCMIIS, 2006)
3. The time slot for the teaching of Mathematics in the basic schools should be increased to allow for the use of various child-centred activities that go into a problem-solving approach.
4. School authorities in the Berekum West District should help to promote mentoring programs that will help teachers to be abreast with problem-solving in the District.
5. Circuit supervisors in the Berekum West District should ensure periodic visitation of schools and together with school heads to make sure Mathematics

teachers comply with activities and processes outlined by the training they attended.

6. District Directorate of Education should provide incentives and general improvement of the condition of service of teachers to motivate teachers to do their best, since teaching Mathematics through problem-solving has been identified as time-consuming and as an approach demanding teacher resourcefulness.

5.5 Suggestions for Further Research

Based on the findings of this study, the following suggestions for further research are made:

1. It is suggested that the effect of problem-solving on JHS pupils' academic performance should be investigated in the Berekum West District.
2. Also, a study of how information and communication technology can promote the teaching of Mathematics through problem-solving is recommended for further study.

REFERENCES

Adeleke, O. A. (2017). *An investigation of the extent of automation of public libraries in South West Nigeria* (Doctoral dissertation).

Adesoji, F. A. (2008). Students' ability levels and effectiveness of problem-solving instructional strategy. *Journal of Social Sciences*, *17*(1), 5-8.

Adiguzel, T., & Akpinar, Y. (2004). Improving school children's mathematical word problem-solving skills through computer-based multiple representations. *Association for Educational Communications and Technology, 27*(1), 19-23.

Alhassan, S. (2006). *Modern approaches to research in educational administration for research students.* Kumasi: Payless Publications Ltd.

Ali, R., Hukamdad, Akhter, A., & Khan, A. (2010). Effect of using problem-solving method in Teaching Mathematics on the Achievement of Mathematics students. *Asian Social Science, 6*(2), 67-72.

Amedahe, F. K. (2002). *Fundamentals of educational research methods*. UCC, Cape Coast: Mimeograph.

Anamuah-Mensah, J., & Mereku, D. K. (2005). Ghanaian junior secondary school two student's abysmal mathematics achievement in TIMSS 2003: A consequence of the Basic school Mathematics. *Mathematics Connection*, *5*(1), 1-11.

Anamuah-Mensah, J., & Mereku, D. K. (2005). Ghanaian JSS2 students' abysmal mathematics achievement in TIMSS 2003: A consequence of the basic school mathematics curriculum. *Mathematics connection*, *5*(1), 1-13.

Anderson, J. A. (2005). *Implementing problem-solving in Mathematics classroom*: *What support do teachers need?* Retrieved on 11-08-2019 fromhttp://www.merga.net.au/documents/RP42005.pdf

Anderson, J. A. (2009). *Mathematics curriculum development and the role of problem-solving.* Sydney: ACSA.

Anderson, J., & White, P. (2004). The influence of perceived constraints on teachers' problem-solving beliefs and practices. In I. R. Faragher, & M. McLean (Eds.), *Mathematics education for the third millennium: Towards 2010* (pp. 39-46), Proceeding of the 27th annual conference of the Mathematics education research group of Australasia, Townsville, Queensland.

Andrews, P., & Xenofontos, C. (2014). Defining mathematical problems and problem-solving: prospective primary teachers' beliefs in Cyprus and England. *Mathematics Education Research Journal*, *26*(2), 279-299.

Andrews, P., & Xenofontos, C. (2015). Analysing the relationship between the problem-solving-related beliefs, competence and teaching of three Cypriot primary teachers. *Journal of Mathematics Teacher Education*, *18*(4), 299-325.

Artzt, A. F., Amour-Thomas, E., & Curcio, F. R. (2009). *Becoming a reflective Mathematics teacher: A guide for observation and self-assessmen*t (2nd ed.). New York: Lawrence Erlbaum Associates.

Awanta, E. K. & Asiedu-Addo, S. K. (2008). *Essential statistical techniques in research for universities, colleges and research institutions.* Accra, Ghana: Salt "N" Light Publishers.

Baki, A. (2004). Problem-solving experiences of students Mathematics teachers through Cabri. A case study. *Teaching Mathematics and Its Application, 23*(4), 172-180.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). *Content knowledge for teaching: What makes it Special?* Retrieved on 21-01-2020. From citeseerx.ist.psu.edu/viewdoc/download? doi=l0.]. l.493.2271&rep=repl& type=pdf

Ball, D., & Bass, H. (2000). Interweaving content and pedagogy in teaching and learning to G Teach: Knowing and using Mathematics. In J. Boaler (ed.), *Multiple perspectives on mathematics teaching and learning.* Westport, CT: Ablex.

Banerjee, A., & Chaudhury, S. (2010). Statistics without tears: Populations and samples. *Industrial Psychiatry Journal*, *19*(1), 60.

Banks, H. T., Hu, S., & Kenz, Z. R. (2011). A brief review of elasticity and viscoelasticity for solids. *Advances in Applied Mathematics and Mechanics*, *3*(1), 1-51.

Baumrind, D. (1991). The influence of parenting style on adolescent competence and substance use. *The Journal of Early Adolescence*, *11*(1), 56-95.

Bay, J. M. (2000). Linking problem-solving to student achievement in Mathematics: Issues and outcomes. *Journal of School Improvement*, *1*(2), 8 - 13.

Bennett, J. M. (2007). *Holt Mathematics course 1*. Orlando, FL: Harcourt.

Bereiter, C. & Scardamalia, M. (1982). From conversation to composition: the role of instruction in a developmental process. In: R. Glaser. R (ed.), *Advances in instructional psychology* (Vol. 2). Hillsdale NJ: Lawrence Erlbaum Associates.

Biddlecomb, B., & Carr, M. (2011). A longitudinal study of the development of Mathematics strategies and underlying counting schemes. *International Journal of Science and Mathematics Education*, *9*(1), 1-24.

Bley, N. S., & Thornton, C. A. (2001). *Teaching Mathematics to students with learning disabilities*. PRO-ED, Inc., 8700 Shoal Creek Blvd., Austin, TX 48757-6897.

