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EFFECTS OF GEOGEBRA ON SENIOR HIGH SCHOOL STUDENTS' MATHEMATICAL UNDERSTANDING OF RIGID MOTION

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2016

DECLARATION

CANDIDATE DECLARATION

I, Christoph Yao Gadese declare that this thesis, with the exception of quotations and references contained in published works which have all been identified and acknowledged is entirely my own original work, and it has not been submitted, either in part or whole, for another degree elsewhere.

SIGNATURE……………………………………………..DATE…………

SUPERVISOR'S DECLARATION

We hereby declare that the preparation and presentation of the thesis was supervised in accordance with the guidelines on the supervision of thesis laid down by the University of Education, Winneba.

SUPERVISOR'S NAME: DR. CHARLES ASSUAH

SIGNATURE…………………………………..DATE…………….…………

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DEDICATION

To my wife Mrs. Jennifer Gadese, my daughter and son Catherine Deladem Gadese and Caleb Mawugbe Gadese respectively.

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ABSTRACT

This study sought to determine effects of GeoGebra on Senior High School students�really mathematical understanding of rigid motion. The study was conducted using students and mathematics teachers of Senior High School ‗A' in Ho in the Volta Region of Ghana. The sample size was made up of 100 students and four mathematics teachers. The control group was made up of 50 students and the experimental group comprised 50 students. The study was conducted using non-equivalent control group with pre-test and post-test design, a quasi-experimental approach using both qualitative and quantitative method. Instruments used for gathering data on the study were rigid motion achievement tests (pre-test & post-test), observations and questionnaires. Findings made with regards to difficulties SHS students have in performing rigid motion were: description of the line of reflection of reflected figures in a given diagram, interpretation of equations of lines of reflection, substitution of object points into formulae to arrive at image points, interpretation of translation vectors, and interpretation of clockwise and anticlockwise rotation. Other findings from the study further showed that these difficulties of students were as a result of factors such as the use of traditional approach to teach students rigid motion at the Junior High School level where formulae were given for the students to use without any practical knowledge of the concept, little or no integration of ICT in the teaching of rigid motion in order to show learners what to do through worked-out and modelled examples, in addition to practice for successful learning. Again, the results showed that there was a significant difference between the mean performances of the experimental group (*M* = 12.10, *SD* = 5.987) and control group (M = 5.98, *SD* =4.298), *t* (98) =5.872, *p* =.000 in the post-test. The average mean performances of each of the two groups showed that the experimental group performed better than the

control group. It was recommended that mathematics teachers blend technology, pedagogy and content plus the relationships between them (Koehler & Mishra, 2008) in teaching geometry. This would help improve students' understanding of the concept. Also, in-service training and workshops should be organized for Senior High School teachers on mathematics specific software packages such as GeoGebra. GeoGebra introductory book, and instructional materials about GeoGebra and its integration into mathematics classrooms should also be developed and distributed to teachers. This would improve teachers' competence and confidence level to integrate ICT in the classroom.

CHAPTER ONE

INTRODUCTION

1.1 Overview

This chapter presents the background to the study, statement of the problem, objectives, research questions, delimitation, definitions of terms, importance of the study, and organization of the rest of the text.

1.2 Background to the Study

Geometry is a concept that has been part of Ghanaian curriculum right from the basic school through to the tertiary level. Over the years, chief examiners' reports in Mathematics have indicated that students' performance in geometry at both Junior High and Senior High School levels have been rather weak. In the past years, chief examiners' reports on the Basic Education Certificate Examinations (BECE) have indicated that students lack sufficient knowledge in geometry and application of geometrical concepts of which geometrical transformations form a part (WAEC, 2005, 2006). This poor performance was manifested by Ghanaian students' performance in their participation in Trends in International Mathematics and Science Study (TIMSS) over the years. The 2007 TIMSS report indicated that Ghanaian students' performance in Geometry was not only low, but also relatively lower than the country average. The Ghanaian JHS 2 students' performances in the four mathematics content domains in TIMSS 2007 when compared to that of TIMSS 2003 showed that with the exception of geometry, there were improvements, though little in other content domains (Anamuah - Mensah, Mereku, & Ghartey-Ampiah, 2008). At the 2011 TIMSS, Ghanaian students' performance in geometry (average score of 315) was again lower than the country average of 331 (Mullis, Michael, Pierre, & Alka, 2012).

Again, chief examiners' reports in core Mathematics for the Senior High School students over the years, indicated that performance of students in geometry had also been very weak (WAEC, 2004, 2005, 2006, 2007, 2008, 2009). This may largely be attributed to the observations made by researchers that in Ghana, teaching is largely by exposition with little opportunities for learners to engage in practical and problem solving activities, which generate deeper understanding (Anamuah-Mensah et al., 2008).

Geometry is a broad area in Mathematics and most of its aspects are practically oriented. Rigid motion for instance, is an aspect of Geometry that should naturally be taught using practical activities and without necessarily resorting to recall of any relation. Yet, students seem to perform poorly when it comes to answering questions on rigid motion.

Teaching and learning in Ghana has over the years been limited to chalk and the chalkboard. Meanwhile, the world is currently drifting very fast towards technology in all spheres of life and Mathematics teaching and learning is no exception. Research findings show that, the low performance in Mathematics generally is partly due to the current teaching strategies used in Senior High Schools and lack of teachers' knowledge of ways to integrate technology into instruction (Agyei & Voogt, 2012). A study conducted by Mereku et al. (2009), indicated that technology is used in typing examination questions in all institutions and in some cases educators use technology in processing students' examination results. Their findings further indicated that very few teachers in Ghanaian Senior High Schools (SHSs) use technology in their teaching. There is minimal use of visualization tools such as Dynamic Geometrical Tools and graphing tools in Mathematics classrooms (Handal, Herrington, & Chinnappan, 2004).

Kurz, Middleton, and Yanik (2004), sugested that exposure of some categories of Mathematics based software can lead to conceptual change. In a study they conducted on preservice teachers, they showed that the thoughts of the preservice teachers became more developed and comprehensive after experiencing and reflecting on the affordances and constraints of tool-based Mathematics software. Khalid (2009), found that teachers have strong desire for the integration of ICT into education but they encountered many barriers, one of which is lack of access to resources. However, some of these resources could be accessed from the internet by teachers and students. Computer graphics make the teaching of Mathematics concepts attractive and easy to understand. Computer graphics describe any use of computers to create or manipulate images (Peter, Michael, Kelvin, & Peter, 2005). Computer graphics inevitably require some knowledge of specific software packages. Dynamic Mathematics software such as GeoGebra is designed to combine certain features of dynamic geometry software, computer algebra systems, and also spreadsheets into a single package (Hohenwarter, Hohenwarter, & Lavicza, 2009). GeoGebra, therefore, becomes the most appropriate software that can help improve Senior High School students' mathematical attainment in Geometry.

This research work therefore, aims at determining the effect that dynamic Mathematics software (GeoGebra) used with the discovery strategy of teaching could have on the mathematical understanding of students in geometry (rigid motion) in the Senior High Schools (SHS) and also encourage teachers to integrate ICT in teaching and learning Mathematics.

1.3 Statement of the Problem

Rigid motion is a concept that is tested in West African Senior School Certificate Examination (WASSCE) and it comes normally as a complete question with sub-questions for a maximum score of 12 in Paper 2 (subjective) (WAEC, 2007; WAEC, 2008; WAEC, 2011 and WAEC, 2012). A number of them are also found in Paper 1(objective) which is compulsory for all candidates. If taught practically and properly, students who attempt questions on rigid motion should score all 12 marks in the subjective and rightly answer all or majority of the objective questions. This would then boost their overall performance in WASSCE. There are reports of low and abysmal Mathematics achievement in WASSCE. It was reported that 31.6% of the candidates obtained grade F9 in Mathematics in the May/June 2014 West African Senior Secondary Certificate Examination (WAEC, 2014). Again, results released by the West African Examination Council (WAEC) indicated that only 25.29% of candidates who took the May/June 2015 West African Senior School Certificate Examination (WASSCE) obtained A1-C6 in Mathematics while 29.75% had D7-E8 and 37.17% had F9 (WAEC, 2015)

A contributing factor to the low performance is students‘ inability to answer satisfactorily questions on rigid motion. After going through marked scripts of 2014/2015 final year students' mock Mathematics scripts in a school (pseudonymously labelled as school A) for the purposes of this research, the researcher realized that majority of them did not answer questions on rigid motion. The few that attempted these questions answered them wrongly. Further scrutiny of the marked scripts revealed that students tried to recall relations to enable them calculate the image points of given object points. Meanwhile, effective teaching of rigid motion just like any topic in mathematics must include not only facts to be mastered, but also an appropriate and logical system of cognitive activity. In other words, what is required is for the student to acquire relational understanding more than procedural understanding. However, the teaching and learning of Mathematics in schools is still dominated by teacher-centred and textbook oriented approach (Lim & Hwa, 2007). In the last four decades, several Ghanaian authors have been involved in curriculum development for schools. It has been noted that teachers continue to teach by merely transmitting mathematical facts, principles and algorithms, and students are commanded to learn them in a passive and fearful manner (Mereku, 2010). This is clearly outmoded and teachers need to stand up to the challenge by employing more innovative and technologically driven methods of teaching the subject. Specifically, the method of teaching rigid motion should be enhanced to make it practical. This can be done through the integration of ICT since an effective pedagogy can utilize technology as a vehicle or medium to achieve its intended purposes (Assuah, 2010).

Integration of technology in education is becoming widespread not only in the developed countries, but in developing countries as well and the quality of mathematical software packages is improving rapidly. Mathematics teachers in Ghana are encouraged to use the calculator and the computer for problem solving and investigations of real life situations (CRDD, 2010). But there are Mathematics specific software packages aside the calculator that could be used together with the computer to make teaching and learning of Mathematics easier for students to understand.

The teachers' role in the integration process is a matter of great concern. Research conducted by several researchers indicated that teachers lack the confidence to integrate ICT in their teaching. Becta (2004) argued that ―many teachers who do not consider themselves to be well skilled in using ICT feel anxious about using it in front of a class of children who perhaps know more than they do" (P.7). Linked directly to teachers' confidence or lack of it to integrate ICT in the teaching and learning process is competency. Christensen and Knezek (2006) described computer self - efficacy as computer confidence in competence. Teachers' confidence to integrate ICT in teaching and learning depends largely on their knowledge and competence in the use of Mathematics specific software packages in addition to being a computer literate.

Meanwhile, most research works likened ICT integration to the use of computer only and little effort is made to delve into software packages. Educationists and curriculum planners in Ghana have been concerned about how teachers and students use computers in schools and how their use supports learning (Boakye & Banini, 2008). Majority of the students also know of only the computer as an ICT tool since they are not exposed to Mathematics specific software tools. Sarfo and Ansong-Gyimah (2010), argued that in Ghana students have strong perceptions that the computer can promote the first five principles of instruction for succesful learning better than the teacher, and that in the perspective of the Ghanaian students it is necessary to pay more attention to the expansion of computers in the classroom in order to enhance quality teaching and learning. But there are Mathematics specific software that teachers can use with the computer to improve teaching and learning. The students' perception about the computer as a typing tool will change once they are exposed extensively to the software that can be used to do Mathematics among other things. There are five general categories of software that utilize tool-based conception of Mathematics software (Kurz, Middleton, & Yanik, 2005). All of these categories can be used as part of a (more or less) complete Mathematics curriculum. According to Kurz et al. (2005), the five categories are Review and Practice

Software, General Software, Specific Software, Environment Software, and Communication Software.

Different packages support teaching at a variety of curriculum levels and they require different amounts of classroom time for students to become proficient with the software (Hohenwarter, Hohenwarter, Kreis, & Lavicza, 2008). While computer algebra systems involve a considerable time commitment and their sophistication enables its use in upper level education, dynamic geometry software can be used as early as in elementary schools due to its mouse-driven user interface. According to Kurz, Middleton, and Yanik (2005), each type of software offers both affordances and constraints to learning in the Mathematics classroom. However, it is believed that the affordances embodied in a software tool enable the teacher to engage students in fundamentally different Mathematics than they could have approached had the software not been present. It is necessary, therefore, to take advantage of the affordances offered by the Mathematics specific software in order to improve the mathematical ability of Senior High School students especially in geometry.

1.4 Purpose of the study

The purpose of this research was to investigate the effect of GeoGebra on Senior High School students‘ mathematical understanding of rigid motion. It is expected to bring out students‘ difficulties in rigid motion and determine how useful GeoGebra could be in addressing these difficulties.

1.5 Objectives

This study was expected to achieve the following objectives.

1. To examine difficulties Senior High School students have in understanding rigid motion.

2. To design a lesson using GeoGebra and use it to teach rigid motion at Senior High Schools level.
3. To find out how effective SHS Mathematics teachers find the use of GeoGebra in teaching rigid motion.

1.6 Research Questions

The research was expected to answer the following questions.

1. What difficulties do Senior High School students have in understanding rigid motion?
2. How effective does the use of GeoGebra application software enhance students‘ achievement in rigid motion?
3. How effective do SHS Mathematics teachers find the use of GeoGebra in teaching rigid motion?

1.7 Hypothesis

The following hypothesis was tested

1. There is no statistically significant difference in the mean performances of experimental and control groups‘ scores in rigid motion.

1.8 Delimitation

Emphasis was laid on rigid motion and multiple transformations of two dimensional figures. The study was restricted only to one SHS in the region. It is hoped that, findings made would be useful to all SHS in Ghana.

1.9 Limitations

According to Best and Kahn (1993) limitations are conditions beyond the control of the researcher that places restrictions on the conclusion of the study and its application. One limitation was the inability to generalise the results of the study to all SHSs in Ghana. The researcher could not cover all the government assisted SHSs in Ghana due to financial constraints and time. The researcher, therefore, used only School ‗A' which represented a very small portion of the entire population.

Another limitation was that the study did not focus on all areas of geometry. Geometry is a very broad area but the aspect covered by the study is limited to only rigid motion.

1.10 Significance of the study

This study is significant because it would enlighten students, teachers, policy makers and school administrators on the availability of Mathematics specific software packages and how they can be used to improve upon the mathematical ability of students.

The study would provide students with a transformative tool that will enable them visualize and manipulate mathematics objects freely. This would then lead to better understanding of rigid motion and other concepts of mathematics.

The study would expose teachers to GeoGebra and its availability for instruction on most concepts in mathematics. It would help teachers to design enhanced mathematics lessons to meet the current demands of mathematics instructions.

The study would serve as a source of reference to curriculum planners so that in planning the Mathematics curriculum in future, Mathematics specific software are included. The existing Mathematics syllabus mentioned only the calculator and the computer in the integration of ICT at Senior High School level.

The study would inform school administrators of the need to provide more computers and install Mathematics specific software packages such as GeoGebra on them for both students and teachers to use in the teaching and learning process.

1.11 Organization of the Study

This study basically is made up of five chapters. Chapter one was devoted solely to the background to the study, purpose of the study, statement of the problem, significance of the study, research questions, delimitation and organisation of the study. Chapter two is on literature review which highlights relevant views and ideas on the topic from other authors. Chapter three describes the research design, sample and sampling techniques, instrumentation used and mode of analysis of data. Chapter four presents analysis of data and its findings. Chapter five focuses primarily on the conclusion, suggestions and recommendations to the study.

CHAPTER TWO

LITERATURE REVIEW

2.1 Overview

The idea of integration of ICT in teaching and learning of mathematics has been researched into by many scholars both local and foreign. The methods of teaching and learning of geometry in general and transformation in particular have also been researched into by many scholars. However, there is limited research in the area of teaching and learning of rigid motion using GeoGebra. Therefore, related literature to geometry and GeoGebra are reviewed to support this write up. The review covers the following thematic areas: Learners' understanding and learning of transformation geometry; Theoretical Framework; Instructional Strategies in teaching and learning geometry; History of Geometry; The importance of studying Geometry; Information Communication Technology (ICT); History of GeoGebra; Mathematics specific software packages and teaching geometry; Barriers to ICT integration and Final chapter summary.

2.2 Learners' understanding and learning of transformation geometry

Basic rigid motion is about a translation, a reflection, or a rotation in the plane. Rigid motion concepts require a bit of preparation about the concept of transformations that are one-to-one and ―onto", separation properties of lines in the plane, distance in the plane, and other concepts that are necessary for a more formal development (Wu, 2013). Transformation geometry provides an ample opportunity for learners to develop their spatial visualization skills and geometrical reasoning ability (Edwards, 1997).

Edwards (2003) identified a particular misconception about rotations. She found that instead of seeing rotation as mapping all the points of the plane around a centre point, the students in her study expected the shape to slide to the given centre point and then turn around it- showing that the students had a hard time seeing rotation as occurring ‗at a distance' from the object (Edwards, 2003). On reflection (Sproule, 2005), carried out a research with Grade 7 learners, and sought to identify the strategies that were best able to support learners in correctly completing reflection tasks. He found that although measuring distances in the diagrams was the most common strategy used by the learners, those participants who folded along the axis of symmetry were the most successful.

While (Sproule, 2005) utilised the use of grid lines, measuring, drawing in marks, turning the figure, mental folds and using the mirror investigating something similar in solving the problems on reflections, (Bansilal and Naidoo, 2012) combined learners' use of both visualisation and analytic strategies in solving transformation geometry problems. The visual approach is one which advocates investigations and discovery of properties via concrete manipulations, models and diagrams and the analytic approach is characterised by general formulae to describe the results of transformations on figures that are situated within the Cartesian plane. According to Bansilal and Naidoo (2012), the analytic approach seemed to be compelling to learners because of the ready availability of formulae. Nevertheless, using generalised formulae to work out problems based on transformation geometry is not as simple as learners perceive it to be.

The visual approach is supported by the ―Van Hiele model of geometric thought" which specifies a visual reasoning level as an initial level of geometric understanding. This is followed by a level of analysis at a higher level, then informal deduction, formal deduction and finally the highest level of rigour (Crowley, 1987). All levels are sequential and hierarchical and teaching activities are often designed according to the levels. The descriptions of the first three levels are given below (Crowley, 1987).

Level 1 (Visualisation): The object is seen as a whole, individual properties are therefore not distinguished.

Level 2 (Analysis): The object can be identified by the properties; each property is seen in isolation so properties of figures are not compared.

Level 3 (Informal Deduction): The objects are still determined by their properties; however the relationships between properties and figures evolve. Whilst the van Hiele levels of thinking are focused on geometry, they are valuable when working with transformation geometry, which deals with transformation of geometric figures. In this study, learners were asked to carry out transformation on triangles and quadrilaterals focusing on the shape and properties before and after the transformation.

According to Koehler and Mishra (2008), at the heart of good teaching with technology are three core components: technology, pedagogy and content plus the relationships between them. In this study, students in their various groups were allowed to draw the shapes and manipulate them using graphics window of GeoGebra to discover the rules of reflection, translation and rotation.

2.3 Theoretical Framework

The study was essentially concerned with how Senior High School students' geometric transformation concepts (rigid motion) at the SHS level could be improved through the use GeoGebra. Various researches have been done in the area of Geometry, especially on the teaching and learning of the concept. The researcher would want to adopt the constructivist theory and discovery approach with respect to the teaching and learning of geometric transformation as the theoretical framework.

2.3.1 Constructivist Theory

Clement and Battista (1990) define constructivism as an epistemology which follows basic tenets:

1. Knowledge is actively created by the student.
2. New mathematical knowledge is created by reflection on physical and mental actions.
3. There is no one true reality. Each person has their own reality based upon their interpretation.
4. Learning is a social process; meaning is negotiated.
5. Students learn when allowed to explore. They tend to memorize when knowledge is "dished out" to them.

The constructivist view of learning is reflected in the developmental theories of (Bruner, 1961; Piaget, 1972 and Vygotsky, 1978) among others. In cognitive constructivism, which originated primarily in the work of Piaget, an individual's reactions to experiences lead to (or fail to lead to) learning. In social constructivism, whose principal proponent was Vygotsky, language and interactions with others such as family, peers and teachers play a primary role in the construction of meaning from experience. Meaning is not simply constructed, it is co-constructed. Proponents of constructivism (Biggs, 1997) offered variations of the following principles for effective instruction:

1. Instruction should require students to fill in gaps and extrapolate material presented by the instructor. The goal should be to wean the students away

from dependence on instructors as primary sources of required information, helping them to become self- learners.

2. Instruction should involve students working together in small groups. This attribute which is considered desirable in all forms of constructivism and essential in social constructivism supports the use of collaborative and cooperative learning.

The study which involves the use of GeoGebra was considered in a discovery learning setting. Senior High School students would have to discover relations of geometric transformation concepts with less interference from the teachers. The intention of the researcher in this regard was guided by the constructivist theory of learning espoused by (Clements & Battista, 1990).

2.3.2 Discovery Approach

For effective geometry instruction, the method should not be the same as in teaching number, algebra or probability. Instead instruction should emphasize hands-on explorations, developing geometric thinking and reasoning, making conjectures and even carrying out geometry projects (Strutchens, et al., 2001; van Hide, 1999). According to Strutchens, Harris, and Martin (2001), students learn geometry by memorizing geometric properties rather than by exploring and discovering the underlying properties.