Bransford, J. D., Brown, A. L., & Cooking, R. R. (2000). *How people learn: Brain, mind, experience, and school: Expanded edition.* Washington. DC: National Academy Press.

Braun, V., & Clarke, V. (2006). Using thematic analysis in psychology. *Qualitative Research in Psychology*, *3*(2), 77-101.

Broudy, H. S. & Spiro, W. E. (1977). Types of knowledge and purposes of education. In R. C. Anderson, R. J. Spiro, & W. E. Montague (Eds.), *Schooling and the acquisition of knowledge* (1-17). Hillsdale, NJ: Erlbaum.

Brown, M. (1978). Emotional intelligence compensates for low IQ and boosts low emotionality individuals in a self-presentation task. *Personality and Individual Differences*, *81*, 169-173.

Bryant, J. (2009). *Problem-solving through communication.* Yale: New Haven Teacher Institute.

Bryman, A. (2016). *Social research methods*. London: Oxford University Press.

Burns, N., & Grove, S. K. (2010). *Understanding nursing research-eBook: Building an evidence-based practice*. New York: Elsevier Health Sciences.

Buschman, L. (2004). Teaching problem-solving in Mathematics. *National Council of Teachers of Mathematics, 10*(6), 302-309.

Cai, J. (2003). Singaporean students' mathematical thinking in problem-solving and problem posing: an exploratory study. *International journal of mathematical Education in Science and Technology*, *34*(5), 719-737.

Cai, J., & Lester, F. (2012). *Research Brief*. Retrieved August 30, 2019, from Council of Teachers of Mathematics Web Site: www.ntcm.org.

Cañadas, M. C., & Castro, E. (2009). Using a model to describe students' inductive reasoning in problem-solving. *Electronic Journal of Research in Educational Psychology, 7*(1), 261–278.

Chapman, O. (2012). Prospective elementary school teachers' ways of making sense of mathematical problem posing, *PNA*, 6(4), 135 146.

Chimni, S., Sankar, S., & Tripathi, A. (2016). On integers that are uniquely representable by modified arithmetic progressions. *Notes on Number Theory and Discrete Mathematics*, *22*(3), 36-44.

Christenson, S. L., Reschly, A. L., & Wylie, C. (Eds.). (2012). *Handbook of research on student engagement*. Springer Science & Business Media.

Cohen, L., Manion, L., & Morrison, K. (2012). *Research methods in education* (5th ed.). London: Routledge Palmer.

Corte, E. A. (2000). *Connecting mathematics problem-solving to the real world. Conference on Mathematics Education into 21st Century: Mathematics for living* (pp. 18-23). Amman, Jordan: ICME.

Cotic, M., & Zuljan, M. (2009). Problem-based instruction in Mathematics and its impact on the cognitive results of the students and on affective-motivational aspects. *Educational Studies, 35*(3), 297-310.

Creswell, J. W. (2009). *Research design: Qualitative and mixed methods approach.* London and Thousand Oaks: Sage Publications.

Creswell, J. W. (2014). *Research design: Qualitative, quantitative, and mixed methods approach*. London: Sage Publications.

Creswell, J. W., & Creswell, J. D. (2017). *Research design: Qualitative, quantitative, and mixed methods approach*. London: Sage publications.

Creswell, J. W., & Miller, D. (2000). Determining validity in qualitative inquiry. *Theory into Practice, 39*(3), 124–130.

Creswell, J. W., & Plano Clark, V. L. (2014). *Designing and conducting mixed methods research.* (2nd ed.). Thousand Oaks, CA: Sage.

Dewalt, K. M., & DeWalt, B. R. (2002). *Participant observation.* Walnut Creek. CA: AltaMira.

Dillman, D. A. (2000). *Mail and internet surveys: The tailored design method*. Brisbane: Wiley.

Dollah, M.U. (2006). *Mathematics teaching and learning through problem-solving.* Kuala Lumpur: Dewan Bahasa & Pustaka.

Donaldson, S. E. (2011). *Teaching through problem-solving: Practices of four High School Mathematics Teachers*. A Dissertation Submitted to the Graduate Faculty of the University of Georgia: Athens, Georgia.

Dvorak, J. (2004, Autumn). *Management znalosti.* Retrieved on 13-03-2019 from http://kariera.ihned.cz/c125 632030-kurzy-vzdelavani-box

Ellison, R. (2009). *Task-based language learning and teaching*. London: Oxford University Press.

Fetters, M. D., Curry, L. A., & Creswell, J. W. (2013). Achieving integration in mixed methods designs—principles and practices. *Health Services Research*, *48*(6pt2), 2134-2156.

Fletcher, M. & Santoli, S. (2003). Reading to learn concepts in Mathematics: An action research project. (ERIC Document Reproductive Service No. ED 482001).

Florence, H. (2012). *The effects of Mathematics manipulatives*. Retrieved September, 19, 2012 from http://www.ehow.com/info_8040301_effects-Mathematics-manipulatives.html

Foong, P. Y. (2002). Roles of problems to enhance pedagogical practices in the Singapore classrooms. *The Mathematics Educator*, *6*(2), 15-31.

Foong, P. Y., Yap, S. F., & Koay, P. L. (1996). Teachers' concerns about the revised Mathematics curriculum. *The Mathematics Educator*, *1*(1), 99-110.

Foshay, R. & Kirkley, J. (1998). *Principles of teaching problem-solving*. PLATO Learning, Inc., Bloomington, MN. Retrieved on 11-010-2019 from www.plato.com/whitepapers.asps

Fosnot, C. T. (2013). *Constructivism: Theory, perspectives, and practice*. New York: Teachers College Press.

Fredricks, J. A., (2004). School engagement: Potential of the concept, state of the evidence. *Review of Educational Research*, *74*(1), 59-109.

Frensch, P. A. (2014). Complex problem-solving: The European perspective. London: First Published 1995 by Lawrence Erlbaum Associates, Inc.

Ghana Statistical Service. (2012). *2010 population and housing census summary report of final results*. Accra Ghana: Sakao Press Limited.

Goldberg, P. D. (2005). Metacognition and art production as problem-solving: A study of third grade students. *Visual Arts Research*, 67-75.

Goldman, A. M. (1989). Onset of superconductivity in the two-dimensional limit. *Physical Review Letters*, *62*(18), 2180.

Grouws, D. A. (2003). The teacher's role in teaching Mathematics through problem-solving. *Teaching Mathematics through problem-solving: Grades*, 6-12.

Hahkiomiemi, A., Leppaaho, R. C., & Francisco, L. (2012). Teaching and learning argumentation, *The Elementary School Journal*, *107*(5), 449-472.

Hanson, J. L., Nacewicz, B. M., Sutterer, M. J., Cayo, A. A., Schaefer, S. M., Rudolph, K. D., ... & Davidson, R. J. (2015). Behavioral problems after early life stress: contributions of the hippocampus and amygdala. *Biological psychiatry*, *77*(4), 314-323.

Hammond, M., & Wellington, J. J. (2013). *Research methods the key concepts*. London; New York: Routledge.

Hartas, D. (2010). Quantitative research as a method of inquiry in education. *Educational Research and Inquiry-Qualitative and Quantitative Approaches*, 65-81.

Hattori, K. (2008). Comparison of classroom assessment practices: A case of Selected Ghanaian and Japanese mathematics lessons. *NUE Journal of International Educational Cooperation*, *3*, 95-105.

Henderson, C. R. (2002). *Faculty conceptions about the teaching and learning of problem-solving in introductory calculus-based physics*. University of Minnesota.

Hiebert, J., & Wearne, D. (2003). Developing understanding through problem-solving. *Teaching Mathematics through problem-solving: Grades*, *6*(12), 3-14.

Hiebert, J., Gallimore, R., Ganier, H., Garnier, H., Givvin, K. B., Hollingsworth, H., & Jacobs, J. (2003). *Teaching Mathematics in seven countries: Results from the TIMSS 1999 video study*. Washington, DC: National Centre for Education Statistics, Institute.

Hsieh, H. F., & Shannon, S. E. (2005). Three approaches to qualitative content analysis. *Qualitative Health Research*, *15*(9), 1277-1288.

Institute for Advanced Study/Park City Mathematics Institute International seminar. (2006, July 3-8). *Bridging Policy and Practice in the Context of Reasoning and Proof*. Retrieved November19, 2019, from http://mathforum.org/pcmi/beliefs/Preparation_Teaching_Problem_Solving_Reasoning_Proof.pdf

Jonassen, D. H (2004). Using cognitive tools to represent problems. *Journal of Research on Technology in Education, 35*(3), 362-381.

Jonassen, D. H. (2000). Toward a design theory of problem-solving. *Educational Technology Research and Development*, *48*(4), 63-85.

Jonassen, D. H. (2010). *Learning to solve problems: A handbook for designing problem-solving learning environments*. New York: Routledge.

Kargas, C. A., & Stephens, M. (2014). *Using coaching to improve the teaching of problem-solving to year 8 students in Mathematics*. Mathematics Education Research Group of Australasia.

Karuku, S. M. (2013). *Being there for the other*: *Towards a phenomenology of help in mathematics* (Unpublished doctoral thesis). University of Alberta, Edmonton, Canada.

Kayan, F. (2007). An imaginative approach to teaching. *Eurasia Journal of Mathematics, Science, & Technology*, *3*(3), 247-248.

Kolar, V., Mastnak, A. & Cadez, H. (2012). Primary teacher students' competences in inductive reasoning. In T. Berqvist (Ed), *Problem-solving in Mathematics education:* Proceeding from 13th ProMath conference: Learning through problem-solving. 54-68.

Kong, Q. P., Wong, N. Y., & Lam, C. C. (2003). Student engagement in Mathematics: Development of instrument and validation of construct. *Mathematics Education Research Journal*, *15*(1), 4-21.

Kusi, H. (2012). *Doing qualitative research: A guide for researchers.* Accra-New Town: Emmpong Press.

Kuzle, A. (2013). Patterns of metacognitive behavior during Mathematics problem-solving in a dynamic geometry environment. *International Electronic Journal of Mathematics Education*, *8*(1), 20-40.

Lambdin, D. V. (2003). Benefits of teaching through problem-solving. In F.K. Lester & R. Charles (Eds.), *Teaching Mathematics through problem-solving*: *Prekindergarten grade 6* (3-13). Reston, VA: National Council of Teachers of Mathematics.

Lee, C. Y., & Chen, M. P. (2009). A computer game as a context for non-routine mathematical problem-solving: the effects of type of question prompt and level of prior knowledge. *Computers & Education, 52*(3), 530-542.

Lesh, R. & Zawojewski, J. S. (2007). Problem-solving and modeling. In F. Lester (Ed.), *Second Handbook of research on Mathematics teaching and learning* (pp. 763-804). Greenwich, CT: Information Age Publishing.

Lester Jr, F. K. (2013). Thoughts about research on mathematical problem-solving instruction. *The Mathematics Enthusiast*, *10*(1), 245-278.

Leung, A. K., Loricchio, D., Brison, R. J., & Graham, K. (2019). Perspectives of ambulatory patients visiting the emergency department during the Christmas and New Year holiday period: Descriptive survey. *Canadian Family Physician*, *65*(7), e305-e310.

Liamputtong, P., & Ezzy, D. (2005). *Qualitative research methods* (2nd ed.). Melbourne: Oxford university press.

Lincoln, Y. (2000). Paradigmatic Controversies, Contradictions, and Emerging Confluences. In Denzin, N and Lincoln, Y (ed.), *Handbook of qualitative research*. London: Sage Publication Inc.

Lincoln, Y. S., & Guba, E. G. (2007). *Naturalistic inquiry*. Newbury Park, CA: sage Publications

Lopez, L. (2008). *Helping at-risk students solve mathematical word problems through the use of direct instruction and problem-solving strategies* (Doctoral dissertation, University of Central Florida Orlando, Florida).

Manuel, S. T. (1998). Instructional qualities of a successful Mathematical problem-solving class. *International Journal of Mathematics Education in Science & Technology, 29*(5), 631-645.

Mason, J. (2006). Mixing methods in a qualitatively driven way. *Qualitative research*, *6*(1), 9-25.