The discovery approach is considered as a situation where learners practically manipulate geometric figures to identify their properties as well as using the Cartesian plane to determine images from pre- image (Sarra, 1999). Usually the discovery approach involves the teacher presenting a series of structured situations to the pupils. The pupils then study these situations in order to discover some concept or generalization. As opposed to exposition, the learner is not told the rule or generalization by the teacher and then asked to practice similar problems. Instead learners are asked to identify the rule or generalization.

Not all learners find it easy to ‗discover' under all circumstances and this may lead to frustration and lack of interest in the activity. To avoid this, it may be necessary to have cards available with additional clues. These clues will assist the learners, through guidance, to discover the rule or generalization. There are other approaches to the teaching and learning of Geometry. Some of them are reviewed in the next section.

2.4 Instructional Strategies in teaching geometry

Geometry is a wonderful area of mathematics to teach. It is full of interesting problems and surprising theorems. It is open to many different approaches. According to Jones (2002) , teaching geometry well can mean enabling more students to find success in mathematics, knowing how to recognise interesting geometrical problems and theorems, appreciating the history and cultural context of geometry, and understanding the many and varied uses to which geometry is put. It means appreciating what a full and rich geometry education can offer to learners when the mathematics curriculum is often dominated by other considerations (the demands of numeracy and algebra in particular). It means being able to put over all these things to learners in a way that is stimulating and engaging, and leads to understanding, and success in mathematics assessments. It is relevant therefore, to review some approaches used in teaching geometry.

2.4.1 Traditional Approach of Teaching Geometry

With this approach, students learn mathematics by listening to their teacher and copying from the chalkboard rather than asking questions for clarifications and justification, discussing, and negotiating meanings and conjectures (Fredua-Kwarteng & Ahia, 2015). Consequently, students learn mathematics as a body of objective facts rather than a product of human invention. When starting a new lesson, the teacher usually revises the previous lesson by writing the important rules or procedures after which she / he asks students / learners to do similar exercises, from the textbook, which were worked in previous lesson or write a new rule or definition. The teacher usually use a ruler for drawing lines and figures, and protractor for measuring angles while the students also do the same in their notebooks using the same tools. He / she assign home work from the textbook each time a topic is completed.

Jones (2002), refers to this approach, when he mentioned that there is the tendency to teach geometry by informing students of the properties associated with plane and solid shapes, requiring them to learn the properties and then to complete exercise which show that they have learned the facts. Such an approach can mean that little attempt is made to encourage students to make logical connection and explain their reasoning. Whiles it is important that students have a good knowledge of geometrical facts, if they are to develop their spatial thinking and geometrical intuition, a variety of approaches are beneficial.

2.4.2 ICT integration approach of teaching geometry

Jones (2011), identified three approaches to ICT integration. In the teacher - demonstration approach, the teacher engages students in discussing an on - screen geometric construction by, perhaps, asking questions about the objects on the screen in order to get the learners to explain what they might expect would happen if some parts of the configuration were moved or changed. This approach was found to allow teachers with little experience of using technology in the classroom to experiment with the technology with relatively small risk.

The second approach entails teachers providing previously created interactive files for their lessons. With such teacher created files, students can experiment with dynamic objects. This provides clear boundaries for learners and time is not spent setting up the task; rather, learners can spend time exploring the mathematics that is central to each task. No doubt, there is quite some teacher control over the material, but the approach can bring in opportunities for creative thinking and problem solving by learners.

The third approach involves learners creating their own files, perhaps for other learners to tackle. This approach provides some learner ownership of the work and engages a different sense of problem solving (and problem posing) by creating that ownership. There is also the development of independence - in learning how to use software, and with additional scope for students' creativity and discovery.

When technology is used appropriately and accurately by teachers, it can provide a very rich environment in which students' geometric understanding and intuition can be easily developed (Hohenwarter, et al., 2009). Computer-based learning environments with appropriate software can transform Mathematics classrooms into many science classes, where students use technology to investigate, conjecture, and verify their findings (Preiner, 2008) .

It is useful to consider geometry as a practical subject and provide opportunities for students to use a range of resources to explore and investigate properties of shapes and geometrical facts. Particular consideration should be given to ways in which the ICT resources, which are increasingly available in schools, can be used to enhance the teaching and learning geometry. The use of dynamic geometry enables the teacher or individual students to generate and manipulate geometrical diagram quickly and explore relationships using a range of examples (Jones, 2002).

Also, in using dynamic geometry software, teachers are giving students hands-on experiences that allow them to visualize and come to an understanding of what is happening in their own minds and teaching them how to apply that understanding to the concepts of geometry in particular and Mathematics in general.

However, Jones (2002), explains that while the use of such software can enliven geometry teaching, it should be noted that it is not always clear what interpretations students make of geometrical objects they encounter in this way. One of the distinguishing features of a dynamic geometry package such as cabri-Geometry is the ability to construct geometrical objects and specify relationships between them. Dynamic geometry software used in geometry classroom is the Geometer‘s sketchpad. This software allows Mathematics to be taught visually to the class as a whole, to small groups or to individuals by creating dynamic and productive three way interaction between teacher, student and computer. Geometer‘s sketchpad enables students and teachers to construct and investigate unlimited geometric shapes. These shapes are first created and then they are explored, manipulated and transformed to ideal concepts. Furthermore, Geometer‘s sketchpad is used for exploration and guided or open-ended discovery which enables students to test their conjectures and be more engaged in their learning. However, the challenge of the teacher is to provide input that serves the learners‘ communicative needs in dynamic geometry environment.

2.4.3 Problem-Solving Approach

Problem solving as used in mathematics education literature refers to the process wherein students encounter a problem – a question for which they have no immediately apparent resolution, nor an algorithm that they can directly apply to get an answer (Schoenfeld, 1992).

Problem-solving is characterized by the teacher "helping students construct a deep understanding of mathematical ideas and processes by engaging them in doing Mathematics: creating, conjecturing, exploring, testing, and verifying" (Lester, et al., 1994). Specific characteristics of a problem-solving approach include:

i. interactions between students/students and teacher/students,
ii. mathematical dialogue and consensus between students,
iii. teachers providing just enough information to establish background/intent of the problem, and students clarifying, interpreting, and attempting to construct one or more solution processes,
iv. teachers accepting right/wrong answers in a non-evaluative way,
v. teachers guiding, coaching, asking insightful questions and sharing in the process of solving problems,
vi. teachers knowing when it is appropriate to intervene, and when to step back and let the pupils make their own way,
vii. a further characteristic is that a problem-solving approach can be used to encourage students to make generalizations about rules and concepts, a process which is central to Mathematics.

Problem-solving approach is considered as very vital in the teaching of geometrical concepts (Wells, 1988). In this approach, a problem is posed to learners and through the solution of such a problem; they begin to form various geometrical concepts which are inherent in the problem. This approach of problem-solving makes use of discovery and direct presentation approaches.

2.4.4 Investigative Approach

A mathematical investigation is an exploration of a topic with the view to discover new ways of thinking and to develop in-depth knowledge about the

Mathematics inherent in the topic rather than to obtain specific answers (Chapin, 1998 cited in Nabie , 2001).

In pursuing an investigation, learners bump into different but related ideas and concepts much earlier, think about in various ways and make connections among the different areas of Mathematics. Through an investigation, a whole class can be involved in a discussion at all levels of lesson delivery. Discussion is a way of sharing ideas. Discussions during and after an investigation can be an effective method for developing a lesson as well as assessing learners' progress (Nabie, Mathematical Investigations in the Reviewed Basic School Curriculum, 2001).

Investigations actively involve students in drawing, measuring, constructing, tracing, folding, cutting, calculating, and comparing geometric objects to look for relationships and formulate conjectures. After an investigation, students may present their results, you might conduct a class discussion to elicit what students learned, or you may summarize the geometrical ideas of the lesson. You might then work through an example of how to apply their new mathematical knowledge, or students might start working on homework in class, utilizing the support of their group members. Investigations in particular, are designed to take advantage of group discussion and multiple students' perspectives. Students will learn best if they discuss what they are discovering, learning, or practicing (Copes, 2008). An investigation is a form of discovery. Vincent as cited in Callingham (2004), suggested that, exploring tessellations was one approach to investigating the properties of two-dimensional shapes.

A major advantage of using investigations is that abstract concepts become meaningful, transferrable and retained because they are attached to performance of an activity. This is supported by Nabie (2004), when he asserted that no matter the operational level of the learner, it is desirable to use vast amounts of materials, which children can manipulate to gain the needed experience for the formation of the appropriate concepts. He advocated that to enable teachers or schools to provide their own materials for practical activities, material development or improvisation should be a main course of study in our teacher training institutions.

Most often a mathematical investigation begins with inductive reasoning which starts with observing a pattern which leads to making a conjecture. Then a question is asked, ―But why is conjecture true for all cases?" Deductive reasoning is now used to look for a reason for proof (Copes, 2008).

2.4.5 Inductive Teaching Method/Approach

The inductive teaching method or process goes from the specific to the general and may be based on specific experiments or experimental learning exercises. In inductive thinking, one considers a number of particular or specific items of information to develop more inclusive or general conceptions (Prabhat, 2009).

Prabhat (2009) used the following examples to illustrate the inductive method

1. Ask students to draw a few sets of parallel lines with two lines in each set. Let them construct and measure the corresponding and alternate angles in each case. They will find them equal in all cases. This conclusion in a good number of cases will enable them to generalize that ―corresponding angles are equal; alternate angles are equal." This is a case where equality of corresponding and alternate angles in a certain specific sets of parallel lines helps us to generalize the conclusion. Thus this is an example of inductive method.
2. Ask students to construct a few triangles. Let them measure and sum up the interior angles in each case. The sum will be same (= 180°) in each case. Thus

they can conclude that ~~t~~the sum of the interior angles of a triangle (= 180°). This is a case where equality of sum of interior angles of a triangle (=180°) in certain number of triangles leads us to generalise the conclusion. Thus this is an example of inductive method.

Induction, according to Sadiq (2009), is that form of reasoning in which a general law is derived from a study of particular objects or specific processes. Students use measurements, manipulators or constructive activities and patterns to discover a relationship. They later formulate a law or rule about that relationship based on their observations, experiences, inferences and conclusions. Sadiq (2009) identified the following as Merits of Inductive method:

1. It is psychological. The student feels interested in experiments, experiences and discoveries.
3. It fosters independence and self-confidence in the pupil which proves very useful in later life.
4. In this method, children discover the solution themselves. Hence it develops and encourages initiative and creative thinking.
5. All that is learnt using inductive method is remembered easily as it is self-acquired.
6. In this method, the pupils observe and analyse particular objects of similar and different nature and try to arrive at general truth.
7. Inductive method takes into consideration all the maxims of good teaching. The process of induction calls for perception, reasoning, judgment and generalization.

2.5 History of Geometry

The history of what we teach and learn is as important as the history of the world. The historical perspective of every issue is very relevant to the total understanding of it details. As educators, whatever we teach in any discipline has a direct link with the history of that descipline. According to Zebrowski (1999), ―we teach what has been learned in the past, hoping that the universe will cooperate in allowing our students to apply this past knowledge to the unfolding future. This is as true in music and art as it is in science and mathematics. We could not teach, nor could anyone learn, if we did not believe that the universe displays a historic continuity". Exploring the history of geometry indicates the following three distinct periods: Intuitive, Classical, and Modern (Yazdani, 2007).

2.5.1 Intuitive geometry (8000 B.C. - 500 B.C.):

Intuitive geometry, archaeological evidence suggests, was born in the Middle East at the time of Sumerians, and was further developed by Babylonians, and then Egyptians. Smith (1953) as cited in Yazdani (2007), writes that the earliest geometry ―was intuitive in its nature; that is, it sought facts relating to mensuration without attempting to demonstrate these facts by any process of deductive reasoning" (p. 270).

One might say that learning the skills of intuitive geometry could continue to thrive without any formal education because it was only through the experience and performance of related professions that intuitive geometry was transferred from one generation to another during the first period of the history of geometry. The first important surviving manuscript about geometry is a document written by an Egyptian priest around 1700 B.C. This manuscript called ‗Ahmes' consists of arithmetic and mensuration, areas of rectangles and circles.

2.5.2. Classical geometry (500 B.C.-1600 A.D.):

It was the result of systemization and formalization of the intuitive geometry and mathematical concepts and ideas. The Greeks established geometric facts based on deductive reasoning not empirical procedures. Thales is credited with being the first mathematician who used deductive reasoning in geometry in the first half of the sixth century B.C. Later, in the second half of the sixth century B.C., Pythagoras attempted to incorporate deductive reasoning in order to systemize geometry. It is reputed that Hippocrates was the first to extend the earlier works of Greek mathematicians by using a few definitions and assumptions to form chains of propositions. These assumptions have become known as ―axioms" and/or ―postulates". Theudius and others followed Hippocrates by further developing a logical presentation of geometry. Finally, Euclid organized and presented the achievements of the Greek mathematicians in his book called the ―Elements". It is also essential to mention the contributions of other philosophers and mathematicians such as Leon, Anaxagoras, Plato, Hypsicles, Apollonius, Hero, Archimedes, Menelaus, Ptolemy, Pappus, and Diophantus to the development of the geometry of this period.

During the period of the Dark Ages that began with the fall of the Roman Empire and extended to the 11th century, the development of geometry like any other branch of mathematics or science was interrupted in Europe. With the rise of the Moslem Empire and its interest in mathematics and science during the seventh century the Greek method of the logical presentation of geometry was preserved. Greek books of mathematics and science were translated into Arabic. During this period, the Indian mathematician Brahmagupta, the Arab mathematician Abulwefa and the Persian mathematician and astronomer Khayyam made some significant contributions to the development of geometry.

The rebirth of learning in Europe in the 11th century called for a translation of the Greek classics that had been preserved outside Europe. Greek classics in science and mathematics such as the ―Elements" were retranslated into Latin. This translation of scholarly work continued during the Renaissance to the 17th century.

2.5.3. Modern geometry (after 1750 A.D.)

Eves (1965) as cited in Yazdani (2007), states that some sources place the beginnings of modern geometry with the work of Saccheri and Lambert as they continued to attempt to prove Euclid‟s fifth postulate from the first four.

Euclid‟s fifth postulate states, ―If a straight line falling on two straight lines make the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles" (Robold, 1969, p. 208) as cited in (Yazdani, 2007). A logically equivalent version of this postulate is as follows: there is exactly one line parallel to a line through a given point not on the line(Playfair‟s postulate). Playfair attempted to substitute his postulate for Euclid‟s fifth postulate but did not succeed. Legendre attempted to prove the parallel postulate as a theorem.

Gauss, Bolyai, Lobachevsky, Beltrami, Klein, Poncelet, Poincare, and Riemann contributed to the development of non-Euclidean geometry by creating models of geometries that obeyed every Euclidean postulate except the fifth. The negation of the fifth postulate in these models showed the independence of Euclid‟s fifth postulate from the first four. Besides the difficulties surrounding the parallel postulate there are some other logical imperfections with Euclid‟s axiomatic system. Euclid established five postulates and five axioms for geometry. Although these initial premises justify the proof of a large number of the propositions in geometry, there are some propositions that cannot be drawn from the Euclid‘s first principles. Some additional postulates are needed. For example, the tacit assumption that the straight line is of infinite extent is such a proposition and was used by Euclid. The Hilbert‘s modern axiomatic development of geometry consists of 21 postulates. Hilbert‘s set of postulates has been incorporated in the instructional content of modern high school geometry textbooks in the United States.

2.6 The importance of studying geometry

Geometry is an important aspect of mathematics that spans from basic school through to the highest level of education all over the world. The need to teach and learn geometry, particularly in the high schools have been researched into by many scholars. González and Herbst (2006), undertook a study that contributes to the historical examination of the justification question for the particular case of the high school geometry course in the United States. They found that the 20th century saw the emergence of competing arguments to justify the geometry course. González and Herbst (2006), discussed four ―modal" arguments which surfaced in the 20th century to offer justification for the geometry course. By modal arguments they mean not necessarily ideologies explicitly promulgated by individuals but central tendencies around which the opinion of various individuals could converge.

The first modal argument was that geometry was justified on the grounds of *a formal argument*—that geometry helped discipline the mental faculties of logical reasoning. The main goal of the geometry course according to proponents of the formal argument was to have students learn to transfer skills and ways of thinking learned in geometry to other domains (Fewcett, 1938).

The second was *a utilitarian argument* which recommended the teaching of applications of geometry. The argument was that geometry would provide tools for students' future work or non-mathematical studies. Proponents of the utilitarian argument considered the geometry students as the future workers that they would become. "Many adults firmly believe that in the training in reasoning and attacking problems in geometry they received something that was of definite value and help to them later in their occupations and professions" (Breslich, 1938).

The third was *a mathematical argument* which justified the geometry course as an opportunity for students to experience the work and ideas of mathematicians. The mathematical argument recommended the study of geometry because of its capacity to engage students in making and proving conjectures or to illustrate for students how dramatic conceptual developments occur in the discipline of mathematics that permit to solve a multitude of new problems. Some proponents, such as Moise (1975), argued that Euclidean geometry is an optimal context for students to engage in making and proving conjectures.One common notion among proponents of the mathematical argument was that the study of geometry remained within the realm of mathematical activity and focused on knowing geometry (González & Herbst, 2006).

Finally, *an intuitive argument* emphasized the role of geometry providing students with an interface language and a representation system to relate to the real world. Proponents of the intuitive argument made a case for geometry as a unique opportunity for students to apply the intuition of the geometric objects to describing the world. Some proponents responded to the need to develop students' basic skills (e.g., calculating perimeter and area of figures) and thus call for developing geometric literacy (Hoffer, 1981). Others tended to go deeper in advocating that the course present alternative mathematical ideas that would be more aligned with students‘ needs (Usiskin & Coxford, 1972).

2.6.1 Inclusion of Geometry in the school mathematics

The learning of geometry contributes to helping students develop the skills of visualization, critical thinking, intuition, perspective, problem-solving, conjecturing, deductive reasoning, logical argument and proof (Jones, 2002). Geometric representations can be a source of help to students to grasp the concepts of other areas of Mathematics such as fractions and multiplication in arithmetic, the relationships between the graphs of functions, graphical representations of data in statistics just to mention a few (Jones, 2002).

Moreover, we live on a solid planet in a three-dimensional world and much of our experience is through visual stimulus which means that the ability to interpret visual information is fundamental to human existence. Geometry offers a rich way of developing visualization skills. Visualization allows students to explore Mathematics and other problems without the need to produce accurate diagrams or use symbolic representations.

Also, geometry is a rich source of opportunities for developing notions of proof. Visual images, particularly those which can be manipulated on the computer screen, invite students to observe and conjecture generalizations. In proving conjectures, students are required to understand how the observed images are related to one another, and in understanding observed images means working with points, circles, polygons, parallel and perpendicular lines, and so on (Jones, 2000). Since much of our cultural life is visual, our aesthetic appreciation of art, architecture, music and many of our cultural artifacts involve geometricprinciples and concepts such as symmetry, perspective, scale, orientation and so on. Our textiles industry for instance, applies a lot of geometric principles and concepts in the design and production of their products.

Geometry provides a culturally and historically rich context within which to do Mathematics. There are many interesting, sometimes surprising results in geometry that can stimulate students to want to know more and to understand why. Presenting geometry in a way that stimulates curiosity and encourages exploration can enhance students‘ learning and their attitudes towards Mathematics. Encouraging students to discuss problems in geometry, articulates their ideas and develop clearly structured arguments to support their intuitions can lead to enhanced communication skills and recognition of the importance of proof. Creativity is a special gift from God and God uses a variety of shapes in His wonderful creation. The expression of creativity through art can be viewed as an act of praising God. The contribution of Mathematics to students‘ spiritual, moral, social and cultural development can be effectively realized through geometry.

Finally, numerous current applications of Mathematics have strong geometric components. In many cases, the problem includes getting ‗geometric‘ information into a computer in a useful format, solving geometric problems, and outputting this solution as a visual or spatial form as a design to be built, as an action to be executed, or an image to entertain. Solving these problems require substantial geometric knowledge (Jones, 2000).

2.7 Information Communication Technology (ICT)

The rapid growth in Information Communication Technologies (ICT) has brought remarkable changes in the twenty-first century and affected demands of the modern society. ICT is becoming increasingly important in our daily lives as well as in educational systems. There is a growing demand on educational institutions to use

ICT to teach the skills and knowledge that students need for the 21st century. According to Buabeng-Andoh (2012), realizing the effect of ICT on the workplace and everyday life, today's educational institutions try to restructure their educational curricula and classroom facilities in order to bridge the existing technology gap in teaching and learning processes.

Government and educational planners in Ghana have responded to the challenge by creating national programs to integrate new technologies (e.g., computers, internet, and intranet) at all educational levels (Sarfo & Ansong-Gyimah, 2010). Morawcyzynski and Ngwenyama (2007), conducted a research in which they explored the interaction amongst investments in ICT, education and healthcare. They further analyzed development in five West African nations: Benin, Cameroon, Senegal, Ivory Coast and Niger. It was found that investments in ICT alone are not enough to significantly impact human development. However, ICT investments were found to be the most important predictors to GDP growth in most West African countries that were studied (Morawcyzynski & Ngwenyama, 2007).

Kaffash, Kargiban, Kargiban, and Remezani (2010), reviewed and examined theoretical approaches and frameworks which are helpful to understand the use of information and communication technology (ICT) in the formal education sector. They found that, first ICT implementation emphasizes different aspects of ICT-related capabilities. Second, theoretical basic ICT in curriculum is based on constructivism; but education in many countries is still based on behaviorism, and need to change the attitudes in educational system. Third, many factors that influence on ICT implementation in education need to be considered. Fourth, many researchers argued that ICT pedagogy need an integrated model. Finally, ICT curriculum model has strong emphasis on interdisciplinary model, and focuses on knowledge and toolcentered learning of ICT capabilities. Thus, teaching and learning processes which have focused on technical ICT knowledge, skills and tool are integrated with other subjects. They integrate ICT technical capacities into basic ICT literacy and core school subjects (Kaffash, et al., 2010).

Technology is not neutral, the penetration of ICT in schools can eventually transform pedagogy and the creation of knowledge. As a result, ICT are contributing to building new relationships between schools and their communities, and to bridging the gap between formal, non-formal and informal education. Eventually, technology may also lead policy-makers to rethink the skills and capacities that children need to become active citizens and workers in a knowledge society. However, ―while many teachers use ICT, they use it primarily to prepare lessons and when they use it in the classroom, it is to support their lecture presentations. Few teachers have their students use ICT regularly in their lessons. Consequently, ICT barely registers on the educational screen" (Kozma, 2011, p.34). However, there are numerous mathematic specific software that teachers can use to improve upon their teaching.

2.7.1 The use of ICT in mathematics education

Educational technology generally refers to the introduction of computers and related pieces of equipment to the classroom (Wenglinsky, 1998). The contemporary mathematics curricula in Ghana expect mathematics teachers to integrate technology in their teaching. But the situation in the schools seem contrary. Agyei and Voogt (2012), indicated that mathematics teachers do not integrate technology in their instruction in spite of government efforts in the procurement of computers and recent establishment of computer labs in most senior high schools. This is disturbing because having a complete infrastructure of the ICT will go meaningless if it is not utilized to the fullest capacity.

Technology can play various instructional roles and it is the responsibility of the instructors to decide how to best use technology to support student learning. In the views of Assuah (2010), varied strategies in pedagogy could be utilized by instructors, if they bring creativity and innovation. He indicates that African instructors should be motivated to incorporate technology in their teaching.

Wenglinsky (1998), undertook a study in which he presents findings from a national study of the relationship between different uses of educational technology and various educational outcomes. Data were drawn from the 1996 National Assessment of Educational Progress (NAEP) in mathematics, consisting of national samples of 6,227 fourth- graders and 7,146 eighth-graders. The study first compared the information about educational technology among different groups of students to discover any possible inequities in technology use. It found that the greatest inequities did not lie in how often computers were used, but in how they were used. In essence, the study found that technology could matter, but that this depended upon how it was used. Haapasalo (2008), suggests that instead of speaking about ‗implementing modern technology into classroom' it might be more appropriate to speak about ‗adapting mathematics teaching to the needs of information technology in modern society'. This means emphasizing more the making of informal than formal mathematics within the framework of eight main activities and motives, which have proved to be sustainable in the history of human thinking processes and making of mathematics.

The use of ICT in teaching mathematics can make the teaching process more effective as well as enhance the students' capabilities in understanding basic concepts (Keong, Horani, & Jacob, 2005). Kurz et al. (2004) sugested that exposure of some categories of mathematics based software can lead to conceptual change. In a study they conducted on preservice teachers, they showed that the thoughts of the preservice teachers became more developed and comprehensive after experiencing and reflecting on the affordances and constraints of tool based mathematics software.

2.7.2 General categories of software

There are five general categories of software that utilize tool-based conception of mathematics software (Kurz, et al., 2005). All of these categories can be used as part of a (more or less) complete mathematics curriculum. According to Kurz et al. (2005), the five categories are Review and Practice Software, General Software, Specific Software Environment Software, and Communication Software.

Review and practice software is more supportive of direct, measurable objectives and emphasizes drill and practice techniques to support the instruction of mathematics. General software allow a student to use common programs to explore and solve problems in a wide variety of mathematical topics. This software grows with the child throughout the school year and can often be used across multiple years, making it both powerful and economical. Specific software allows students to use tools to investigate distinct mathematical topic(s), providing insight and knowledge into a specific domain. Environment software affords a range of possible investigations, alowing students to experience "real" world applications of mathematics interactively. Such software allows for much more students control over problem solving and interpretation than other types of software and also may support the development of logical mathematics argument. Communication software enables dicourse among students, collaborative learning and out of class learning (Kurz, et al., 2005).

2.7.3 Types of Software Tools Used for Mathematics Education

General tools for mathematics education according to Preiner (2008) include for example dynamic geometry software, computer algebra systems, spreadsheets, and dynamic mathematics software.

In general, *computer algebra systems* mainly deal with the symbolic and numeric representation of mathematical objects. They allow for manipulating a variety of algebraic expressions and functions, and can deal for example with basic mathematical operations, simplification, factorization, derivatives, integrals, sequences, and matrices (Fuchs, 2007). Additionally, they allow for plotting graphs of functions and equations. Moreso, most computer algebra systems allow for graphically displaying explicit and sometimes even implicit equations, whereby those graphical representations usually can't be modified directly by using the mouse (Hohenwarter, 2002). As a result of their feasibility, computer algebra systems have reached to a spectrum consisting of a wider age group, thus they have started to appear as an educational tool in primary and secondary education (Güyer, 2008). Examples of computer algebra systems are *Derive*, *Maple*, and *Mathematica.*

Pure *dynamic geometry software* is operated mainly with the mouse by activating different geometric tools and applying them to the drawing pad or already existing objects. In general dynamic geometry software provides three main features that usually can't be found in computer algebra systems or spreadsheets: drag mode, customizable tools, and trace or locus of objects (Graumann, H¨olzl, Krainer, Neubrand, & Struve, 1996).

Drag mode: Dynamic geometry software allows the creation of geometric constructions and other dynamic figures (e.g. function graphs) by using the computer mouse and a variety of geometric tools and menu items. Relations and dependencies between objects are maintained while an object is dragged with the mouse by updating their positions dynamically. The so called *drag test* is an important concept that enables users not only to check the robustness of a construction by dragging different objects with the mouse, but also to explore a variety of similar constructions and special cases which is not possible in a traditional paper-and-pencil construction. Apart from dynamic movements, a DGS also allows the user to apply transformations to objects and measure lengths and angles. Additional features include the insertion of text and sometimes images into the drawing pad, which can be used to enhance a dynamic construction.

Customized tools: The available geometric tools are usually organized in toolboxes and can be activated by clicking on the corresponding icon in the toolbar or by selecting appropriate commands from the menu. Additionally, a sequence of construction steps can be grouped and saved as a new tool. Thus, users can define their own geometric construction tools and save them in the toolbar.

Trace or locus: The trace of an object in respect to a parent object can be displayed allowing users to examine movements and dependencies between mathematical objects. In this way, the locus line can either be created manually by moving corresponding objects with the mouse, or created automatically by the software itself.

Dynamic geometry software usually provides the following basic mathematical objects: points, segments, lines, circles, vectors, and conic sections. Additionally, it is possible (a) to do analytic geometry using a coordinate system, and (b) to work with function graphs by creating the locus of a given point whose *y*-coordinate is calculated using a given expression. Although keyboard input of .numbers and expressions is possible in most dynamic geometry software programs, it is usually limited to a range of special commands and predefined expressions. Such input is mainly used to carry out calculations whose results can be integrated into the construction process.Examples of dynamic geometry software are *Cabri Geometry* and *Geometer's Sketchpad*.

Spreadsheets are computer applications that allow the display of alphanumeric text or numeric values in table cells which are organized in rows and columns. Formulas can be used to calculate new values by referring to other cells. Whenever the content of one cell is modified, all other related cells are updated automatically. Therefore, electronic spreadsheets are principally used as tools for mathematical and statistical calculations, allowing students to focus on the mathematical reasoning by freeing them from the burden of calculations and algebraic manipulations (Ozgun-Koca, 2000). Spreadsheets are usually operated using keyboard input, formulas, and commands. They allow for plotting data in different types of charts which automatically adapt to modifications of the data. Examples of spreadsheets are *MS Excel* and *Calc*.

Dynamic mathematics software is designed to combine certain features of dynamic geometry software, computer algebra systems, and also spreadsheets into a single package. The resulting new dynamic mathematics software packages differ in their range of combined features, as well as in the degree of dynamic interaction between those features. In the case of *GeoGebra*, different representations of the same mathematical object are connected dynamically, allowing users to go back and forth between them thereby making relationships among those representations more easily comprehendible for students . Whenever one of the representations is modified, all others adapt automatically in order to maintain the relations between the different objects. New objects can be created either by using dynamic geometry tools or algebraic keyboard input. By its provision for keyboard input, a range of pre-defined commands can be used in GeoGebra and Mathematical topics other than geometry can be treated as well (e.g. algebra, calculus). Examples of dynamic mathematics software are *GeoGebra and GEONExT*

2.8 Short History of GeoGebra

According to Preiner (2008), the development of GeoGebra began in 2001 as Markus Hohenwarter‗s Master‗s thesis project at the University of Salzburg, Austria. After studying mathematics education as well as computer engineering, he started to implement his idea of programming a software that joins dynamic geometry and computer algebra, two mathematics disciplines that other software packages tend to treat separately. His main goal was to create an educational software that combines the ease of use of a dynamic geometry software with the power and features of a computer algebra system, which could be used by teachers and students from secondary school up to college level.

After publishing a prototype of the software on the Internet in 2002, teachers in Austria and Germany started to use GeoGebra for teaching mathematics, which was, at this point, rather unexpected by the creator, who got a lot of enthusiastic emails and positive feedback from those teachers (Hohenwarter & Lavicza, 2007). In 2002, Hohenwarter received the European Academic Software Award EASA in Ronneby, Sweden, which finally inspired him to go on with the development of GeoGebra in order to enhance its usability and extend its functionality. Further development of GeoGebra was funded by a DOC scholarship awarded to Hohenwarter by the Austrian Academy of Sciences, which also allowed him to earn his PhD in a project that examined pedagogical applications of GeoGebra in Austrian secondary schools.

Since 2006, GeoGebra‘s ongoing development has continued at Florida Atlantic University, USA, where Hohenwarter works in a teacher training project funded by the National Science Foundation‘s Math and Science Partnership initiative. During the last two years of close collaboration with a number of middle and high school mathematics teachers, GeoGebra was enhanced by including a range of important features. This enhanced functionality enabled the creation of user defined tools and significant simplification in the steps required for user creation of interactive instructional materials, the so called dynamic worksheets. Future plans to further extend and enhance GeoGebra involve the implementation of a dynamically linked spreadsheet, as well as a computer algebra extension, pushing the software further towards the goal of being a versatile and easy to use software package that can be used for a wide range of different grade levels and mathematical contents by students and teachers around the world (Preiner, 2008).

2.8.1 GeoGebra’s User Interface

Since GeoGebra joins dynamic geometry with computer algebra, its user interface contains additional components that cannot be found in pure dynamic geometry software. Apart from providing two windows containing the algebraic and graphical representation of objects, components that enable the user to input objects in both representations as well as a menubar are part of the user interface (see figure 1).

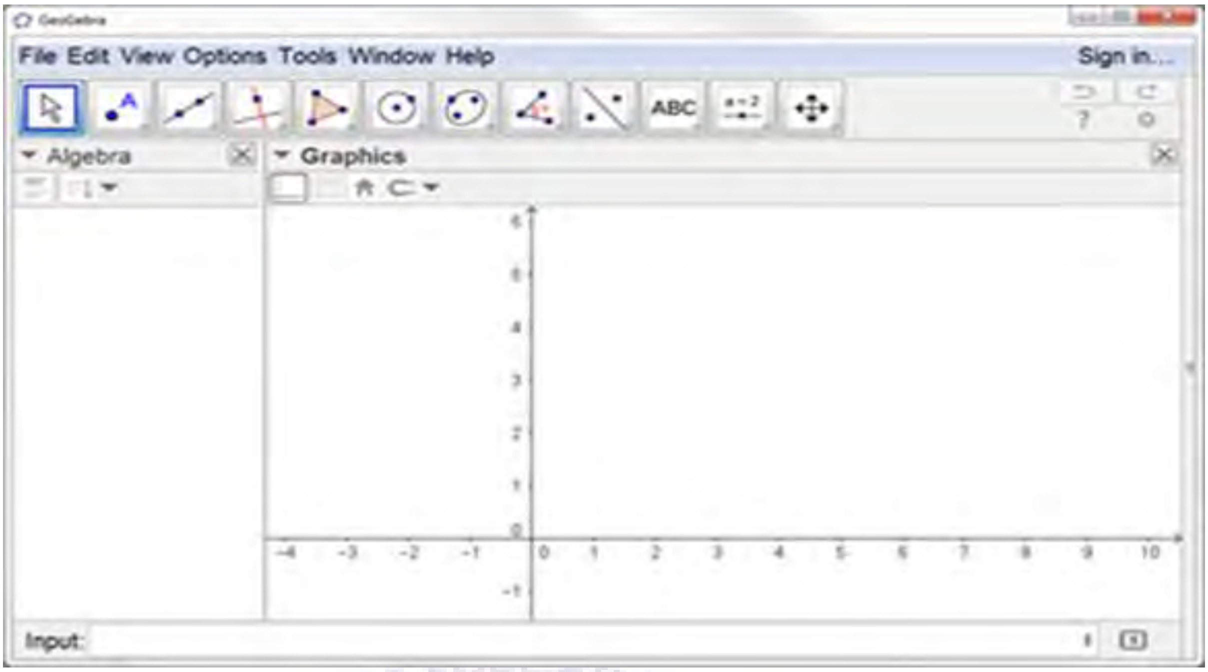

Figure 1: GeoGebra's user interface

2.8.2 Graphics window:

The graphics window is placed on the right hand side of the GeoGebra window. It contains a drawing pad on which the geometric representations of objects are displayed. The coordinate axes can be hidden and a coordinate grid can be displayed by the user. In the graphics window, existing objects can be modified directly by dragging them with the mouse, while new objects can be created using the dynamic geometry tools provided in the toolbar.

2.8.3 Toolbar:

The toolbar consists of a set of toolboxes in which GeoGebra‘s dynamic geometry tools are organized. Tools can be activated and applied by using the mouse in a very intuitive way. Both the name of the activated tool as well as the toolbar help, which is placed right next to the toolbar, give useful information on how to operate the corresponding tool and, therefore, how to create new objects. In the right corner of the toolbar the Undo and Redo buttons can be found, which enable the user to undo mistakes step-by-step.

2.8.4 Algebra window:

The algebra window is placed on the left hand side of the GeoGebra window. It contains the numeric and algebraic representations of objects which are organized into two groups:

- Free objects can be modified directly by the user and don't depend on any other objects.
- Dependant objects are the results of construction processes and depend on _parent objects'. Although they can't be modified directly, changing their parent objects influences the dependant objects. Additionally, the definition of a dependant object can be changed at any time.

Additionally, both types of objects can be defined as auxiliary objects, which means that they can be removed from the algebra window in order to keep the list of objects clearly arranged. Algebraic expressions can be changed directly in the algebra window, whereby different display formats are available (e.g. Cartesian and polar coordinates for points). If not needed, the algebra window can be hidden using the View menu.

2.8.5 Input field:

The input field is placed at the bottom of the GeoGebra window. It permits the input of algebraic expressions directly by using the keyboard. By this means awide range of pre-defined commands are available which can be applied to already existing objects in order to create new ones.

2.8.6 Menu bar:

The menubar is placed above the toolbar. It provides a wide range of menu items allowing the user to save, print, and export constructions, as well as to change default settings of the program, create custom tools, and customize the toolbar.

2.8.7 Construction protocol and Navigation bar:

Using the View menu, a dynamic construction protocol can be displayed in an additional window. It allows the user to redo a construction step-by-step by using the buttons of a navigation bar. This feature is very useful in terms of finding out how a construction was done or finding and fixing errors within a construction. The order of construction steps can be changedas long as this doesn't violate the relations between dependant objects. Furthermore, additional objects can be inserted at any position in order to change, extend, or enhance an already existing construction. Additionally, the Navigation bar for construction steps can be displayed at the bottom of the graphics window, allowing repetition of a construction without giving away the required construction steps ahead of time.

2.8.8 Why is GeoGebra different?

Currently, there are two types of educational software that connect the mathematical fields of geometry and algebra and are used for mathematics teaching and learning (Preiner, 2008). On the one hand, there is dynamic geometry software (DGS) that allows users to create and dynamically modify Euclidian constructions. Geometric properties and relations between objects used within a construction are maintained because manipulating an object also modifies dependant objects accordingly. Some dynamic geometry programs even provide basic algebraic features by displaying the equations of lines or conic sections, as well as other mathematical expressions which usually can't be modified directly by the user.

On the other hand, there are computer algebra systems (CAS) which symbolically perform algebra, analytic geometry, and calculus. Using equations of geometric objects, a computer algebra system can decide about their relative position to each other, and display their graphical representations. Many computer algebra systems are also able to plot explicit and sometimes even implicit equations. Generally, the geometric representation of objects cannot be directly modified by the user.

GeoGebra is an attempt to join these two types of software, whereby geometry, algebra, and calculus are treated as equal partners. The software offers two representations of every object: the numeric algebraic component shows either coordinates, an explicit or implicit equation, or an equation in parametric form, while the geometric component displays the corresponding solution set (Hohenwarter, 2002).

In GeoGebra both representations can be influenced directly by the user. On the one hand, the geometric representation can be modified by dragging it with the mouse, whereby the algebraic representation is changed dynamically. On the other hand, the algebraic representation can be changed using the keyboard causing GeoGebra to automatically adjust the related geometric representation.This new bidirectional dynamic connection between multiple representation of mathematical objects opens up a wide range of new application possibilities of dynamic mathematics software for teaching and learning mathematics while fostering student understanding of mathematical concepts in a way that was not possible several years ago.

2.9 Mathematics specific software packages and teaching geometry

Research on designing teaching scenarios based on various forms of geometry software, and of integrating them into the regular pattern of classroom teaching, shows that it can take quite a long time to reach the point where tasks genuinely take advantage of the computer environment (Christou, Jones, Mousoulides, & Pittalis, 2006). Such research indicates that geometry tasks selected for use in the classroom should, as far as possible, be chosen to be useful, interesting and/or surprising to pupils (Jones, 2011). In addition, it can be helpful if classroom tasks expect pupils to explain, justify or reason, and be critical of their own and their peers‘ explanations. Jones et al. (2009) identified the following framework of teaching approaches with geometry software as ways of providing professional development and support for teachers. The framework is represented by fig. 2.

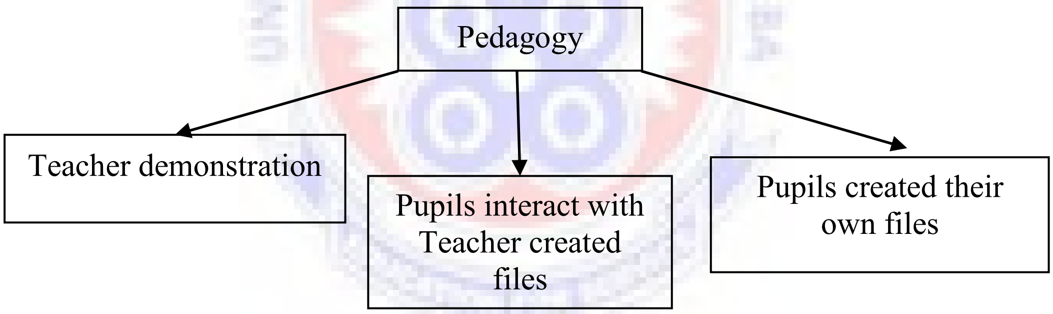

Figure 2: Framework of teaching approaches with geometry software

2.9.1 Teacher - demonstration approach

In the teacher-demonstration approach, the teacher engages students in discussing an on-screen geometric construction by, perhaps, asking questions about the objects on the screen in order to get the learners to explain what they might expect would happen if some parts of the configuration were moved or changed. In the project, this approach was found to allow teachers with little experience of using technology in the classroom to experiment with the technology with relatively small risk. In addition, this kind of use requires less change in the classroom setting and needs fewer resources than either organizing classes into a computer room or using a class set of laptops in the regular classroom.

2.9.2 Teacher - created files

The second approach entails teachers providing previously created interactive files for their learners. With such teacher-created files, students can experiment with dynamic objects. This provides clear boundaries for learners and time is not spent setting up the tasks; rather, learners can spend time exploring the mathematics that is central to each task. No doubt there is quite some teacher control over the material, but the approach can bring in opportunities for creative thinking and problem solving by learners.