Matlala, S. J. (2015). *The experiences of secondary Mathematics teachers teaching Mathematics through problem-solving.* A Thesis presented Graduate School: Stellenbosch University.

Mayer, R. E. & Wittrock, M. C. (2006). Problem-solving. In P. A. Alexander & P. H. Winne (Eds.), *Handbook of educational psychology* (pp. 287-303). Mahwah, NJ: Erlbaum.

McIntosh, R., Jarrett, D., & Peixotto, K. (2000). *Teaching mathematical problem-solving: Implementing the vision*. Portland, Oregon: Mathematics and Science Education Center, North West Regional Laboratory.

McQueen, R. (1999). Four views of Knowledge and Knowledge Management. *Proceedings of Americas Conference of AIS*, August 1998, 609-611.

Mereku, A. G. (2015). Assisting Ejura Presbyterian basic school students in solving linear equations in one variable using activity method. Department of Basic Education, UEW, Unpublished.

Mesel, T. (2013). The necessary distinction between methodology and philosophical assumptions in healthcare research. *Scandinavian Journal* 0/ Caring Sciences, 27, 750-756.

Ministry of Education [MOE]. (2019). *Teaching syllabus for JHS Mathematics.* Accra: CRDD.

Ministry of Education [Ontario] (2007). The Ontario Curriculum: Grades 9 and 10 Mathematics. Toronto, ON: Ontario Ministry of Education & Training, Queen's Park children's mathematical modeling processes. *Australian Primary Mathematics Classroom, 10*(1), 4-8.

Mitchell, M. L., & Jolley, J. M. (2012). *Research design explained*. Belmont, CA: Wadsworth Cengage Learning.

Morrison, K. (2012). *School leadership and complexity theory*. Routledge.

Murat, A. & Memnun, D. (2008). Mathematics teacher trainees' skills and opinions on solving non-routine mathematical problems. *Journal of Theory and Practice in Education*, *4*(2), 213-238.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school Mathematics.* Reston, VA: Author.

National Educational Assessment. (2016). Finding report Accra: Assessment Service Unit

National Research Council. (1989). Improving risk communication. *http://www. nap. edu/openbook. php? isbn= 0309039436.*

Nohda, R. (2000). *Teaching of Mathematics*. New Delhi India: A.P.H. Publishing Corporation.

Norton, S., McRobbie, C. J., & Cooper, T. J. (2002). Teachers' responses to an investigative Mathematics syllabus: Their goals and practices. *Mathematics Education Research Journal*, *14*(1), 37-59.

Olson, A., & Peterson, R. L. (2015). *Student engagement*. Lincoln: University of Nebraska-Lincoln.

Patton, M. Q. (2002). *Qualitative evaluation and research methods* (3rd ed.). Thousand Oaks, CA: Sage Publications, Inc.

Peterson, M. (2016). *Computer games and language learning*. Springer.

Polya G. (1945). *How to solve it*: *A new aspect of mathematical method*. Princeton, New Jersey: Princeton University Press.

Pólya, G. (1954). *Mathematics and plausible reasoning: Induction and analogy in mathematics* (Vol. 1). Princeton University Press.

Polya, G. (1957). *How to solve it* (2nd ed.). New York, NY: Doubleday.

Polya, G. (1962). *Mathematical discovery: On understanding, learning and teaching problem-solving.* New York: John Wi1ey and Sons, 1nc.

Polya, G. (1985). *How to solve it: A new aspect of mathematical method.* Princeton, N.J.: Princeton University Press.

Polya, G. (2004). *How to solve it: A new aspect of mathematical method* (No. 246). Princeton University Press.

Polya, G., & Simon, K. (1966). *Let us teach guessing*. Washington, DC: Mathematical Association of America.

Posamentier, A. S., Smith, B. S., & Stepelman, J. (2006). *Teaching secondary Mathematics: Techniques and enrichment units.* Upper Saddle River, NJ: Pearson Merrill Prentice Hall.

Reicher, S., & Taylor, S. (2005). Similarities and differences between traditions. *Psychologist*, *18*(9), 547-549.

Resnick, L. B., (1976). Problem-solving and intelligence. In L. B. Resnick (ed.), *The nature of intelligence Hillsdale.* NJ: Lawrence Erlbaum

Robert, A. L., Sharman, M. D., Britton, S., & New, P. B. (2009). Identification and Use of Theoretical Frameworks for a Qualitative Understanding of Mathematics Transfer. *CAL-elaborate International, 17*(1), 55-60.

Robson, C., & McCartan, K. (2016). *Real world research*. John Wiley & Sons.

Ross, J. A., & Maynes, F. T., (1983). Development of a test of experimental problem-solving skills. *Journal of Research in Science Teaching, 3,* 63-75.

Rudd, B. (2010). *Talking is for all: How children and teenagers develop emotional literacy.* London: Sage.

Salant, P., & Dillman, D. A. (1994). *How to conduct your own survey.* New York: John Wiley and Sons.

Saleh, F. (2009). *Problem-solving schemes of secondary school Mathematics teachers.* Retrieved on 11/10/19 from http//: math.unipa.it/~grim/Esaleh7.PDE

Sasso, M. T., & Ross, D. B. (2019). The qualitative report 10th Annual Conference, Davie, Florida at Nova Southeastern University, January 16 to 18, 2019. How Narcissists Cannot Hold an Organization Together: A Mixed Method Approach to a Fictitious Puzzle Factory.

Saunders, M., Lewis, P., & Thornhill, A. (2009). *Research methods for business students*. Edinburgh Gate, and Harlow: Financial Times Prentice Hall.

Schoenfeld, A. H. (1985). *Mathematical problem-solving*. Orlando, FL: Academic Press.

Schoenfeld, A. H. (1987). What's all the fuss about metacognition? *Cognitive science and Mathematics Education*, *189*, 215.

Schroeder, T. L., & Lester, F. K. (1989). Developing understanding in Mathematics via problem-solving. *New Directions for Elementary School Mathematics*, *31*, 42.

Sepeng, P. (2014). Mathematical reasoning and common-sense in word problem-solving. *International Journal of Educational Sciences*, *7*(3), 755-763.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational researcher*, *15*(2), 4-14

Siernon, D., & Booker, G. (1990). Teaching and learning FOR, ABOUT and THROUGH problem-solving. *Vinculum*, *27*(2), 4-12.