2.9.3 Learner - created files

The third approach involves learners creating their own files, perhaps for other learners to tackle. This approach provides some learner ownership of the work and engages a different sense of problem solving (and problem posing) by creating that ownership. There is also the development of independence – in learning how to use the software, and with additional scope for student creativity and discovery.

2.10 Barriers to ICT integration

The act of integrating ICT into teaching and learning is a complex process and one that comes with a number of difficulties. Many of these challenges are related to costs or infrastructural and technical issues, such as lack of access to technology or poor connectivity (Kozma, 2011). This is particularly the case in low-income countries where the citizenry is struggling to make ends meet. According to Jones, as cited in Becta (2004), teachers feel reluctant to use computer if they lack confidence.

"Fear of failure" and "lack of ICT knowledge" have been found as some of the reasons for teachers' lack of confidence for adopting and integrating ICT into their teaching (Balanskat, Blamire, & Kefala, 2006). Similarly, in a survey conducted by Becta (2004), approximately 21% of the teachers who were surveyed, reported that lack of confidence influence their use of computers in their classrooms. Becta (2004), stated that "many teachers who do not consider themselves to be well skilled in using ICT feel anxious about using it in front of a class of children who perhaps know more than they do"(p.7). Becta (2004), grouped the barriers according to whether they relate to the individual (teacher-level barriers), such as lack of time, lack of confidence and resistance to change, or to the institution (school-level barriers), such as lack of effective training in solving technical problems and lack of access to resources.

To broaden the classification, Balanskat et al. (2006), divided the barriers into three levels; micro, meso and macro barriers. The micro level barriers include those related to teachers' attitudes and approach to ICT, the meso level barriers include those related to the institutional context while the macro (system-level barriers) include those related to the wider educational framework. Buabeng-Andoh (2012), classified the barriers as teacher-level, school-level and system-level factors that prevent teachers from ICT use. These barriers, according to Buabeng-Andoh (2012), include lack of teacher ICT skills; lack of teacher confidence; lack of pedagogical teacher training; lack of suitable educational software; limited access to ICT; rigid structure of traditional education systems; restrictive curricula.

Agyei and Voogt (2012), identified as major barriers to technology integration the current teaching strategies used in senior high schools, and lack of teachers' and pre-service teachers' knowledge of ways to integrate technology in instruction.

However, in 2011,UNESCO believes the main challenge, including in most advanced education systems, lies in teachers' capacities to use technology effectively in the classroom.

2.11 Problems/Difficulties in Learning Geometry

According to Strutchens, Harris, and Martin (2001), students learn geometry by memorizing geometric properties rather than by exploring and discovering the underlying properties. Geometry knowledge learned in this way is limited and superficial. For example, if students memorize that a square has four equal sides, they will be unable to distinguish between a square and a rhombus. Eventually these students find difficulty in applying that limited geometry knowledge in problem solving. This lack of understanding often discourages the students, invariably leading to poor performance in geometry tests.

A number of factors have been proposed to explain what makes geometry learning difficult. First, the geometry language, which involves specific terminology, is unique and needs particular attention and understanding before it can be used meaningfully. Misuse of geometry terminology can lead to misconceptions of geometric knowledge (Bishop, 1986). Next, geometry requires visualizing abilities but many students cannot visualize three-dimensional objects in a two-dimensional perspective. Visualizing cross- sections of solids is very difficult for students lacking ample prior concrete experiences with solid objects. Due to their limited geometric experiences, students may not have had enough opportunities to develop and exercise their spatial thinking skills for effective geometry learning.

Another problem is that traditional approaches of geometry instruction do not seem to help students achieve the intended learning outcomes in the curriculum. By using just textbooks and chalkboards, classroom geometry experiences hamper optimal learning.

There is an urgent need to change the traditional mode of geometry instruction to one that is more rewarding for both teachers and students. Specifically, learners must be given opportunities to personally investigate and discover geometry to enable understanding of the subject in depth and also in relation to other fields of Mathematics.

A pertinent problem with many geometry students is their weakness in the language of geometry (Bishop, 1986). The vocabulary in geometry is specific and carries meaning, descriptions and even properties. Knowing a geometric name like "triangles" and "squares" may not imply the student understands their exact meanings or their properties involving angle sums, perimeter or area. Noraini (1999), observed that some 13 and l4 year old Malaysian students were unable to explain simple terms like "perimeter" and "triangle". Words like "area, isosceles, scalene, and equilateral" gave rise to much confusion among her sampled subjects. Evidently, geometry language, especially in the comprehension of geometry terms, plays a very important role in learning and understanding of geometric concepts (Khoo & Clements, 2001). In a study on the van Hiele levels of thinking in geometry among sixth and ninth graders Fuys, Geddes, and Tischler (1988), found that the inability to advance in level of thinking may be related to students' deficiencies in language, both in knowledge of geometry vocabulary and ability to use it precisely and consistently.

Another problem of geometry learning involves the ability to visualize. Many concepts in geometry require students to visually perceive the objects and identify their properties by comparing them with their previous experiences involving similar objects. These geometrical concepts also require visual interpretations as many geometry problems are presented in a two-dimensional format on paper. Thus students who are unable to extract geometric information about three-dimensional solid objects drawn on paper will face difficulty in interpreting questions involving solid geometry.

Some Mathematics educators recommend more visual activities in the classroom to help students understand geometric concepts. It would, therefore, seem helpful for students if geometry lessons could be carried out with hands-on activities. By being able to "touch-see-and-do" and interacting with the objects of their learning, students can learn geometry in a more imaginative and successful way (Bishop, 1986).

In the past, geometry lessons were pictured as students copying diagrams and properties of figures and shapes from blackboards and doing repetitive exercises to calculate angles, lengths, and areas of geometric figures. This approach posed problems to both teachers and students, and both groups began to dread geometry. Teachers became frustrated because their poor conceptual understanding led to poor geometry achievement.

Even in many geometry classrooms in Ghana today, teachers introduce students to facts about Euclidean geometry and then drill them with concepts in deductive reasoning. Students are seldom given opportunity to discover and conceptualize geometry on their own.

However, for effective geometry instruction, the method should not be the same as in teaching number, algebra or probability. Instead instruction should emphasize hands-on explorations, developing geometric thinking and reasoning, making conjectures and even carrying out geometry projects (Strutchens, et al., 2001; van Hide, 1999).

2.12 The Senior High School (SHS) Curriculum

The Senior High School mathematics syllabus is designed to put great deal of emphases on the development and use of basic mathematical knowledge and skills (CRDD, 2010).

The major areas of content covered in all the SHS classes are as follows: Numbers and numeration, Plane Geometry, Mensuration, Algebra, Statistics and Probability, Trigonometry, Vectors and Transformation in a Plane and Problem solving and application (mathematical processes).

Transformation deals with rigid motion and enlargement including scale drawing and its application (CRDD, 2010). The syllabus deals with transformation in two parts. In the first year (form one), it is named rigid motion I and in the second year (form two), is rigid motion II and enlargement. For rigid motion I, the following specific objectives are to be achieved.

The student will be able to:

1.11.1 identify and translate an object or point by a translating vector and describe the image;

1.11.2 identify and explain the reflection of an object in a mirror line;

1.11.3 describe the image points of shapes in a reflection.

For rigid motion II, the following objectives are to be achieved.

The student will be able to:

2.13.1 identify shapes with rotational symmetry;

2.13.2 identify the image of an object (or point) after a rotation about the origin (or point).

The syllabus also requires students to as a form of evaluation identify some Ghanaian (or adinkrah) symbols that have rotational symmetry and state the order of rotational symmetry.

2.12.1 Meaning of rigid motions

Rigid Motions: motions that do not change the size of the object, but rather the object‘s orientation or position in space. In rigid motion, there is pre-image (object or point that undergoes motion) and image (resultant of the motion). That is, if a point *P* has been transformed into a point P^1 (*read P prime*)**,** then *P* is called the pre-image or object and the point P^1 is called the image.

2.12.2 Types of rigid motion

a. Translations: moving every point a constant distance in a specified direction.
b. Reflections: produces a mirror image in the axis or line of reflection.
c. Rotations: a transformation in space that describes the motion of a rigid body around a fixed point.
d. Combinations of translations, rotations, and/or reflections.

Two or three motions can be combined. Combination of motions produce the same image if done in the opposite order.

i. Combination of 2 **motions**

A. Translation and Rotation
B. Translation and Reflection
C. Reflection and Rotation

ii. Combination of 3 **motions**

Combinations of 3 rigid motions can be done in 6 diferent ways **(**3! = 6).

A. Translation, Rotation, Reflection

B. Translation, Reflection, Rotation

C. Reflection, Translation, Rotation

D. Reflection, Rotation, Translation

E. Rotation, Reflection, Translation

F. Rotation, Translation, Reflection

2.12.3 Non – rigid motion

Non-rigid Motions: motions that change the size of the object while preserving dimensions (i.e. no warping), but not the orientation in space.

Dilations: a transformation that produces an image that is the same shape as the original, but is a different size. It stretches or shrinks the original figure. The pre-image and image are in direct proportion by a proportionality constant. Transformation therefore combines rigid and non rigid motions.

2.13 Summary

Teaching and learning of geometry in general and rigid motion in particular requires the combinations of approaches for effective formation of concepts on the part of students. Literature reviewed so far have focussed on a theoretical framework underlining geometry, history and importance of studying geometry, history and relevance of GeoGebra to teaching and learning geometry. Some approaches that are relevant to the teaching and learning of geometry in general and geometrical transformations in particular have also been reviewed. Approaches of teaching geometric transformations that were reviewed in the literature included traditional approach, ICT integration approach, problem solving approach, discovery approach, investigative approach, and inductive teaching approach. All these approaches have different modes of application depending on the needs and levels of students involve in the lesson. They also come with advantages and disadvantages. It is, therefore, necessary that teachers weigh the advantages and disadvantages of each and select the most appropriate one for teaching and learning of geometric transformations concepts.

ICT integration approach has been reviewed extensively to highlight different techniques that are available for mathematics instructors to use to enhance the teaching and learning of geometry. The teacher-demonstration technique, when used with GeoGebra for instance, could allow students to discuss various lines of reflection by explaining what they expect to happen if the orientation of the lines were changed. ICT integration, however, has some challenges embedded in its application. It is, therefore, necessary for teachers to do a lot of research and practice to effectively deliver mathematical concept using ICT.

The structure of the Senior High school syllabus use in Ghana is spiral in nature and as such, learners' understanding in a particular topical area invariably enhances the understanding of other topical areas as well. It is, therefore, the responsibility of all teachers to spend time in developing understanding of their learners in topics of the mathematics syllabus which have linkages to other topics in the syllabus through the other approaches that have been reviewed. In helping students develop geometrical concepts, consideration must be given to level and ability of the students together with selection of appropriate approaches required. The right choices of approaches by the teacher which are favourable to the students to a larger extent go a long way to help students form the expected concepts.

CHAPTER THREE

METHODOLOGY

3.1 Overview

This chapter discusses the research design, population, sample and sampling procedure, research instruments, data collection and data analysis procedures.

3.2 Research Design

A quasi experimental design with both qualitative and quantitative methods was used. Quasi experimental approach was used because two intact classes were used and it took place in a real - life setting as opposed to a laboratory (Vanderstoep & Johnston, 2009). Also, quasi-experiments are based on creative design techniques to reduce the various threats that may cause a study‘s findings to be invalid or unreliable (Green, Camilli, & Elmore, 2006). The researcher used non-equivalent control group design with pre-test and post-test which is one of the most commonly used quasi-experimental designs in educational research. This is often the case since students are naturally organized in groups as classes within schools and are considered to share similar characteristics (Best & Kahn, 2006). The non-equivalent control group design with pre-test and post-test is represented as:

Non-equivalent Pre-test-Post-test Control Group Design

Group	Pre-test	Treatment	Post-test
A →	O_1 →	X →	O_2
B →	O_1 →	→	O_2

Time →

In this design, A represents the experimental group, B represents the control group, O_1 represents pre-tests, X represents the treatment implemented, and O_2

represents post-tests (Cohen et al., 2007). So while both the control and treatment group complete a pre-test and post-test, the treatment group is the only group that receives the research treatment. The researcher depended on participants‘ pre-tests to be certain that any selection biases are controlled. The pre-test scores were used to put the students into the experimental and control groups. It was done using a matched - groups design, so that participants are placed into groups based on a pre-existing characteristic (Vanderstoep & Johnston, 2009) which is their pre-tests. While pre-tests may be an option in the design of true-experiments, pre-tests are essential in the design of quasi- experiments (Slavin, 2007).

The researcher used both qualitative and quantitative methods to collect data. Qualitative method is used to understand, in depth, the viewpoint of research participants concerning the difficulties they face doing a task since qualitative research is more descriptive than predictive (Vanderstoep & Johnston, 2009). Quantitative method quantifies data and converts it to numerical scores and allows for generalization and answering of research questions (Fraenkel & Wallen, 2009).

3.3 Population

The target population for the study was made up of Senior High School Mathematics teachers and students in the government assisted Senior High Schools (SHS) in Ho Township. The researcher selected this region because of his familiarity with the academic environment and challenges of the region. The researcher has been teaching in this region for the past six years. There are three government assisted Senior High Schools in Ho Township. They are pseudonymously represented as school ‗A‘, school ‗B‘ and school ‗C‘. School ‗A‘ and ‗B‘ are single sex schools (all female) while School ‗C‘ is made up of both boys and girls (mixed). Since the problem is pervasive more with girls, the focus was on School A because that was where the researcher originally identified the problem.

School ‗A' runs programs in General arts, General science, Business, Agricultural science and Visual arts. The total enrolment of the school is 1,891. This is made up of 694 first year students, 719 second year students, and 478 third year students. There are also 13 teachers in the Mathematics department of the school. The population is clearly unreachable considering the limited time within which the study should be completed. The researcher therefore, used the second year group and the accessible population was made up of 100 General arts students from two classes (2B and 2D) and four mathematics teachers

3.4 Sample and Sampling technique

3.4.1 Sample

Students of General arts 2A and 2D classes, and four Mathematics teachers at Senior High School ‗A' in Ho formed the sample. Senior High School A was chosen because of its familiarity and accessibility to the researcher. Simple random sampling was used to select the two intact classes involved. Pre-test was conducted for all the students in the two selected classes and the result used to put them into experimental and control groups. The sample was made up of 100 students and 4 Mathematics teachers. The control group was made up of 50 students and the experimental group comprised 50 students.

3.4.2 Sampling technique

The researcher adopted simple random sampling technique to select the students to form the sample. Convenient sampling technique was employed to select four teachers to be part of the study. Convenient sampling technique was used because teachers selected should be accessible throughout the duration of the study and should be willing to help carry out the research.

3.5 Data Collection Instruments

Considering the mixed method approach of quantitative and qualitative nature of this study, the research was conducted using observation guide, achievement tests (pre-test and post-test) and a questionnaire.

3.5.1 Observation Guide

Observation guide was prepared and used during instructional periods to collect information on the difficulties students face in doing rigid motion. The observation guide was made up of two main columns (see Appendix F). Column 1 was made up of the sections for the instructional periods and column 2 was for recording misconceptions/difficulties associated with doing rigid motion. Column 2 was sub-divided into four columns to take care of each of the four groups. The difficulties were recorded under the groups that experienced them.

3.5.2 Achievement test

A teacher/researcher-made geometry achievement test was prepared and used to investigate second year students‘ performance in geometry, specifically on rigid motion. The test items were based on all aspects of rigid motion in the JHS and SHS syllabi. The pre-test was in four sections. Question one of Section A was made up of six sub-items on reflection. In this section, the pre-image and the image were drawn on grid paper and students were required to describe fully equation of line of reflection involved. There were two other questions under this section that required students to find the image point of given object point (see Appendix A).

Section B was made up of six sub-items on reflection. In this section, the pre-image and the image were drawn on grid paper and students were required to determine the translation vector involved. There was one other question under this section that required students to find the image point of given object point under a given translation vector.

Section C was made up of six sub-items on rotation. In this section, the pre-image and the image were drawn on grid paper and students were required to describe fully the rotation by stating the angle of rotation involved. There were two other questions under this section that required students to find the image point of given object point under given angle of rotation about the origin and a point other than the origin respectively.

Section D was made up of four sub-items on composition of transformation. In this section, the pre-image and the image were drawn on grid paper and students were required to describe fully the type of transformation involved. The post-test followed the same format and design of the pre-test.

3.5.3 Questionnaire

The questionnaire was made up of three parts (see Appendix E). The first part was made up of six items that were both open and close ended. The second part of the questionnaire investigated Mathematics teachers' background knowledge and experience in teaching rigid motion over the years. There were six items covering this part as well. The last part of the questionnaire made up of thirteen items was on how effective teachers found the use of GeoGebra to teach rigid motion.

3.6 Data collection procedure

Observation was used to collect data on areas where students were facing difficulties in rigid motion. Observation guide was given to each of the teachers in charge of the groups to record their observations. The teachers had two different lessons with the students a week for three weeks to teach the experimental group using GeoGebra. The same procedure was used to teach the control group using the traditional approach. Each lesson was for a duration of one hour. The teachers carried out their observations during each of the six lessons for both groups. Each specific difficulty was recorded under the group that experienced it. Out of the four groups, the number that experienced a particular problem was recorded and analysed.

Pre-test was also used together with observation to collect data on areas where students were facing difficulties in rigid motion. Post-test was used to determine how effective the use of GeoGebra application software enhance students‘ achievement in rigid motion.

Test 1 which was made up of three sections (A-D) altogether was administered to the students before the intervention activities as a diagnostic test (pre-test) while a parallel test 2 was administered to them as post-test after the intervention activities to determine the effectiveness of the intervention.

The questions required students to demonstrate their knowledge of reflection, translation rotation and a combination of them. Each student was given a printed question paper with spaces provided for the correct answers to be supplied. See Appendix A and B for the pre-test and post-test respectively. Answers of students to the pre-test questions were marked using a marking scheme that the researcher prepared. The researcher examined wrong answers given by the students to find out the possible misconceptions they held about rigid motion. Answers of students to the post-test questions were also treated the same as the pre-test questions. See Appendix C and D for marking scheme of pre-test and post-test respectively.

The researcher used a questionnaire guide to collect information from the teachers involved in the implementation of the intervention. The first part which was made up of six items basically sought to collect information about gender, age, teaching experience, university attended, major area of study, second area of study (minor) and area being taught (core and/or elective mathematics). The second part of the questionnaire investigated Mathematics teachers‘ background knowledge and experience in teaching rigid motion over the years. There were six items covering this part as well. The last part of the questionnaire made up of thirteen items was on how effective teachers found the use of GeoGebra to teach rigid motion. Twelve of the questions contained items on a five-point Likert scale (1 = strongly agree, 5 = strongly disagree) and the thirteenth question was an open ended question about the teachers‘ perception on the effectiveness of technology use in teaching rigid motion. Respondents scoring less than 3.0 on the scale were labelled as having agreed or strongly agreed to the statement in question and those scoring above 3.0 were labelled as having disagreed or strongly disagreed to the statement. Respondents scoring 3.0 on the scale were labelled as being undecided.

A Likert scale provides a range of responses to a given question or statement, and there should be unidimensionality in the scale; the scale should be measuring only one thing at a time (Oppenheim, 2007) . The data obtained from the Likert–type rating scale was computed to find the mean score for each item with regards to the use of GeoGebra to teach rigid motion. The questionnaire was administered after the intervention.

3.7 Intervention

Students in the experimental group were put into four groups. Each group chose its secretary and chairperson. The groups were guided to carry out selected activities by one of the four selected teachers. The teaching strategy used was the discovery approach. After each activity the four groups came together and engaged in discussions based on the activity. Students in each group discussed and agreed on their findings after which each group shared its findings with the whole class during which corrections and inputs were made to fine tune the findings under the supervision of the teachers. The researcher acted as the resource person during the discussion and was always available to offer explanations and clarifications to the whole class or any group needing assistance.

3.7.1 Learning by discovery using GeoGebra

The students were made to explore shapes using GeoGebra to carry out reflections, translations and rotations. They made sketches of their explorations on grid paper (graph sheet), study the relationship between the pre-images and the images to practically discover the rules. The discovery approach is considered as a situation where learners practically manipulate geometric figures to identify their properties as well as using the Cartesian plane to determine images from pre- image (Sarra, 1999). According to Strutchens, Harris, and Martin (2001), students learn geometry by memorizing geometric properties rather than by exploring and discovering the underlying properties. The following activities were used.

Reflections

(a) Using "Polygon" tool, make a polygon. Can be triangle or quadrilateral. The polygon is completed when you click on the original point.

(b) Construct a line of reflection ($x = 0$) using the "Line" tool.

(c) Click "Reflect object" tool, select object to be reflected, and the line about which it is to be reflected.

(d) Locate the image polygon.

(e) Using convenient scale, draw the polygon and its image on the graph sheet. Study the object points and the image points to discover the rule that maps the object points to the image points.

(f) Repeat the steps in a, b, c, d, e, using the lines $y = 0, x = y, x = -y, x = a, y = a$.