Silver, E. A. (1982). Knowledge organization and mathematical problem-solving. In F. K. Lester & J. Garofalo (Eds.), *Mathematical problem-solving: Issues in research* (pp. 15-25). Philadelphia, PA: FranklinSinclair & Lehr, 2004

Simon, M. K., & Goes, J. (2013). Assumptions, limitations, delimitations, and scope of the study. *Retrieved from dissertation recipes. Com*.

Sinclair, M. F., Lehr, C. A., & Anderson, A. R. (2003). Facilitating student engagement: Lessons learned from Check & Connect longitudinal studies. *The California School Psychologist*, *8*(1), 29-41.

Singh, S. K. (2008). Role of leadership in knowledge management: a study. *Journal of Knowledge Management*, *12*(4), 3-15.

Stemberg, R. J. (2002). Human intelligence. In E. Tulving (Ed), *Memory, consciousness, and the brain: The Tallinn Conference* (363 - 373). Philadelphia, PA: Taylor and Francis, Psychology Group.

Taplin, M. (2010). *Mathematics through problem-solving*. Institute of Sathya Sai Education.

Tashakkori, A., & Teddie, C. (1998). *Mixed methodology: Combining qualitative and quantitative approaches*. Thousand Oaks, CA: Sage.

Thomas, R. M. (2003). *Blending qualitative and quantitative research methods in theses and dissertations.* London: A sage publications Ltd.

Tratton, P. R., & Midgett, C. (2001). *Learning through problems: A powerful approach to University of Minnesota*. Retrieved on September 2019 from http://groups.physics.umn.edu/physed/People/charles.htm

Van de Walle, J. A. (2007). Designing and selecting problem-based tasks. *Teaching mathematics through problem solving: Prekindergarten-grade*, *6*, 67-80

Springer, Cham.Verschaffel, L., Greer, B., & De Corte, E. (2002). Everyday knowledge and mathematical modeling of school word problems. In *Symbolizing, modeling and tool use in Mathematics education* (pp. 257-276). Springer, Dordrecht.

Walle, J. A. V. D. (2007). Elementary and middle school Mathematics. In *USA, Library of Congress Cataloging-in-publication data* (Vol. 6).

Wang, Y., & Chiew, V. (2010). On the cognitive process of human problem-solving. *Cognitive Systems Research*, *11*(1), 81-92.

Webb, P., & Sepeng, P. (2012). Exploring mathematical discussion in word problem-solving. *Pythagoras*, *33*(1), 1-8.

Wilkinson, D., & Birmingham, P. (2003). *Using research instruments: A guide for researchers*. New York: Psychology Press.

Wilson, J. W., Fernandez, M. L., & Hadaway, N. (1993). Mathematical problem-solving. *Research ideas for the classroom: High school Mathematics*, *57*, 78.

Zanzali, N.A. (2003). Implementing the intended Mathematics curriculum: Teachers' belief about the meaning and relevance of problem-solving. *Proceeding of the International Conference The Decidable and the Undecidable in Mathematics Education* (pp. 34-37).

APPENDICES

APPENDIX A

QUESTIONNAIRE FOR TEACHERS

UNIVERSITY OF EDUCATION, WINNEBA

DEPARTMENT OF BASIC EDUCATION

Thank you for taking your precious time to complete this questionnaire. The researcher is an M.Phil. Student in the Department of Basic Education, University Education, Winneba. This questionnaire is not meant to assess you. Instead, it seeks to elicit information on basic school Mathematics teachers' perceived knowledge and practice of problem-solving in teaching Mathematics.

Your name and other personal identifications are not required and will not at any point time be associated with your responses. Once again, the confidentiality of your responses is highly assured.

Thank you for your time, patience and participation.

SECTION A: GENERAL INFORMATION

Please tick [√] in the appropriate space provided below and supply answers where required. If you want to change an item you have already ticked, put [×] over the selected item and tick the new item.

1. Gender

a. Male [ ]

b. Female [ ]

2. Age

a. 20-30 [ ]

b. 31-40 [ ]

c. 41-50 [ ]

d. 51 and above [ ]

3. Academic Qualification

a. Cert A [ ]
b. Diploma/HND [ ]

c. Bachelor's Degree/ Post Dip [ ]
d. Master's Degree (MPhil., Med, MA) [ ]

e. Others [ ]
f. Specify ....

4. How long have you been teaching Mathematics?

a. 1 - 5 year(s) [ ]

b. 6 – 10 years [ ]

c. 11-15 years [ ]

d. 16 - 20 years [ ]

e. 21 years and above [ ]

5. Teachers' Rank

a. Untrained

b. Superintendent

c. Senior Superintendent

d. Principal Superintendent

e. Assistant Director and above

SECTION B: TEACHERS' PERCEIVED KNOWLEDGE OF PROBLEM-SOLVING

Directions: For each statement below use the following key to indicate how you respond to the statement regarding your perceptions of classroom assessment.

Please tick [√] in the appropriate box. If you want to change an item you have already ticked, put a cross [×] over the selected item and tick the new item. **Rating Scale: Strongly Disagree (SD = l), Disagree (D = 2), Agree (A = 3), Strongly Agree (SA = 4).**

SN	STATEMENT	SD	D	A	SA
1	Problem-solving involves tasks that challenges pupils' ability				
2	Mathematics problems should task pupils to reason logically and critically				
3	Mathematics problem should require pupils to conjecture their strategies in solving it				
4	Mathematics lessons should guide pupils to self-develop strategies in solving problems				
5	Mathematics problems should have connection with pupils' real-life situation				
6	Mathematics problem should challenge pupils to apply their daily skills in solving it				

SECTION C: TEACHERS' INSTRUCTIONAL STRATEGIES OF PROBLEM-SOLVING

Directions: For each statement below use the following key to indicate how you respond to the statement regarding your perceptions of classroom assessment. Please tick [√] in the appropriate box. If you want to change an item you have already ticked, put a cross [×] over the selected item and tick the new item. **Rating Scale: Strongly Disagree (SD = l), Disagree (D = 2), Agree (A = 3), Strongly Agree (SA = 4).**

SN	STATEMENT	SD	D	A	SA
1	I task pupils on problems that are within their ability based on what they have been taught.				
2	I explain the problem for pupils to understand.				
3	I pose an open-ended problem for pupils to solve.				
4	I provide adequate time for pupils to use to solve Mathematics problems in class.				
5	I promptly react to every response from the pupils.				
6	I allow each pupil to explain the procedure used to arrive at an answer.				
7	I frame the problem so that the pupils can fully understand it.				
8	I give problem only based on what pupils have been taught in class.				
9	I allow pupils to confer, consult and cooperate with friends when solving Mathematics problems.				
10	I adopt other motivational techniques to boost pupils' morale up and also encourage them to solve problems.				
11	I finally discuss the various solutions with the whole class and allow suggestions from pupils.				