Translation

(a) Using "Polygon" tool, make a polygon. Can be triangle or quadrilateral. The polygon is completed when you click on the original point.

(b) Create a "Vector between two points" along which translation will occur.

(c) Click "Translate object by vector", select the polygon, then the vector along which it will be translated.

(d) Play with the vector length/direction or the polygon using the selection tool.

(e) Locate the image polygon.

(f) Using convenient scale, draw the polygon and its image on the graph sheet. Study the object points and the image points to discover the rule that maps the object points to the image points.

(g) Repeat the steps in a, b, c, d, e, f using two other vectors.

Rotation

(a) Using "Polygon" tool, make a polygon. Can be triangle or quadrilateral. The polygon is completed when you click on the original point.

(b) Select "Rotate" tool, choose a point of rotation (origin), and an angle (90^0)

(c) Play with the point of rotation (inside/outside the polygon, vertices) and the angle.

(d) Locate the image polygon.

(e) Using convenient scale, draw the polygon and its image on the graph sheet. Study the object points and the image points to discover the rule that maps the object points to the image points.

(f) Repeat the steps in a, b, c, d, e, two other points of rotation and the angles 180^0 and 270^0.

3.8 GeoGebra Introductory Workshop

A two day workshop was organised to take the four selected teachers through the application of GeoGebra interface. The activities below were used to introduce GeoGebra to the teachers.

3.8.1 Introduction

Rigid motion is motion that does not change the size of the object. It only changes the orientation of the object in space. Pre - image is the object that will undergo motion and the image is what is formed after the pre - image has undergone rigid motion. Rigid motion could be taught using a variety of tools. One of such tools is GeoGebra.

GeoGebra has two main windows; algebraic and graphic windows. The algebraic window is placed on the left hand side while the graphic window is placed on the right hand side. The graphic window contains a drawing pad on which geometric representation of objects are displayed. Both the coordinate axes and the coordinate grid can be hidden or displayed by the user. In the graphics window, existing objects can be modified directly by dragging them with the mouse, while new objects can be created using the dynamic geometry tools provided in the toolbar. The toolbar consists of toolboxes in which GeoGebra dynamic geometry tools are organised (Preiner, 2008). The toolbox for transformation has a drop menu comprising 'Reflect about line', 'Reflect about point', 'Reflect about circle', 'Rotate around point', 'Translate by vector' and 'Dilate from point'.

3.8.2 Activities

The teachers were taken through selected activities that prepared them to guide the students (see Appendix G). They were made to explore shapes using GeoGebra to carry out reflections, translations and rotations. They made sketches of their explorations on grid paper (graph sheet), study the relationship between the pre-images and the images to practically discover the rules. The discovery approach is considered as a situation where learners practically manipulate geometric figures to identify their properties as well as using the Cartesian plane to determine images from pre- image (Sarra, 1999). According to Strutchens, Harris, and Martin (2001), students learn geometry by memorizing geometric properties rather than by exploring and discovering the underlying properties.

Figure 3 shows translation of a triangle by a given translation vector using GeoGebra interface.

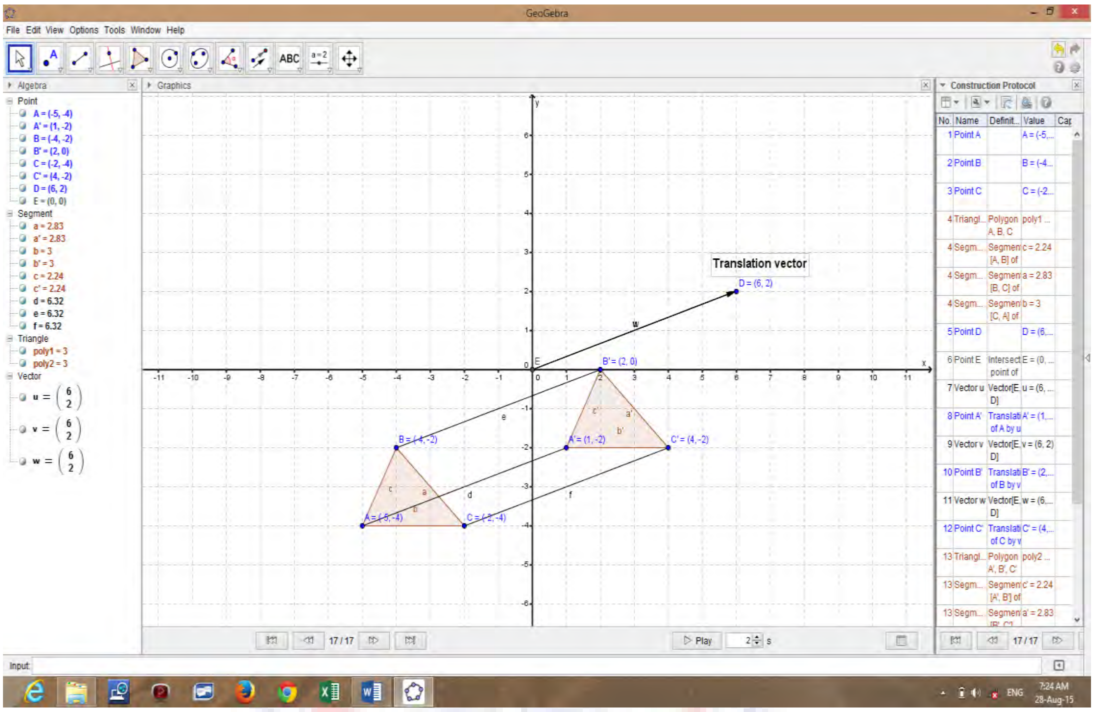

Figure 3: Translation by a vector **Created with GeoGebra**

In figure 3, triangle ABC was translated by the translation vector $\begin{pmatrix} 6 \\ 2 \end{pmatrix}$ to the image triangle $A^1B^1C^1$. The teachers discovered that to translate a point, one need to count units horizontally and vertically respectively to arrive at the image point.

Figure 4 shows a reflection in the line $y = -x$ using GeoGebra interface.

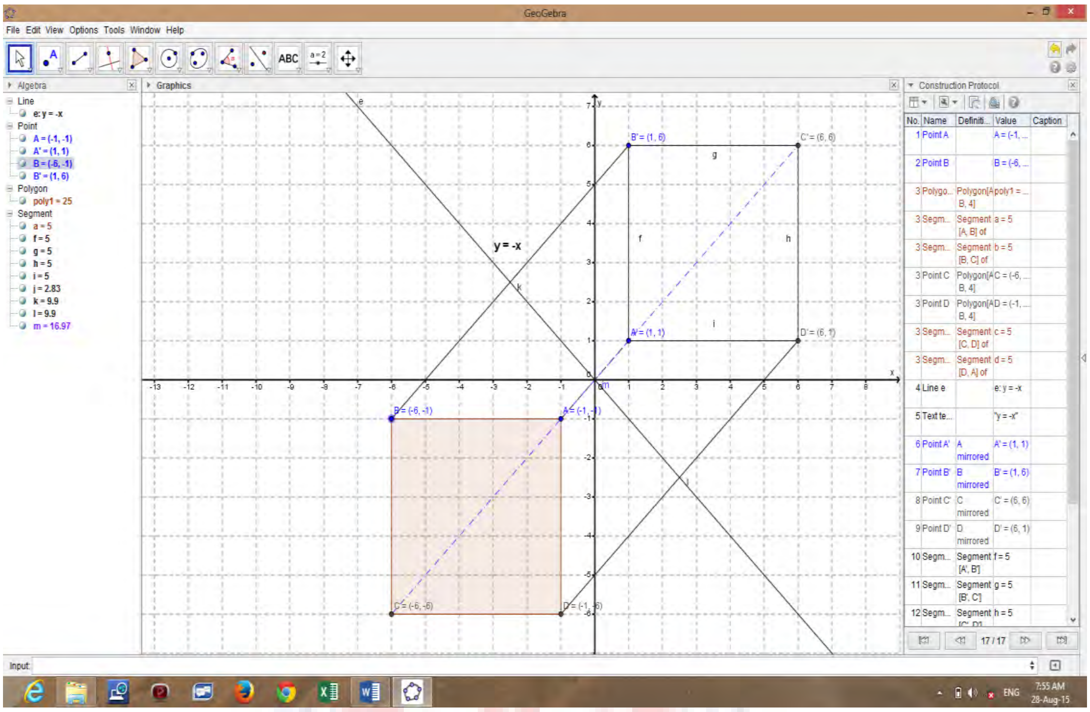

Figure 4: Reflection about the line *y* = - *x* **Created with <u>GeoGebra</u>**

Figure 4 shows the reflection of the square ABCD in the line $y = -x$ to form the image square $A^1B^1C^1D^1$. The teachers discovered that to reflect a point, one need to draw lines perpendicular to the mirror line and away from it equal distances to arrive at the image.

Figure 5 shows rotation about the origin through 90^0 counter clockwise.

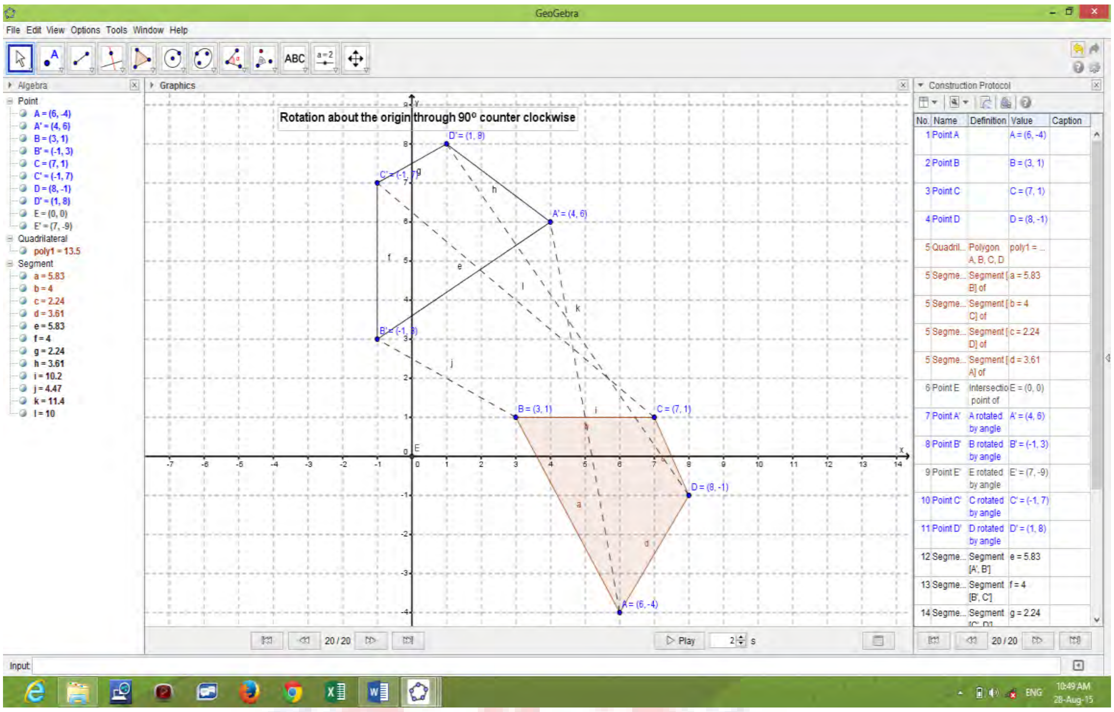

Figure 5: Rotation about the origin through 90o counter clockwise.

Figure 5 shows the rotation of the quadrilateral ABCD through 90^0 counter clockwise about the origin to form the image quadrilateral $A^1B^1C^1D^1$. The teachers discovered that to rotate a point counter clockwise, one need to draw an arc counter clockwise using the origin as the centre and then use the protractor to measure the given angle. The point of intersection of the arc and a line showing the size of the angle is the image point.

Figure 6 shows rotation about the origin through 90^0 counter clockwise.

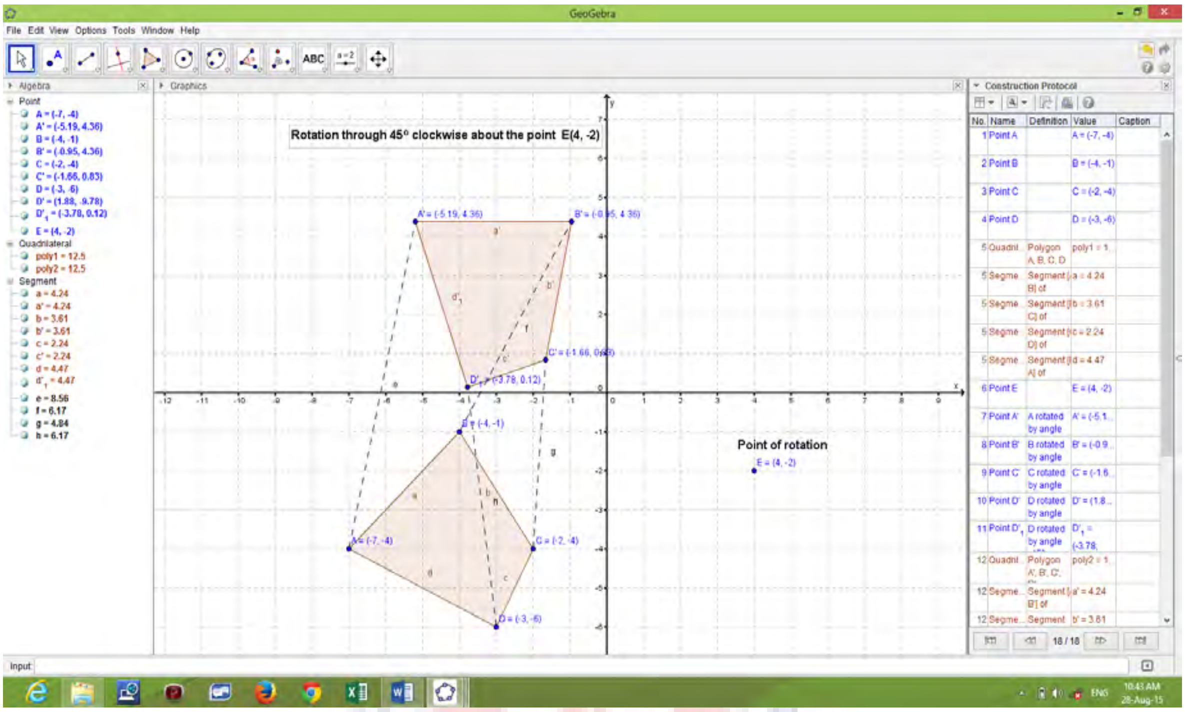

Figure 6: Rotation about (4, - 2) through 45^0 counter clockwise.

Figure 6 shows the rotation of the quadrilateral ABCD through 45^0 counter clockwise about $(4,-2)$ to form the image quadrilateral $A^1B^1C^1D^1$. The teachers discovered that to rotate a point counter clockwise, one need to draw an arc counter clockwise using the given point as the centre and then use the protractor to measure the given angle. The point of intersection of the arc and a line showing the size of the angle is the image point.

3.9 Validity and Reliability

In order to validate the research instruments, the researcher consulted the curriculum as well as some prescribed Mathematics textbooks for SHS students. The purpose was to gain insight into what learners are expected to learn in order to develop the instruments accordingly. The tests were validated by having colleague teachers review the items with respect to course objectives stated in the syllabus.

After constructing the test items, the researcher consulted his supervisor to cross check them for content and construct validity. The researcher carried out piloting of the instruments in a sister school (School B) to help refine these instruments.

In this study, the split-half method was used to check the reliability of the instruments. The spilt-half method requires the construction of a single test consisting of a number of items. These items are then divided or split into two parallel halves (usually, making use of the even-odd item criterion) and compared.

3.10 Data Analysis

The data collected through observation and questionnaire were analysed using descriptive statistics in the form of simple percentages, frequency distribution, charts, and measures of central tendency.

Data from students' marks obtained in the pre-test and post-test on rigid motion were analysed using independent samples t-test statistic. Unlike paired sample t-tests which compare means where the two groups are related, as in before-after, repeated measures, matched-pairs, or case-control studies, independent sample t-tests are used to compare the means of two independently sampled groups (Triola, 2004). It also compares mean scores from a study in which each participant receives only one level of the independent variable (Vanderstoep & Johnson, 2009).

CHAPTER FOUR

RESULTS AND DISCUSSION

4.1 Overview

The purpose of this study was to help Senior High School students build strong and firm content knowledge in rigid motion through the incoporation of GeoGebra. In this chapter, findings from the study are presented and discussed in relation to the three research questions. The discussions of these research questions were based on qualitative and quantitative analysis of data obtained from observations, questionnares and achievement tests (both pre and post tests). The discussions focused on:

1. Demographic information about sampled SHS teachers;
2. Findings related to the research questions and hypotheses;
3. Discussion of findings

4.2 Demographic Information about Sampled SHS Teachers

Information about the demographic background of SHS teachers who were sampled for the purpose of this study covered a wide range of characteristics such as their gender status, age, university attended, major and minor subjects of study, teaching experience, area of Mathematics being taught by the teachers. The coverage of all these areas was to ascertain if they contributed to SHS students‘ difficulties in learning rigid motion at the SHS level.

While some research findings by Kay (2006), revealed that male teachers use more ICT in their teaching and learning processes than their female counterparts, UNESCO in 2009 believed that program objectives are more likely to be reached if interest and experiences of both women and men are taken into account at all stages of a programming process. Two male and two female teachers were used in this study to reduce the gender gap and to give equal opportunity to both sexes. There is still a raging debate of whether experience influences a successful use of ICT. While Niederhauser and Stoddart (2001) reported that teachers' experience in teaching did not influence their use of computer technology in teaching, other research works showed that teaching experience influence the successful use of ICT in classrooms. Gorder (2008) reported that teacher experience is significantly correlated with the actual use of technology. All the teachers who answered the questionnaire claimed to have more than 9 years teaching experience. This immense teaching experience probably might be the reason why they quickly and easily learnt the use of GeoGebra to teach rigid motion at the preparatory workshop.

Among the four teachers, one attended University of cape coast (UCC) and offered statistics as minor and mathematics as major subjects. The rest of the teachers attended University of Education, Winneba (UEW). These teachers offered ICT as minor subject and Mathematics as major. This presupposes that the teachers who attended UEW had some level of computer literacy before the start of the workshop. Their knowledge in ICT was highly valuable in the intervention process.

All four teachers said they were teaching both core and elective Mathematics. Rigid motion which is a topic in core mathematics is applied in elective Mathematics when doing linear transformation under matrices. Therefore, a teacher who is teaching elective Mathematics needs to have a sound knowledge of rigid motion in order to effectively teach linear transformation. With the exception of one teacher who had been teaching at SHS A for two years, the rest of the teachers had been teaching in the school for about 3 to 4 years.

Figure 7 shows the number of times the selected teachers taught rigid motion in their current school.

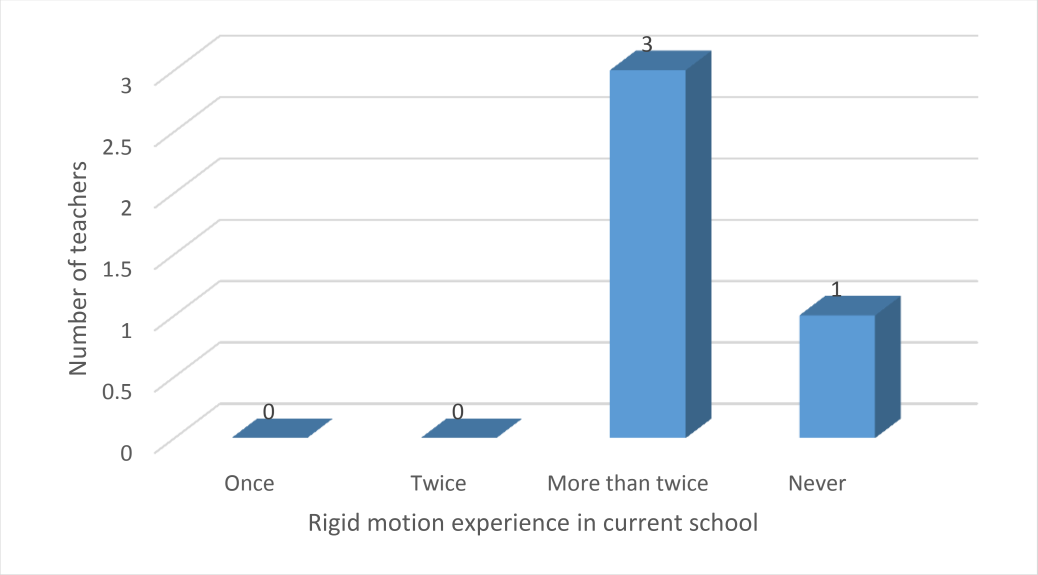

Figure 7: Number of times of teaching rigid motion in current school

Figure 7 revealed that three of the four teachers had taught rigid motion more than twice in the current school (School _A') but one teacher had never taught rigid motion in the current school.