SECTION D: PUPILS' ENGAGEMENT IN PROBLEM-SOLVING

Directions: For each statement below use the following key to indicate how you respond to the statement regarding your perceptions of classroom assessment. Please tick [√] in the appropriate box. If you want to change an item you have already ticked, put a cross [×] over the selected item and tick the new item. **Rating Scale: Strongly Disagree (SD = l), Disagree (D = 2), Agree (A = 3), Strongly Agree (SA = 4).**

SN	STATEMENT	SD	D	A	SA
1	Pupils prefer giving them routine procedures rather than explaining the principles behind them.				
2	Pupils pay much attention to how to pass a test (exercise) than applying the concept in a real-life situation.				
3	Pupils prefer to learn new concepts familiar to them than trying to understand what they ought to know.				
4	Pupils easily give up trying when they are not able to solve a problem immediately.				`
5	Pupils try various ways to get a solution to a mathematical problem.				
6	Pupil's works on Mathematics problems persistently to make sure their answers are correct.				
7	Pupils devote much of their time to solving a Mathematics problem.				
8	Pupils are curious to learn new things in Mathematics and are excited when teaching a new topic.				
9	Even though some Mathematics problems are tough, pupils are happy when they are able to solve them.				

APPENDIX B

INTERVIEW GUIDE FOR TEACHERS

This interview is to give you an option to express your views and experience about the use of problem-solving as an instructional strategy. Your views will remain confidential and will be used for only this research purpose. I am pleased to have you for this interview. The interview shall last for about 35 minutes and I wish you will permit me to audiotape your voice for later transcription. You may also ask for clarification if you are in doubt.

PART A

TEACHERS' PERCEIVED KNOWLEDGE FOR TEACHING PROBLEM-SOLVING

1. In your own opinion, what do you think problem-solving is all about?

   ………………………………………………………………………………………

2. Do you see problem-solving necessary for teaching Mathematics in your classroom?

   Yes [ ]   No   [ ]   If yes explain

   ………………………………………………………………………………………

PART B

TEACHERS' INSTRUCTIONAL STRATEGIES FOR TEACHING PROBLEM-SOLVING

3. Do you use instructional strategies for teaching problem-solving? Yes [ ]
   No [ ]

   If yes explain

   ……………………………………………………………………………………

4. Do you think there are challenges of teaching Mathematics through problem-solving?

   Yes [ ]   No [ ]   If yes
   explain…………………………………………………………

PART C

PUPILS ENGAGEMENT OF PROBLEM-SOLVING

5. Do you incorporate problem-solving into your Mathematics lessons? Yes [ ]
No [ ] If yes explain
....
......

6. Do you engage your pupils in Mathematics problem-solving? Yes [ ] No [ ]
If yes explain
....

PART D

IMPROVING TEACHING OF PROBLEM-SOLVING

7. Do you think there are advantages of teaching Mathematics throng problem-solving?
Yes [ ] No [ ] If yes
explain...
.....

8. Do you think there are disadvantages to teaching Mathematics through Problem-solving?
Yes [ ] No [ ] If yes explain
....

9. In your own opinion, how can teachers be encouraged to teach Mathematics through problem-solving?
....
....

APPENDIX C

OBSERVATION GUIDE FOR TEACHERS

Rating Scale: I = Not at all, 2 = Some evidence, 3 = Clear evidence

SN	Instructional Practices	1	2	3
1	Teacher task pupils on problems that are within their ability and explain the problem for pupils understanding			
2	Teacher pose open-ended problems and provide adequate time for pupils to solve			
3	Teacher promptly react to pupils responses and allow them to explain their procedures			
4	Teacher frame problem so that the pupils can fully understand it			
5	Teacher allow pupils to confer, consult and cooperate with friends when solving Mathematics problems			
6	Teacher prefer giving routine procedures and much attention to pupils as an easy way of passing a test			
7	Teacher easily taught new concept which are familiar to pupils and give up on problems which the pupils cannot solve immediately			
8	Teacher allow pupils to device various ways of getting a solution and persistently make sure their answers are correct			
9	Teachers devote much time for pupils to solve a Mathematics problem and gets excited when solving challenging problems			
10	Teacher provides concrete material for students			
11	Teacher allows students to work in groups			
12	The teacher serves as a facilitator, a guide by allowing students to construct their knowledge during problem-solving lessons.			
13	Teacher encourage students to pose their problem			
14	Teacher makes pupils happy when they can solve difficult Mathematics problems			

APPENDIX D

LETTER OF INTRODUCTION

UNIVERSITY OF EDUCATION, WINNEBA
FACULTY OF EDUCATIONAL STUDIES
DEPARTMENT OF BASIC EDUCATION
P. O. Box 25, Winneba, Ghana
+233 (050) 9212015
beducation@uew.edu.gh

Date: February 4, 2020

The District Director
Berekum West District Education Directorate
Berekum, Bono Region

Dear Sir/ Madam,

LETTER OF INTRODUCTION

We forward to you, a letter from Mr. Collins Asomah, a second year M.Phil student of the Department of Basic Education, University of Education, Winneba, with registration number 8180030024.

Mr. Collins Asomah is to carry out a research on the Topic ***"Mathematics Teachers Knowledge for Teaching Problem Solving in Public Junior High School in the Berekum West District"***.