All the teachers said the common teaching strategy they normally use in teaching rigid motion was to guide students to use graph sheets practically to discover formulae. This means they have never used Mathematics specific computer software to teach rigid motion. It therefore implied that Mathematics teachers do not incorporate ICT in their teaching. This is consistent with the findings of Agyei and Voogt (2012), that Mathematics teachers do not integrate technology in their instruction in spite of government efforts in the procurement of computers and recent establishment of computer laboratories in most Senior High Schools.

4.3 Research question 1:

What difficulties do senior high school students have in understanding rigid motion?

Research question one sought to identify difficulties Senior High School students have when answering questions pertaining to rigid motion. The difficulties were identified from students�250 answers to the pre-test questions and also observations made during the intervention.

4.3.1 Results of research question 1

Difficulties identified from responses to pre-test items

The difficulties students have in carrying out rigid motion were many and varied depending on which type of rigid motion they were confronted with. A detailed diagnosis and careful analysis of students‘ answers to the pretest questions in each section revealed four major difficulties relating to their understanding of rigid motion. These include wrong interpretation of lines of reflection, poor determination of translation vectors, wrong judgement on the angle and center of rotation and misinterpretation of multiple transformations

Lines of reflection

On reflection, students basically had difficulties locating the lines of reflection practically. Section A of the pre-test demanded from students to describe fully the type of reflection involved by providing the line of reflection in each case (See Appendix A). There were six sub questions under question one in section _A‘ that sought to test students‘ knowledge in determining lines of reflection.

Table 4.1 shows the pre-test results of both the control and experimental groups on reflection.

Table 4.1: Pre-test results of control and experimental groups on reflection

	CG Wrong Answer % (n)	EG Wrong Answer % (n)	CG Correct Answer % (n)	EG Correct Answer % (n)	CG No attempt %(n)	EG No attempt %(n)
Q1 (a)	18(9)	22(11)	78(39)	74(37)	4(2)	4(2)
Q1 (b)	64(32)	74(37)	2(1)	6(3)	34(17)	20(10)
Q1(c)	64(32)	72(36)	2(1)	8(4)	34(17)	20(10)
Q1 (d)	28(14)	56(28)	28(14)	30(15)	30(15)	14(7)
Q1 (e)	60(30)	66(33)	2(1)	2(1)	38(19)	32(16)
Q1 (f)	62(31)	74(37)	2(1)	2(1)	36(18)	24(12)

Results from Table 4.1 indicated that apart from the first sub question in the pre-test which recorded 78 % (39) correct answers for the control group and 74 % (37) for the experimental group, the percentage of correct answers for the rest of the sub questions were abysmally low. It ranges from as low as 2 % (1) to 28 % (14) for the control group and 2 % (1) to 30 % (15) for the experimental group. The first sub question recorded percentages above the average of 50% because the line of reflection involved was the commonly used $y-axis$. The fifth sub question of question one in section ‗A' which involved the equation of the line $y=\frac{1}{2}$ was answered wrongly by a large number of students, 30 students representing 60 % in the control group and 33 students representing 66 % in the experimental group.

These results showed that majority of the students did not have the concept of equations of straight line. This could be seen on students' answer scripts in the pre-test where they were not able to identify the line of reflection involved in most cases.

Table 4.2 provides a summary statistics on students‘ responses to a question asking them to find the image points of given object points when given the line of reflection.

Table 4.1: Frequencies and percentages of students' responses to finding the image points of given object points when given the line of reflection.

Responses to Q3	CG Freq.	CG %	EG Freq.	EG %
Wrong	16	32	21	42
Correct	4	8	2	4
No attempt	30	60	27	54
Total	50	100	50	100

The second and third questions under the same section ‗A‘ sought to test students‘ ability to find the image point of given object points when given the line of reflection. The difficulty students had in answering these questions included wrong substitution of the constant **k** in the formula some of the students quoted as $\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} 2k-x \\ y \end{pmatrix}$ for the line $x+3=0$. The value of **k** was substituted as 3 instead of -3 This accounted for why 32 % (16) in the control group and 42 % (21) in the experimental group got question three wrong as depicted in Table 4.2. Meanwhile, a whopping 60% in the control group and 54 % among the experimental group did not attempt this particular question and this is very much worrying.

Determination of translation vectors

Question 4 under Section B was on translation (See Appendix A). The questions required students to identify the translation vectors involved in translating one object onto the other.

Table 4.3 shows the number of students who responded to the question rightly or wrongly in both groups. It also shows the number of students who did not attempt the question.

Table 4.2: Pre-test results of control and experimental group on translation

	CG	EG	CG	EG	CG	EG
	Wrong Answer % (n)	Wrong Answer % (n)	Correct Answer % (n)	Correct Answer % (n)	No attempt %(n)	No attempt %(n)
Q4 (a)	22(11)	22(11)	0(0)	2(1)	20(10)	76(38)
Q4 (b)	20(10)	18(9)	0(0)	0(0)	26(13)	82(41)
Q4(c)	12(24)	18(9)	0(0)	0(0)	28(14)	82(41)
Q4 (d)	20(10)	12(6)	0(0)	0(0)	30(15)	88(44)
Q4 (e)	18(9)	16(8)	0(0)	0(0)	22(11)	84(42)
Q4 (f)	20(10)	8(4)	0(0)	2(1)	22(11)	90(45)

In Table 4.3, it was revealed that only sub question one and six of question four in section ‗B' recorded 2% (1) each while the rest of the sub questions recorded 0% (0) in the experimental group. On the part of the control group, no student had any of the six sub questions right. What was more revealing was the high number of students ranging from 76 % (38) to 90 % (45) who did not attempt these questions in both groups.

The difficulties students had with the translation was that students did not demonstrate any practical knowledge of this concept. They used formulae which they could not handle effectively. They attempted to substitute object points and the image points into the formula $\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} x+a \\ y+b \end{pmatrix}$ but, while some could not substitute the values correctly, others could not make the translation vector $\begin{pmatrix} a \\ b \end{pmatrix}$ the subject of the relation. Figure 8 shows the summary statistics of the number of students who provided right or wrong answers to the question that asked them to find the image of a point when the translation vector is given.

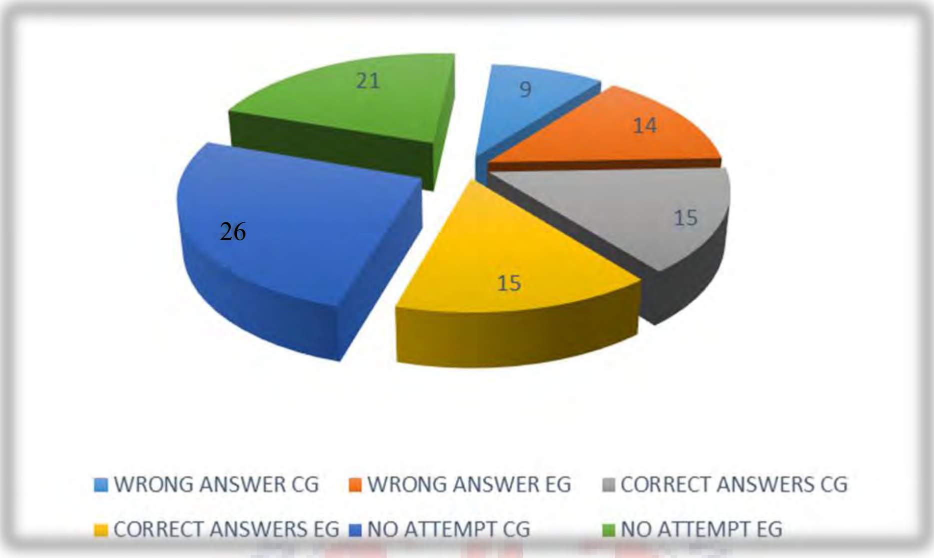

Figure 8: Responses of senior high school students to translation in both experimental and control groups

Data from figure 8 indicated that, for question 5, slightly more students recorded correct answers 30 % (15) than wrong answers 28 % (14) within the experimental group and 30 % (15) right answers than wrong answers 18 % (9) in the control group. This can be attributable to the simple addition of the translation vector and the object point involved. Yet, a staggering 42 % (21) in the experimental group and 52 % (26) in the control group did not attempt this question.

Angles and Centre of rotation

The difficulties students had in rotation seemed to be deep rooted, especially in the experimental group. This is because most of the students did not attempt the questions. There were six sub questions under question 6 of section ‗C' (See Appendix A).

Table 4.4 shows the summary statistics of the number of students who provided right or wrong answers to the six sub-questions that asked them to find the angle of rotation in each case. The pre-images and the images were provided.

Table.4. 3: Pre-test results of control and experimental group on rotation

	CG Wrong Answers % (n)	EG Wrong Answers % (n)	CG Correct Answers % (n)	EG Correct Answers % (n)	CG No attempt %(n)	EG No attempt %(n)
Q6 (a)	34(17)	30(15)	0(0)	0(0)	66(33)	70(35)
Q6 (b)	34(17)	26(13)	0(0)	0(0)	66(33)	74(37)
Q6 (c)	32(16)	26(13)	0(0)	0(0)	68(34)	74(37)
Q6 (d)	24(12)	24(12)	2(1)	0(0)	74(37)	76(38)
Q6 (e)	24(12)	20(10)	2(1)	0(0)	74(37)	80(40)
Q6 (f)	24(12)	20(10)	0(0)	0(0)	76(38)	80(40)

It was surprising to note from Table 4.4 that none of the students in the experimental group got any of the six sub questions correct while majority of the students ranging from 70 % (35) to 80 % (40) in the experimental group and 66 % (33) to 74 % (37) in the control group did not even attempt to answer them. The number of students in the control group who answered these questions correctly were still very low, 2 % (1). Further analysis of the wrong answers of students revealed their difficulty in the interpretation of clockwise and anticlockwise rotation. For instance, $+90^0$ was interpreted as clockwise 90^0 instead of anticlockwise 90^0.

Another difficulty discovered was that students had no idea on how to perform rotation practically without the recall of formulae, so formulae were memorized but written and used wrongly. According to Strutchens et al. (2001), students learn geometry by memorizing geometric properties rather than by exploring and discovering the underlying properties. Geometry knowledge learned in this way is limited and superficial.

On center of rotation, students seemed to know of only the origin. This was so pervasive that on answering sub question six in section ‗C' where the center of origin was (2, 1) , one student concluded in her answer, ―This is not rotation". This explains why no student from both groups got this particular sub question correct.

Table 4.5 provides information on students' responses on finding the image point of given object points when given the angle of rotation

Table 4.4: Frequency and percentage presentation of students' responses on finding the image point of given object points when given the angle of rotation.

Responses to Q7	**CG** Freq.	**CG** %	**EG** Freq.	**EG** %
Wrong	14	28	8	16
Correct	2	4	5	10
No attempt	34	68	37	74
Total	50	100	50	100

Questions seven and eight sought to find out whether students could determine image points of given object points. Students used formulae in answering these questions. The difficulty students encountered was their inability to recall the formulae correctly. Question seven for instance, could be solved using the formula $\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} -y \\ x \end{pmatrix}$. But students stated the formula as $\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} y \\ x \end{pmatrix}$ or $\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} -x \\ y \end{pmatrix}$. This explains why only 10 % (5) from the experimental group and 4 % (2) from the control group had it right as can be seen in Table4. 5.

Figure 9 shows the summary statistics of the number of students who provided right or wrong answers to the question that asked them to find the image of a point when given the angle of rotation about a point other than the origin.

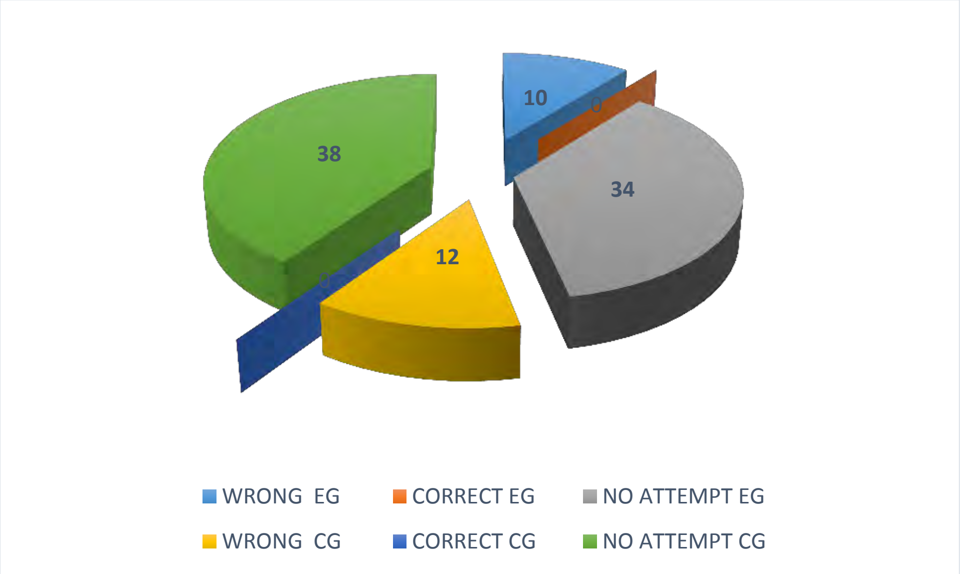

Figure 9: Statistics of students' responses to rotation about a center other than the origin.

Question seven involved a centre of rotation other than the origin. Here, no students from both the control group and the experimental group got it right. Figure 9 showed that more students from the control group 76 % (38) than the experimental group 68 % (34) did not attempt the question.

Interpretation of multiple transformations

The questions under section D of the pre-test were crafted to involve reflection, rotation, translations and a multiple of them altogether in the same question as is normally the case in WASSCE.

Figure 10 shows the summary statistics of the number of students who provided right or wrong answers to the multiple transformation.

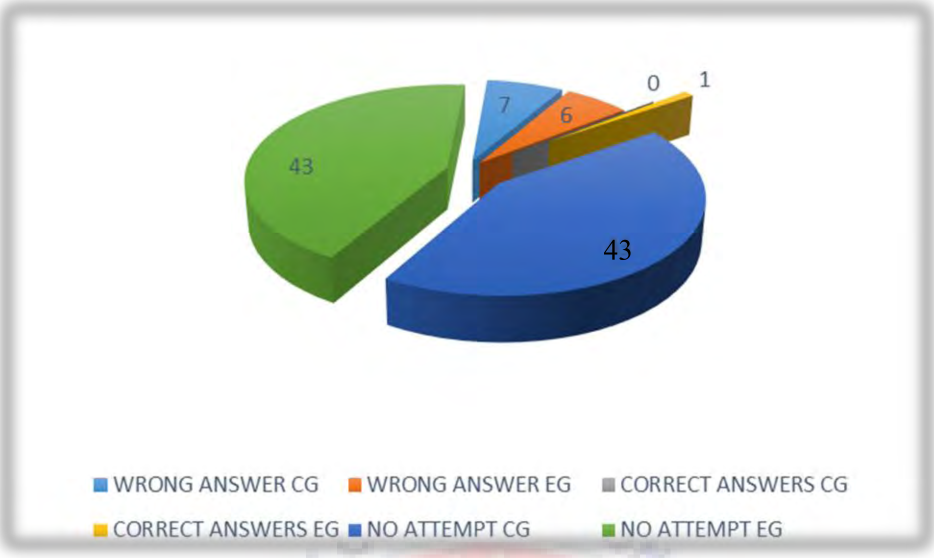

Figure 10: Responses of students to multiple transformations

Data from Figure 10 revealed a paltry 2 % (1) correct response and 12 % (6) wrong answers by the experimental group to sub question four of question nine in section ‗D' that involved multiple transformation (See Appendix A). As high as 86 % (43) of the experimental group did not attempt this question. With regards to the control group, the responses were no better. While no student answered this question rightly, 14%(7) got it wrong and 86%(43) refrained from answering it.

The students' difficulties in the single transformations include confusing reflections (especially those involving $x = y$ or $x = -y$) for rotation or translation for rotation and vice versa.

This finding revealed that students have difficulties learning concepts and their interconnections (Chiu, Robert, & Klassen, 2008).

Observation

Table 4.6 shows the difficulties that each of the groups had as observed by the teachers and recorded in the observation guide (see Appendix G)

Table 4.5: Frequency and percentage representation of difficulties of students in performing rigid motion

Difficulty	Number of groups	Percentage
Counting units to and away from mirror line	3	75
Drawing the lines $x = y$ $x = -y$ and getting perpendicular distances to and from mirror lines.	4	100
Interpretation of translation vectors.	2	50
Interpretation of clockwise and anticlockwise rotation.	4	100
Measuring of angles of rotation with protractor.	2	50
Rotation about a point other than the origin.	4	100

During the intervention, the researcher observed that the students had difficulties with locating lines of reflection, most especially those involving $x = k$, $y = k$, $x = y$ and $x = -y$. They also found it difficult to draw perpendicular lines to and from these same lines. All the four groups representing 100% had these difficulties as can be seen in Table 4.6. This confirmed the students' low performance in the pre-test. Another difficulty observed was the students' inability to easily determine the relationship between the object and the image distances. The students were made to measure the object distance and the image distance using rule. This activity helped the students‘ to establish that the object distance is equal to the image distance.

On translation, it was observed during the intervention that, students seemed comfortable counting units on the y-axis first before the x-axis. Two of the groups representing 50% had this problem as can be seen in table 2. This difficulty was so endemic that students engaged in it unconsciously.

The observation made with respect to rotation was that students had difficulties interpreting clockwise and anticlockwise rotation especially those involving $+90^0$ and -90^0. They interpreted $+90^0$ as clockwise and -90^0 as anticlockwise. All the groups (100%) experienced this difficulty. Two of the groups representing 50% had problems in using the protractor to measure the angles involved. This is consistent with research findings that students have a lot of misconceptions while they are using the protractor to measure angles (Ayşen, 2012). Also students had a hard time seeing rotation as occurring ‗at a distance‘ from the object (Edwards, 2003).

4.3.2 Discussion of research question 1

Statistics gathered from the pre-test revealed that very low percentages ranging from 2 % (1) to 28 % (14) for the control group and 2 % (1) to 30 % (15) for the experimental group rightly answered questions on reflection. The only question on reflection answered rightly by over 50% in both groups was the commonly used line of reflection $y-axis$. This was alluded to by (Fort, 2014) when he revealed that in the past, teachers taught rigid motion solely by using the coordinate plane with ―nice" transformations such as reflection across the y axis.

The question which involved the line $y = \frac{1}{2}$ was answered wrongly by a large number of students, 30 students representing 60 % in the control group and 33 students representing 66 % in the experimental group. The study revealed that the students had difficulties with the identification of the lines of reflection. These difficulties could be cured by designing activities involving the on-screen geometric construction approach (Jones, 2011). This could be done by drawing lines on the screen and asking learners to explain what they expect to happen if some parts of the line were moved. Learners could also be made to measure distances in the diagrams or fold along the axis of symmetry (Sproule, 2005).

On translation, students were expected to identify the translation vectors in each of the sub questions of question 4. It was observed that high number of students ranging from 76 %(38) to 90 %(45) did not attempt these questions in the experimental group. This shows the high level of difficulties students have when it comes to translation.

For question 5, slightly more students recorded correct answers 30 %(15) than wrong answers 28 % (14) within the experimental group and 30 %(15) right answers than wrong answers 18 %(9) in the control group. This can be attributable to the simple addition of the translation vector and the object point involved. Yet, a staggering 42 % (21) in the experimental group and 52 %(26) in the control group did not attempt this question.

It was revealed that students lacked practical knowledge of this concept. Instead of counting units horizontally to the left or right for the x-coordinates and vertically upwards or downwards for the y-coordinates depending on whether the coordinate is positive or negative, they rather used formula. They however, could not handle the formula correctly. While some could not substitute the values correctly, others could not make the translation vector $\begin{pmatrix} a \\ b \end{pmatrix}$ the subject of the relation. This algorithmic reasoning approach which seems to be completely dominating in most part of the students' solutions rather adds to their difficulties. This is consistent with the findings of Lithner (2011) that algorithmic reasoning is surely an essential part of mathematical competence, but if it dominates completely, then learning difficulties are only made worse in the long term perspective.

Data collected on rotation revealed students had difficulties with locating the centre of rotation. They performed all rotations about the origin including rotations about a centre other than the origin and sometimes confused the centre of rotation with the object points being rotated. This was so pervasive that on answering sub question six in section ‗C' where the center of origin was (2, 1) , one student concluded in her answer, ―This is not rotation". This explains why no student from both groups got this particular sub question correct. This may be attributed to the methods used to teach the students at the JHS level. Mostly the traditional approaches to the teaching of geometry do not seem to help students achieve the intended outcomes in the curriculum. The GeoGebra approach used with the discovery method of teaching adopted by the researcher resolved this difficulty to a large extent. Although Christou et al. (2006), revealed that it can take quite a long time to reach the point where tasks genuinely take advantage of the computer envirnment, Jones (2011) found that geometry tasks selected for use in the classroom should, as far as possible, be chosen to be useful, interesting and/or surprising to the students.

There were six (6) sub questions of question 6 on rotation. None of the students in the experimental group got any of the six sub questions correct while majority of the students ranging from 70 %(35) to 80 %(40) in the experimental group and 66% (33) to 74 %(37) in the control group did not even attempt to answer them. The number of students in the control group who answered these questions correctly were still very low, 2 %(1). Further analysis of the wrong answers of students revealed their difficulty in the interpretation of clockwise and anticlockwise rotation. For instance, $+90^0$ was interpreted as clockwise 90^0 instead of anticlockwise 90^0. This lends credence to the findings of Bishop (1986) that a pertinent problem with geometry students is their weakness in the language of geometry. He asserted that vocabulary in geometry is specific and carries meaning, descriptions and even properties. Khoo and Clements (2001) supported these findings by revealing that geometry language, especially in the comprehension of geometry terms, plays a very important role in learning and understanding of geometric concepts.

Another difficulty discovered was that students had no idea on how to perform rotation practically without the recall of formulae. So formulae were memorized but written and used wrongly. This finding agrees with Strutchens et al. (2001), that students learn geometry by memorizing geometric properties rather than by exploring and discovering the underlying properties through conceptual understanding and problem solving abilities (Lithner, 2011) which are not developed by imitative reasoning. It was also found that students had difficulties with the measurement of angles using protractor. This is in tandem with the findings of Ayşen (2012), that students have a lot of misconceptions while they are using the protractor.

These difficulties were identified in the students' pre-test and observations made during the intervention lessons. It was clear that the students were taught rigid motion at the Junior High School using the traditional approach where formulae were given for the students to use without any practical knowledge of the concept.

According to Yeo (2007), traditional approaches of geometry instruction do not seem to help students achieve the intended learning outcomes in the curriculum. There is an urgent need to change the traditional mode of geometry instruction to one that is more rewarding for both teachers and students. Specifically, learners must be given opportunities to personally investigate and discover geometry to enable understanding of the subject in-depth and also in relation to other fields of mathematics. One way to achieve this is to integrate ICT in the teaching of geometry in order to show learners what to do through worked-out and modelled examples, in addition to practice for successful learning.

Prepared introductory materials to facilitate students' first contact with GeoGebra should be introduced to the students as well as knowledge about the most common problems and impediments that arise during the introductory process of dynamic mathematics software (Preiner, 2008). The teacher factor in the integration process is very crucial. According to Balanskat et al. (2006) inadequate and inappropriate training leads to teachers being neither sufficiently prepared nor sufficiently confident to carry out full integration of ICT in the classroom.

4.4 Research question 2

How effective does the use of GeoGebra application software enhances students' achievement in rigid motion?

Research question two essentially focused on the effect of GeoGebra on the teaching and learning of rigid motion at the SHS level. The pre-test and post-test were analysed to determine if there was any improvement in the students' performance.

4.4.1 Results of research question 2

Analysis of Pre-test and Post-test results

Table 4.7 compares the control groups' performance in the pre-test and post-test on questions in rigid motion.

Table 4.6: Comparison of Control group's Pre-test and Post-test in rigid motion

	Pre-test			Post-test		
	CG Wrong Answer % (n)	CG Correct Answer % (n)	CG No attempt % (n)	CG Wrong Answer % (n)	CG Correct Answer % (n)	CG No attempt %(n)
Q1	52(26)	20(10)	28(14)	74(37)	24(12)	2 (1)
Q2	40(20)	10(5)	50(25)	64(32)	26(13)	10 (5)
Q3	32(16)	8(4)	60(30)	66(33)	18(9)	16 (8)
Q4	20(10)	0(0)	80(40)	60(30)	16(8)	24 (12)
Q5	18(9)	30(15)	52(26)	76(38)	12(6)	12 (6)
Q6	28(14)	2(1)	70(35)	62(31)	22(11)	16 (8)
Q7	28(14)	4(2)	68(34)	48(24)	30 (15)	22 (11)
Q8	24(12)	0(0)	76(38)	68(34)	6(3)	26 (13)
Q9	18(9)	0(0)	82(41)	66(33)	10(5)	2 (12)
Q10	22(11)	28(14)	50(25)	48(24)	48(24)	4 (2)

As shown in Table 4.7, the control group recorded 20% and 10% correct responses in the pre-test for questions 1 and 2 respectively, but recorded 24% and 26% respectively in the post-test. Although, there was some slight increases, these percentages were still low.

Similarly, in questions 3 and 4, although there was some level of improvement in the percentage of students who answered them rightly, it was again very marginal.

Apart from question 5 where the control group recorded a decrease from 30% in the pre-test to 12% in the post-test, the same trend of percentage increases from the pre-test to the post-test in questions answered rightly were observed for questions 6,7,8,9 and 10 in both groups.

Table 4.8 compares the experimental groups' performance in the pre-test and post-test on questions in rigid motion.

Table 4.7: Comparison of Experimental group's Pre-test and Post-test in rigid motion.

	Pre-test			Post-test		
	EG Wrong Answer % (n)	EG Correct Answer % (n)	EG No attempt % (n)	EG Wrong Answer % (n)	EG Correct Answer % (n)	EG No attempt % (n)
Q1	60(30)	20(10)	20(10)	40(20)	48(24)	12(6)
Q2	34(17)	10(5)	56(28)	60(30)	28(14)	12(6)
Q3	42(21)	4(2)	54(27)	54(27)	30(15)	16(8)
Q4	16(8)	0(0)	84(42)	50(25)	38(19)	12(6)
Q5	28(14)	30(15)	42(21)	18(9)	66(33)	16(8)
Q6	24(12)	0(0)	76(38)	38(19)	46(23)	16(8)
Q7	16(8)	10(5)	74(37)	54(27)	12(6)	34(17)
Q8	32(16)	0(0)	68(34)	52(26)	0(0)	48(14)
Q9	16(8)	0(0)	84(42)	60(30)	16(8)	24(12)
Q10	8(4)	40(20)	52(26)	8(4)	74(37)	18(9)

As shown in Table 4.8, the experimental group recorded 20% and 10% correct responses in the pre-test for questions 1 and 2 respectively, but recorded higher percentages of 48% and 28% respectively in the post-test. These increases were higher than the increase recorded by the control group.

Again, for questions 3 and 4, there were some level of improvement in the percentage of students who answered them rightly. The percentages changed from 4% and zero percent to 30% and 38% respectively in the experimental group as shown in Table 4.8.

Apart from question 8 where the experimental group recorded no change in the pre-test and post-test, the same trend of percentage increases from the pre-test to the post-test in questions answered rightly were observed for questions 5,6,7,9 and 10.

Hypotheses Testing

In all, one hypothesis was formulated to address this research question.

1. **H_o:** There is no statistically significant difference in the mean performances of the experimental and control groups' post-test scores in rigid motion.

   H_1: There is statistically significant difference in the mean performances of the experimental and control groups' post-test scores in rigid motion.

An independent samples t-test was conducted to verify if there was any difference in the mean performance of the experimental and control groups in rigid motion post-test. Table 4.9 shows the Independent Sample T-Test of Post-Test scores of SHS students.

Table 4.8: Independent Sample T-Test of Post-Test of SHS students

	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
						Lower	Upper
Post-test	5.872	98	.000	6.120	1.042	4.052	8.188

Results of the independent samples t-test (df = 98, t =5.872, and p = 0.000) in Table 4.9 indicated that the difference in means was significant at $p < 0.05$. Hence the null hypothesis that both experimental and control groups' performance in the post-test on rigid motion did not differ was rejected in favour of the alternative hypothesis. This means that there was a significant difference between the mean performances of the experimental and control groups.

Table 4.10 shows Group Statistics of control group (CG) and experimental group (EG) in the post-test scores

Table 4.9: Group Statistics of CG and EG in the post-test scores

	N	Mean	Std. Deviation	Std. Error Mean
EG	100	12.10	5.987	.847
CG	100	5.98	4.298	.608

The results, as shown in Table 4.10, indicated that the mean score of the experimental group was 12.10 while that of the control group was 5.98. This means the average score of the experimental group was higher than that of the control group.

4.4.2 Discussion of research question 2

The results of this analysis revealed that, although the means of both experimental and control groups saw a leap in the post-test, it was higher in the experimental group than in the control group. The analysis showed a consistent increase in the percentage of questions answered rightly from pre-test to post-test in both groups. However, the increases were more significant in the experimental group than in the control group. These significant increases in percentages in nine of the ten questions in the post-test with regards to the experimental group may partly be due to the use of GeoGebra integration and the discovery approach of instructions which made the teaching and learning of rigid motion very practical with relatively small risk (Jones et al., 2009). In other words because the experimental group were engaged in practical activities using GeoGebra on the computer coupled with the use of grid paper to discover rules, it afforded them the opportunity to effectively address questions on rigid motion using practical understanding.

Again, the mean performance of the experimental group in the post-test, 12.10, was far higher than that of the control group that had mean of 5.98 as shown in Table 4. Once again, the high mean of the experimental group in the post-test compared to that of the control group indicated a far better performance of SHS students in the experimental group. This could be attributed to the influence of the use of GeoGebra in the teaching and learning of the concept rigid motion. An independent samples t-test conducted also revealed that, there is statistically significant difference in the mean performances of the experimental and control groups in the post-test scores.

4.5 Research question 3:

How effective do SHS mathematics teachers find the use of GeoGebra in teaching rigid motion?

This research question sought to determine how effective SHS mathematics teachers found the use of GeoGebra to teach rigid motion.

4.5.1 Results of research question 3

A questionnaire was used to address this research question. Section C of the questionnaire (see appendix E) was on effectiveness of use of GeoGebra to teach rigid motion. Respondents were asked to indicate on a five-point scale ranging from strongly agree (1) to strongly disagree (5) their views on the effectiveness of the use of GeoGebra in their teaching and learning approach.

Table 4.11 shows the percentage presentation of Teachers‘ Perceptions about the use of GeoGebra in teaching rigid motion.

Table 4.10: Percentage presentation of Teachers' Perceptions about the use of GeoGebra in teaching rigid motion.

S/N	Variable	SA (%)	A (%)	U (%)	SD (%)	D (%)
1	GeoGebra is user friendly.	100	0	0	0	0
2	GeoGebra is easy and intuitive to use.	50	50	0	0	0
3	Teaching reflection with GeoGebra enhances students' understanding.	50	50	0	0	0
4	Teaching translation with GeoGebra enhances students' understanding.	50	25	25	0	0
5	Teaching translation with GeoGebra enhances students' understanding.	50	50	0	0	0
6	Teaching rigid motion with GeoGebra is the best practical method.	75	25	0	0	0
7	Students easily deduced formulae for rigid motion after using GeoGebra.	50	25	0	0	25
8	Students' attitude towards GeoGebra designed lessons was positive and encouraging.	0	75	25	0	0
9	It is potentially helpful to use GeoGebra to teach SHS students.	100	0	0	0	0
10	The use of GeoGebra to teach rigid motion is highly very effective	100	0	0	0	0

Scale: SD=Strongly Disagree D=Disagree U=Undecided A=Agree SA=Strongly Agree

As shown in Table 4.11, all respondents (100%) strongly agreed (1) or agreed (1 and 2) that GeoGebra is user friendly, easy and intuitive to use. Apart from rotation where a respondent (25%) was undecided (3), all respondents agreed or strongly agreed that teaching reflection and translation with GeoGebra enhances students understanding of the concept. Majority of the respondents (75%) strongly agreed (1) that the best practical approach to the teaching and learning of rigid motion is the incorporation of GeoGebra.

However, while 75% of the respondents agreed (2) that students' attitude towards the GeoGebra designed lessons was positive and encouraging, 25% of the respondents was undecided (3). Meanwhile, all respondents agreed strongly that it is potentially helpful to use GeoGebra to teach rigid motion in the Senior High Schools and that its use is very effective.

The last question of this section was an open ended question which sought the views of the respondents on how they perceived the use of GeoGebra in teaching and learning of rigid motion. Each of the respondents perceived the use of GeoGebra as the most effective approach they have ever come across and recommended that it should be introduced to all SHS Mathematics teachers in the region and beyond.

4.5.2 Discussion of research question 3

On the composition of the sample, all the student participants were female students offering General Arts. The selected teachers who took part in the research were 50% male and 50% female. This gender balance of teachers is in tandem with the assertion by UNESCO in 2009 that program objectives are more likely to be reached if interest and experiences of both women and men are taken into account at all stages of a programming process.

Data from the questionnaire administered to the teachers revealed that GeoGebra is user friendly, easy and intuitive to use (Hohenwarter & Lavicza, 2007) and that students' attitude towards the GeoGebra designed lessons was positive and encouraging (Hohenwarter, Hohenwarter, & LavicZa, 2008). The teachers concluded that integration of GeoGebra into the teaching of rigid motion is the most effective approach they have ever come across and recommended that it should be introduced to all SHS Mathematics teachers in the region and beyond.

CHAPTER FIVE

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

5.1 Overview of the Study

This chapter presents the conclusion of the whole research study. It includes a summary of the findings and highlights on its educational implications. It further outlines some limitations, recommendations and avenues for further research studies.

The study aimed at determining the effect that Mathematics dynamic software (GeoGebra) could have on the mathematical attainment of students in rigid motion in the Senior High Schools (SHS).

The study was conducted using non-equivalent control group design with pre-test and post-test design, a quasi-experimental approach using both qualitative and quantitative method. Quantitative data was used to investigate the difficulties students have in the study of rigid motion and the effects of GeoGebra on students' understanding of rigid motion. Qualitative data on the other hand was used to investigate the views of selected Mathematics teachers in the sampled school on the effectiveness of the use of GeoGebra in the teaching and learning of rigid motion.

The target population for the study was made up of Senior High School Mathematics teachers and students in the government assisted Senior High Schools (SHS) in Ho Township. The accessible population was made up of 100 General arts students from two classes (2B and 2D) and four Mathematics teachers. The two classes sampled for the study were reshuffled and put into experimental group (EG) and control group (CG). The experimental group was taken through GeoGebra designed lessons in reflection, rotation, translation and their combinations. The study was guided by research questions and hypothesis were also tested. Instruments that were used in the collection of data were basically questionnaire, observation and geometric test (pre and post-test). Analysis of data was based on both descriptive and inferential statistics.

5.2 Summary of key findings

5.2.1 Research question 1: What difficulties do Senior High School students have in understanding rigid motion?

The researcher identified a number of difficulties students have in their learning of the concepts of rigid motion. The difficulties identified in the study were:

1. Description of the line of reflection of reflected figures in a given diagram.
2. Interpretation of equations of lines of reflection.
3. Substitution of values into formulae to arrive at image points.
4. Interpretation of translation vectors.
5. Interpretation of clockwise and anticlockwise rotation.
6. Rotation of figure with different center apart from the origin.
7. Multiple transformations.

5.2.2 Research question 2: How effective does the use of GeoGebra application software enhance students' achievement in rigid motion?

Findings in respect of research question two indicated that the use of GeoGebra designed lessons to teach the experimental group led to a rather remarkable performance in their post-test. The performances of the students of the experimental group in the post-test suggested that the use GeoGebra to teach rigid motion was useful in the teaching and learning process.

5.2.3 Research question 3:

How effective do SHS Mathematics teachers find the use of GeoGebra in teaching rigid motion?

The findings in the study indicated that sampled teachers supported the view that the use of GeoGebra in teaching rigid motion was very important and could be employed by teachers in Senior High Schools in the teaching and learning of rigid motion at this level. The teachers were unanimous on the following:

1. GeoGebra is user friendly, easy and intuitive to use.
2. Apart from rotation, all the teachers sampled concluded strongly that teaching reflection and translation with GeoGebra enhances students understanding of the concept.
3. The best practical approach to the teaching and learning of rigid motion is the incorporation of GeoGebra.
4. Students‘ attitude towards the GeoGebra designed lessons was positive and encouraging.
5. It is potentially helpful to use GeoGebra to teach rigid motion in the Senior High Schools and that its use is very effective.
6. GeoGebra is the most effective approach they have ever come across and recommended that it should be introduced to all SHS mathematics teachers in the region and beyond.

For more teachers to use GeoGebra in their teaching, they must be given adequate training. GeoGebra cannot be used in isolation. According to Koehler and Mishra (2008), at the heart of good teaching with technology are three core components: technology, pedagogy and content plus the relationships between them. Buabeng-Andoh (2012) recommended that teachers be given sufficient training on how to use

ICT in teaching and learning processes to acquire the requisite knowledge and skills in integrating the technology in classrooms. This will provide opportunities for teachers to support student-centered learning. Studies revealed that offering high-quality professional development for teachers is essential for successful technology integration (Hohenwarter, Hohenwarter, & Lavicza, 2008).

5.3 Educational implication of the study for mathematics teaching

Jamieson-Proctor, Burnett, Finger, and Watson (2006) revealed that rapid technological change and global communication are facts of life in the 21st century. According to them, the closing decades of the 20th century and the beginning of the 21st century were and would be characterised by change in almost every aspect of people's working, public and private lives. Consequently, the appearance of information and communication technology (ICT) in schools through improved provision of computer hardware, infrastructure and connectivity should not be seen as an isolated example of change.

Contemporary belief in Mathematics education is that learners need to be active learners rather than passive recipients of mathematical concepts to be learnt meaningfully (Kwang, 2002) and Mathematics specific software packages such as GeoGebra provide huge opportunities for students to actively interact with the computer and learn concepts with little or no guidance. In a report by Wenglinsky (1998) computers are neither cure-alls for the problems facing schools, nor mere fads that have no impact on student learning. Rather, when they are properly used, computers may serve as important tools for improving student proficiency in Mathematics, as well as the overall learning environment in the school.

Bearing in mind the complexity of the problems most Mathematics classrooms in Ghana face in terms of ICT infrastructure and lack of application software, an environment with a more generalized application that offer a technology readily available and user friendly among Mathematics classrooms with the potential for supporting students' higher-order thinking in Mathematics (such as spreadsheet) is proposed for use in professional development programs (Agyei & Voogt, 2011). In their view, this will ensure that teachers will be able to use existing hardware and software in creative and situation specific ways to design ICT resources to accomplish their teaching goals.

Integrating ICT in the classroom teaching and learning process helps to foster authentic learning abilities in learners. The use of GeoGebra in this study for instance gave SHS students the opportunity to develop rigid motion concepts in a practical setting. For effective geometry instruction, the method of teaching should not be the same as in teaching number, algebra or probability. Instead instruction should emphasize hands-on explorations, developing geometric thinking and reasoning, making conjectures and even carrying out geometry projects (Strutchens et al., 2001).

Mathematics educators should prioritise the use of Mathematics specific software in their lesson delivery. In the views of Preiner (2008), many teachers and students today have access to computers and although appropriate software is available both in schools and at home, technology is rarely integrated substantially into everyday teaching. Being aware of the vital role that teachers play in a technology-supported Mathematics classroom, professional development opportunities need to be adapted in order to better prepare teachers for this new challenge of eff ectively integrating technology into their teaching practice.

The use of GeoGebra to teach rigid motion in this study revealed the need for educators to incoporate ICT more in their teaching and learning process. Van Voorst (1999) highlighted that technology was ―useful in helping students view Mathematics less passively, as a set of procedures, and more actively as reasoning, exploring, solving problems, generating new information, and asking new questions." Furthermore, he claimed that technology helps students to ―visualize certain Mathematics concepts better" and that it adds ―a new dimension to the teaching of Mathematics" (p.2).

Hohenwarter and Lavicza (2007) revealed that GeoGebra is freely downloadable from the internet and thus it is available both in schools and at home without any limitations. They asserted that GeoGebra not only offers a novel dynamically connected learning environment, but also its development aimed at delivering a software package that can be utilized in a wide range of grade levels. In Ghana, GeoGebra could be used to effevtively teach geometry at the SHS level. This is because developers of GeoGebra emphasised that users should be able to use the software intuitively without having advanced computer skills.

5.4 Conclusion

The study sought to determine the effect of teaching with GeoGebra on students' understanding of rigid motion. Rigid motion is an important concept in the Mathematics curriculum of both Junior High Schools and Senior High Schools in Ghana. Over the years, chief examiners' reports in Mathematics indicated that performance of students in geometry at both Junior High and Senior High School levels had been rather weak. In the past years, chief examiners' reports on the Basic Education Certificate Examinations (WAEC, 2005, 2006) indicated that students lack sufficient knowledge in geometry and application of geometrical concepts of which geometrical transformations are a part. This poor performance was manifested by Ghanaian students' performance in their participation in Trends in International Mathematics and Science Study (TIMSS) over the years. To help improve upon students‘ knowledge in geometry, the researcher employed the approach of ICT integration to determine its effects on students‘ performance in rigid motion. Three research questions were formulated to guide the study.

The findings in the study indicated that the students were taught rigid motion at the Junior High School using the traditional approach where formulae were given for the students to use without any practical knowledge of the concept. The use of GeoGebra in teaching rigid motion in this research showed a marked improvement in students understanding of the concept and therefore, was very effective. It further revealed features of GeoGebra that makes it effective and suitable for teaching rigid motion; user friendly, easy and intuitive to use. It was also revealed that students‘ attitude towards the GeoGebra designed lessons was positive and encouraging. The sampled teachers who participated in this study concluded that GeoGebra is the most effective approach they have ever come across and recommended that it should be introduced to all SHS mathematics teachers in the region and beyond.