We would be grateful if permission is granted him to carry out his studies in the District.

Thank you.

Yours faithfully,

DEPT. OF BASIC EDUCATION
UNIVERSITY OF EDUCATION
WINNEBA, GHANA

....
MRS. SAKINA ACQUAH (PHD)
(Ag. Head of Department)

www.uew.edu.gh

APPENDIX E

TRANSCRIBED DATA FROM THE INTERVIEW

Five teachers were interviewed to capture all the research questions. The five teachers were selected from the eight (80) teachers who responded to the questionnaire. The abbreviation TR followed by a number in the write connotes a teacher and R connotes a researcher.

Nsapor Methodist Junior High School Mathematics Teacher 1 (Female)/23/12/2019

TR 1 had a Bachelor's degree in Mathematics and a Diploma in Education holder and had taught for eleven years. He is between 41-50 years of age. She is the Mathematics teacher at Nsapor for JHS 1, JHS 2 and JHS 3, with a total of 105 pupils in all the three classes.

Jamdede M/A Junior High School Mathematics Teacher 2 (Male)/28/12/2019

TR 2 holds Diploma and B. Ed in Basic Education from the University of Education, Winneba. He falls within the 31–40-year group and has been teaching Mathematics at the Junior High level for nine years. He handled only JHS 2 and JHS 3 and is the form master for JHS 3.

Koraso D/A Junior High School Mathematics Teacher 3 (Male)/ 20/01/2020

TR 3 was a Diploma in Basic Education holder. He had been in the teaching service for about 5 years and has participated in a few in-service training programmes. He was within the 20–30-year age range and was the only teacher who teaches Mathematics in the school.

Jinijini St. Lucy Junior High School Mathematics Teacher 4 (Male) 28/01/2020

TR 4 also holds B. Ed. Mathematics Education certificate from the University of Cape Coast. He is a graduate teacher and has been teaching for about twelve (12) years at the JHS. He is a JHS 2 form master.

Fetentaa Junior High School Mathematics Teacher 5 (Male) /11/02/2020

TR 5 had a Master's degree in Mathematics and a Diploma in Education holder and had taught for fifteen years. He is between 41-50 years of age. He is the Mathematics teacher for JHS 1, JHS 2 and JHS 3, with a total of 115 pupils in all the three classes.

As a follow up on Research Question one ***(what is the public JHS Mathematics teachers' perceived knowledge of problem-solving?).***

R: In your own opinion, what do you think problem-solving is All about?
TR 1 Said "Ah, problem-solving is a way of helping pupils to solve the problem that they encounter in their daily life"

TR 2 Said "ah Mathematics problem-solving has to do with the efforts that one makes to find a solution to a problem that probably you might ahh ahh have not met before".

TR 3 Said "hmmmmm…Problem-solving is all about giving Mathematics problem to children to think and reason very well before getting an answer".

TR 4 Said "from the experience I have, problem-solving has to do with word problem where children are required to think and sometimes work in a group to provide a solution to an unsolved problem".

TR 5 Said "problem-solving is a process of finding a solution to a given problem". Interviewed data''.

R.: Do you see problem-solving necessary for teaching Mathematics?

TR 1: Yes

R.: If yes explain

TR 1 Said "I got to know problem-solving as a focus in teaching Mathematics through some books I have read. I got to know that problem-solving is one of the best approaches to evaluating students' application of Mathematics. So as a teacher I try to relate my lessons to students' daily life".

TR 2: Yes

R.: If yes explain

TR 2 Said "I got to know problem-solving as being one of the ways of testing children understanding. I learnt that way back at college about 10 years ago and I sometimes use it in my lessons".

TR 3 Said "I recognized problem-solving as a focus on teaching Mathematics right from training college. We were taught and when I became a teacher, I sometimes try to also use problem-solving in my lessons".

TR 4 Said "I heard of problem-solving from a workshop I attended some years ago. I got to know that problem-solving challenges children thinking so when I want to children to think deeply, I use problem-solving"

TR 5 Said "I heard problem-solving from college and also from a workshop. It gives an in-depth understanding of both conceptual and procedural knowledge to pupils".

Concerning Research Question two (**To what extents do public JHS Mathematics teachers apply problem-solving instructional strategies in their teaching?),** the interview was to solicit teachers' views on the extent to which the teachers apply problem-solving instructional strategies in their teaching. The following questions were asked:

R: Do you use instructional strategies for teaching problem-solving?

TR 1 Yes

R.: If yes explain

TR 1 Said "I love to give students group work assignments and also I sometimes encourage students to guess their answers before solving it".

TR 2: Yes

R.: If yes explain

TR 2 Said "I discuss the problem with them using drawings and diagrams to illustrate or charts and as well as questions to model the problem. Then I lead students to solve the problem by asking leading questions. Finally, I help them to work backwards to be sure that the answer is correct"

TR 3: Yes

R.: If yes explain

TR 3 Said "I have various ways of adopting problem-solving. For example, I use normal exercise being in the form of puzzles so that students can reason more and write few".

TR 4: Yes

R.: If yes explain

TR 4 Said "I fact, I use a variety of strategies some of which are making a list, making a chart or a table, drawing a diagram, making a model, simplifying the problem, looking for a pattern working backwards. Sometimes, I even use a formula or equation or act out the problem situation, using guess and check".

TR 5: Yes

R.: If yes explain

TR 5 Said "Through shared responsibilities between teacher and student (democratic classroom). Ah, for example, peer teaching, group discussions, group presentations and so on".

The next question was asked to find out from the teachers some of the challenges of teaching Mathematics through problem-solving. The following excerpts are some of the views expressed by the respondents:

R.: Do you think there are challenges of teaching Mathematics through problem-solving?

TR. 1: Yes
R.: If yes explain
TR1 Said "Students prefer to be told Mathematics rather than to be guided by the teacher to explore and construct their understanding".

TR 2: Yes
R.: If yes explain
TR. 2 Said "Teaching through problem-solving requires a lot of time and if time is not sufficient, it is better to teach Mathematics by telling".

TR 3: Yes

R.: If yes explain
TR 3 Said "Teachers have inadequate subject matter knowledge, pedagogical content knowledge and personal problems".

TR 4: Yes

R.: If yes explain
TR. 4 Said "Some teachers lack the requisite knowledge, skills and expertise for teaching Mathematics through problem-solving".

TR 5: Yes

R.: If yes explain
TR. 5 Said "Ah identifying the correct procedure in solving a specific problem is a challenge of teaching Mathematics through problem-solving".

The next research question also sought to **find out how pupils were engaged in problem-solving.** Some of the questions the researcher asked are as follows:

R.: Do you incorporate problem-solving into your Mathematics lessons?
TR.1: Yes
R.: If yes explain
TR. 1: Said "Well in my lessons, I provide the opportunity for students to reason logically and criticised their answer and I do this most of the time".

TR. 2: Yes

R.: If yes explain
TR. 2: Said "Not often as and when what I am teaching calls for it. For example, I write Mathematics word problem form for the children to provide the solution".

TR. 3: Yes

R.: If yes explain
TR. 3: Said "I like giving children projects as well as homework where they can get help from the house".

TR. 4: Yes

R.: If yes explain
TR. 4: Said "By converting word problem into mathematical expressions, equations and so on".

TR. 5: No

R.: If no explain

TR. 5 Said ''I don't have any idea about how to incorporate problem-solving into Mathematics lessons''.

The next question was asked to find out from teachers how they engage pupils in Mathematics problem-solving. The following excerpts are some of the views expressed by the respondents:

R.: Do you engage your pupils in Mathematics problem-solving?

TR. 1: Yes
R.: If yes explain
TR 1 Said "I give them questions I believe they can do to and they also perform".

TR. 2: Yes

R.: If yes explain
TR. 2: Said "I engage pupils in problem-solving by putting them in groups so that they can work together like share ideas and things to solve the problem".

TR. 3: Yes

R.: If yes explain
TR. 3 Said "You know children dislike Mathematics and at times difficult questions might scare them from even coming to school and you know our system too…., So I give them questions I believe they can do it and perform".

TR. 4: Yes

R.: If yes explain
TR. 4: Said "Pupils in Mathematics problem-solving, hmm…. well, I give them questions and I encourage them to follow the procedures I used to solve it. Even if it is difficult for them, I encourage them to reason well and at times some students will get the answer".

TR. 5: Yes

R.: If yes explain
TR. 5: Said "By allowing pupils to attempt problem-solving questions on their own".

The final research question also sought to **find out measures to improve the teaching of problem-solving?** Some of the questions the researcher asked are as follows:

R.: Do you think there are advantages of teaching Mathematics throng problem-solving?
TR. 1: Yes

R.: If yes explain
TR. 1 Said "Ah a lot of benefits, they can be independent when they face a real-life problem because already, they have been given the skills to manage the difficult problem so it helps the students a lot. If you teach problem-solving, it encourages the students to solve problems in real-life situations".

TR. 2: Yes

R.: If yes explain
TR. 2 Said "I think that using a problem-solving approach in teaching Mathematics is interesting and enjoyable because students understanding the lessons better. After going through a lesson, students can be given a problem to help them revise what has been taught".

TR. 3: Yes

R.: If yes explain
TR. 3 Said "If a student is confronted with an external problem at home, he/she uses the problem-solving techniques taught in school to address that situation".

TR. 4: Yes

R.: If yes explain
TR. 4: Said "I must say students gain a lot of understanding when they are taught through a problem-solving approach. As students solve the problem in one topic, other topics came in, therefore problem-solving helps students to practice other topics. When it comes to exams, for example, the experience they gain through the approach helps them to understand the questions. They are also able to answer the questions accordingly. So, the benefits are……that they learn a lot, they acquire knowledge".

TR. 5: Yes

R.: If yes explain
TR. 5: Said "It gives practical understanding to pupils as they solve a problem related to their everyday life".

The next question was asked to find out from teachers the disadvantages of teaching Mathematics through problem-solving. The following excerpts are some of the views expressed by the interviewees:

R. Do you think there are disadvantages to teaching Mathematics through problem-solving?

TR. 1: Yes

R.: If yes explain
TR. 1 Said "If you look at the way examination are set, students are likely not able to answer most of the examination questions well when they are taught through problem-solving. Problem-solving does not meet the demands of examination and students always learn to have at the back of their minds passing examinations".

TR. 2: Yes

R.: If yes explain
TR. 2: Said "Hmm…teachers are not comfortable because he/she does not know much about problem-solving so he/she has little knowledge for the subject to teach".

TR. 3: Yes

R.: If yes explain
TR. 3: Said "Ah…the large number of students in the class also makes it difficult to use a problem-solving approach in teaching because it is not easy going round all the students individually explaining to them or making sure they follow the correct path in solving a problem".

TR. 4: Yes

R.: If yes explain
TR. 4: Said "Problem-solving is very time-consuming"

TR. 5: Yes

R.: If yes explain
TR. 5: Said "Ah…. if the right technique is not used, it tends to confuse pupils".

The final question was asked to find out from teachers in their opinion, how can teachers be encouraged to teach Mathematics through problem-solving. The following excerpts are some of the views expressed by the respondents:

R.: In your opinion, how can teachers be encouraged to teach Mathematics through problem?

TR. 1: Yes

R.: If yes explain
TR. 1: Said "Organising in-service training for teachers to acquire knowledge in problem-solving will encourage teachers to teach Mathematics through problem-solving".

TR. 2: Yes
R.: If yes explain
TR. 2: Said "Most teachers need to be trained to know how to use problem-solving because is like we know of problem-solving only when it comes to a word problem question. If teachers are trained to know how to use it, I think they will be encouraged to use it"

TR. 3: Yes

R.: If yes explain
TR. 3: Said "I think if textbooks provide a lot of problem-solving questions it can help".

TR. 4: Yes
R.: If yes explain
TR. 4: Said "Ghana education service should organise in-service training courses for teachers especially on Mathematics problem-solving because we were brought out by teachers who did not have interest in Mathematics so they made Mathematics to look like it was a very difficult subject"

TR. 5: Yes
R.: If yes explain
TR. 5 Said "By organising seminars and workshops for teachers on problem-solving techniques"