Finally, the most striking finding made from the research is that Mathematics teachers do not incorporate ICT in the teaching of rigid motion. This could be attributed to the fact that teachers have little or no training in how to use mathematics specific software.

5.5 Recommendations

From the findings of this study, it is recommended that;

1. Mathematics teachers blend technology, pedagogy and content plus the relationships between them (Koehler & Mishra, 2008) in teaching geometry. This would help improve students understanding of the concept.

2. In-service training and workshops should be organized for Senior High School teachers on Mathematic specific software packages such as GeoGebra. GeoGebra introductory book, and instructional materials about GeoGebra and its integration into Mathematics classrooms should also be developed and distributed to teachers. This would improve teachers‘ competence and confidence level to integrate ICT in the classroom.
3. Senior High School teachers should adapt technology that is readily available and user friendly with potential for supporting students‘ practical knowledge in geometry. This would ensure teachers adapt teaching styles that make the teaching and learning of geometry purely practical.
4. Mathematics teachers should be encouraged to become designers of technological resources, by learning how to use existing hardware and software in creative and situation specific ways to accomplish their teaching goals. With this, they can integrate available technology in their daily lesson plans and into traditional classroom practice.

5.6 Suggestions for further studies

The educational implication of the findings of this study calls for further research in the area of geometry. The following are suggested for further research:

1. This study centred on only rigid motion. Other areas of geometry such as geometrical constructions, circle theorems and coordinate geometry could be looked at in the light of employing GeoGebra in teaching them.
2. The study covered only two classes in a single sex SHS in the Volta region of Ghana because of proximity, time to complete the study and finance. The research design could be modified so that more SHSs could be used to give a wider view on the use of GeoGebra in teaching and learning of geometry.
3. The study could be extended to look into other areas such as calculus.

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APPENDICES

Appendix A – Rigid Motion (Pre-Test)

Instructions:

(1) Answer all questions in each of the four sections, showing clearly all workings where necessary. Each question carries equal marks.

(2) Please indicate your CODE below on the given question paper.

Code……………………….

SECTION A – REFLECTION

1. Describe fully the following reflections by providing the equation of the line of reflection in figure 1.

a. Shape 1 onto shape 3……………………………………………………….

b. Shape 3 onto shape 7……………………………………………………….

c. Shape 2 onto Shape 4………………………………………………………..

d. Shape 3 onto shape 6…………………………………………………………

e. Shape 3 onto shape 4…………………………………………………………

f. Shape 7 onto shape 5…………………………………………………………

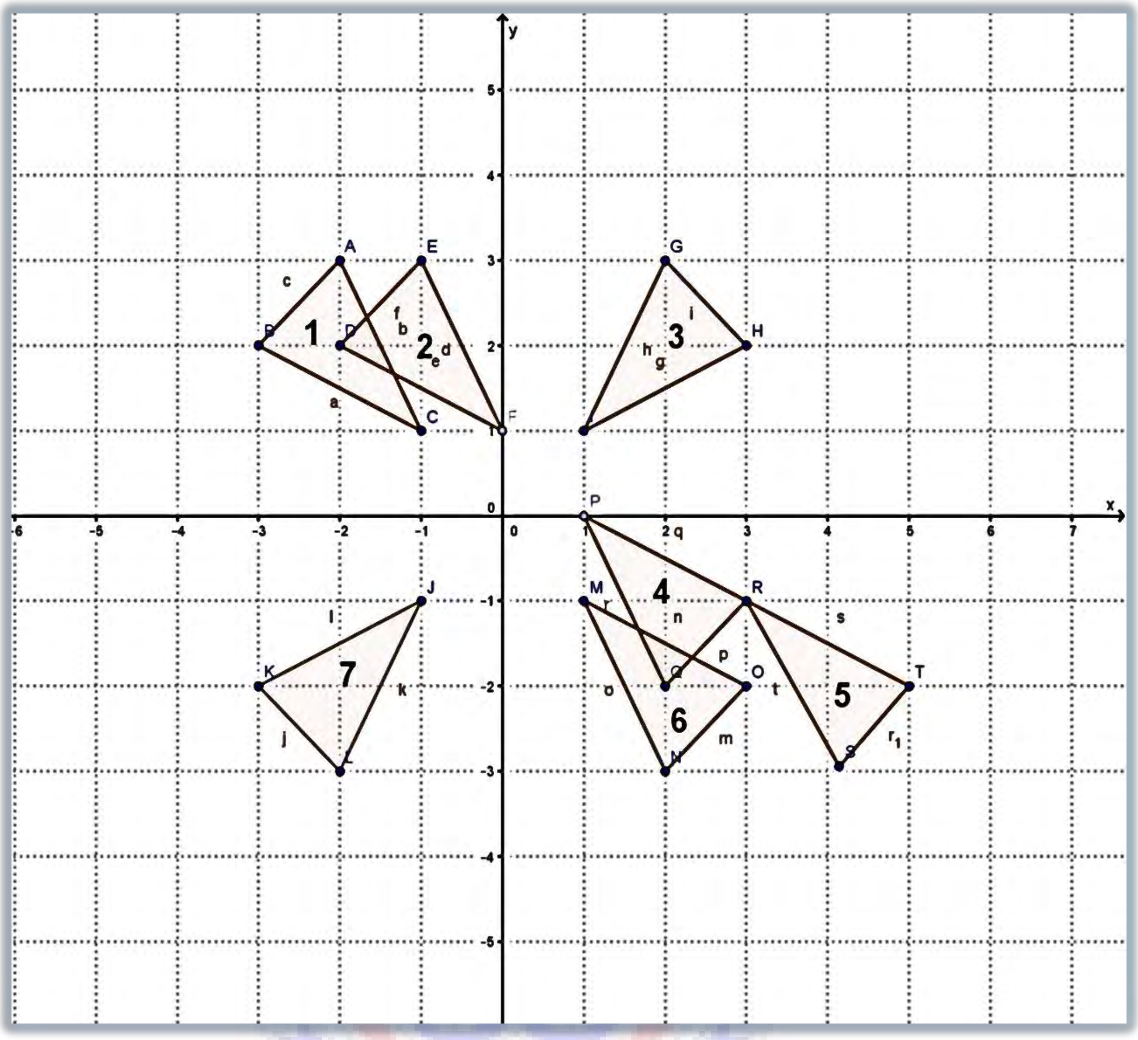

Figure 1: Reflection of pre-image to image

2. Find the image of the point $A(-4,-3)$ when it is reflected in the line $y-1=0$

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3. Find the image of the point $P(-2,0)$ when it is reflected in the line $x+3=0$

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……………………………………………………………………………………………

…………………………………………………………………………………..……

SECTION B-TRANSLATION

4. Determine the translation vector in each of the following translations in figure 2 below:

a. Shape 1 onto shape 3……………………………………………………………

b. Shape 2 onto shape 7……………………………………………………………

c. Shape 3 onto shape 4…………………………………………………………..

d. Shape 5 onto shape 6……………………………………………………………

e. Shape 7 onto shape 1……………………………………………………………

f. Shape 4 onto shape 5……………………………………………………………

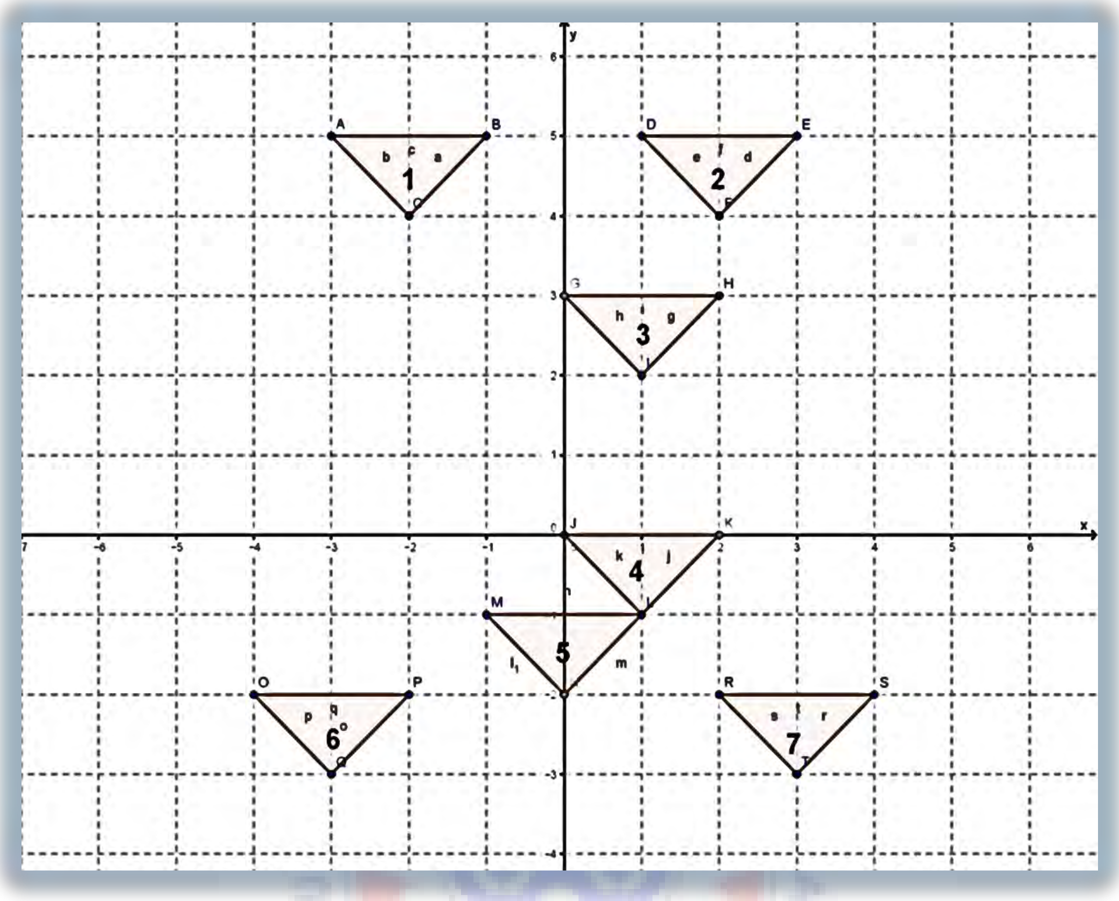

Figure 2: Translation of pre-image to image

5. Find the image of the point $V(3,6)$ under a translation by the vector $T\begin{pmatrix}-3\\-2\end{pmatrix}$

……………………………………………………………………………………………

SECTION C – ROTATION

6. Describe fully each of the rotations in figure 3 below by stating the angle of rotation in each case.

a. Shape 2 onto shape 5……………………………………………………………

b. Shape 2 onto shape 3 …………………………………………………………..

c. Shape 2 onto shape 4……………………………………………………………

d. Shape 3 onto shape 5……………………………………………………………

e. Shape 4 onto shape 5……………………………………………………………

f. Shape 2 onto shape1……………………………………………………………

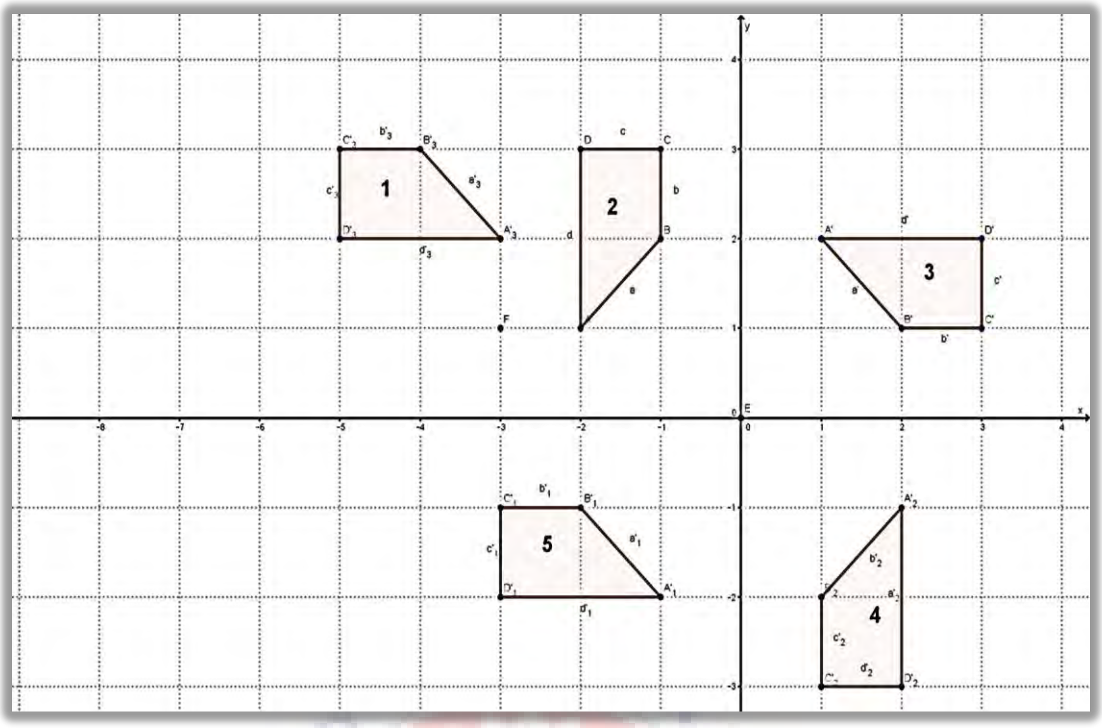

Figure 3: Rotation of pre-image to image

7. If P^{l} is the image of $P(-2,4)$ under a clockwise rotation through 270^{0} about the origin, find the coordinates of P^{l} ……………………….

8. Find the image of the point $R(4,1)$ when it is rotated through $+90^{0}$ about the point $(-1,1)$……………………………………………………………………….

SECTION D- GENERAL/COMPOSITION OF TRANSFORMATION

9. Use figure 4 to answer the following questions.

   a) The single transformation that maps Shape 3 to Shape 5 is

   …………………………………………………………………………………

   b) The single transformation that maps Shape 3 to Shape 1 is………………..…

   ………………………………………………………………………………...

   c) The single transformation that maps Shape 4 to Shape 2 is…………….……

   d) Describe the transformation that will map Shape 5 to Shape 1 ………………

   …………………………………………………………………………………

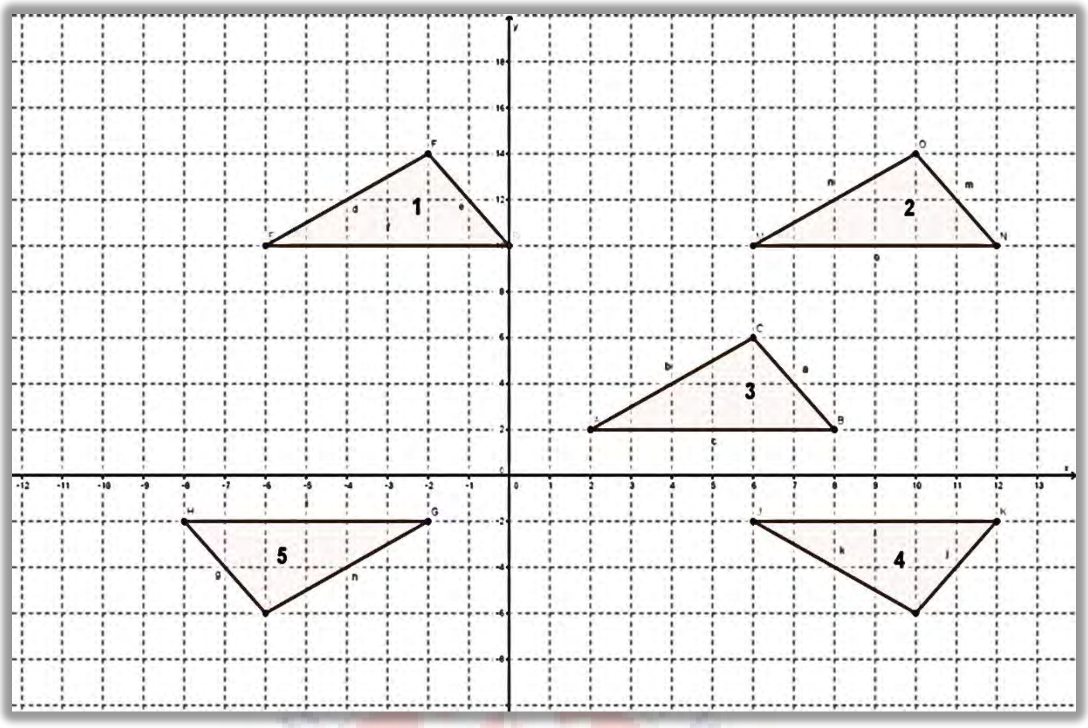

Figure 4: Multiple transformation

10. Find the image of the point $(3,-2)$ under the transformation given by $\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} 3x \\ 3y+2 \end{pmatrix}$.

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APPENDIX B

Rigid Motion Test (Post Test)

Instructions:

(1) Answer all questions in each of the four sections, showing clearly all workings where necessary. Each question carries equal marks.

(2) Please indicate your CODE below on the given question paper.

Code……………………….

SECTION A – REFLECTION

1. Describe fully the following reflections by providing the equation of the line of reflection in figure 1.

a. Shape 2 onto shape 3…………………………………………………..…………

b. Shape 2 onto shape 4…………………………………………………..…………

c. Shape 3 onto Shape 5…………………………………………………..…………

d. Shape 1 onto shape 6………………………………………………………………

e. Shape 3 onto shape 6……………………………………………………

f. Shape 4 onto shape 7……………………………………………………

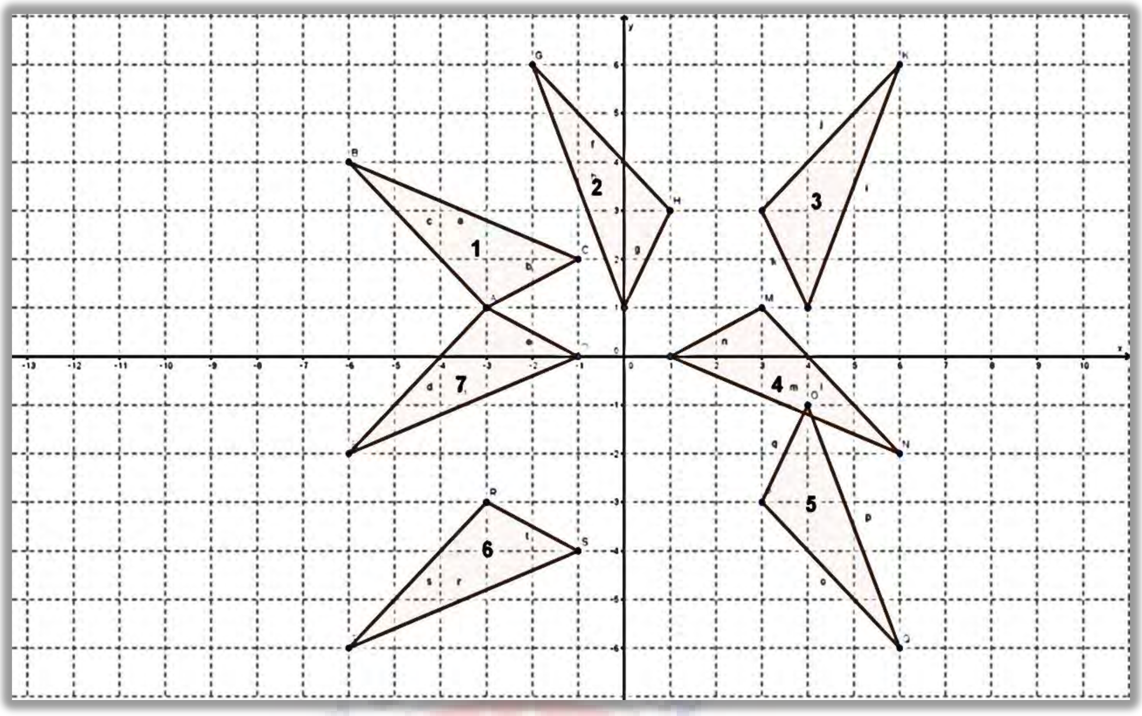

Figure 1: Reflection of pre-image to image

2. Find the image of the point $A(-2,-6)$ when it is reflected in the line $y-1=0$

……………………………………………………………………………………

……………………………………………………………………………………

……………………………………………………………………………………

3. Find the image of the point $P(-3,1)$ when it is reflected in the line $x+2=0$

……………………………………………………………………………………

……………………………………………………………………………………

……………………………………………………………………………………

SECTION B-TRANSLATION

4. Determine the translation vector in each of the following translations in figure 2 below:

a. Shape 2 onto shape 3……………………………………………………………

b. Shape 1 onto shape 4……………………………………………………………

c. Shape 5 onto shape 4……………………………………………………………

d. Shape 6 onto shape 3……………………………………………………………

e. Shape 4 onto shape 3……………………………………………………………

f. Shape 1 onto shape 5…………………………………………………………....

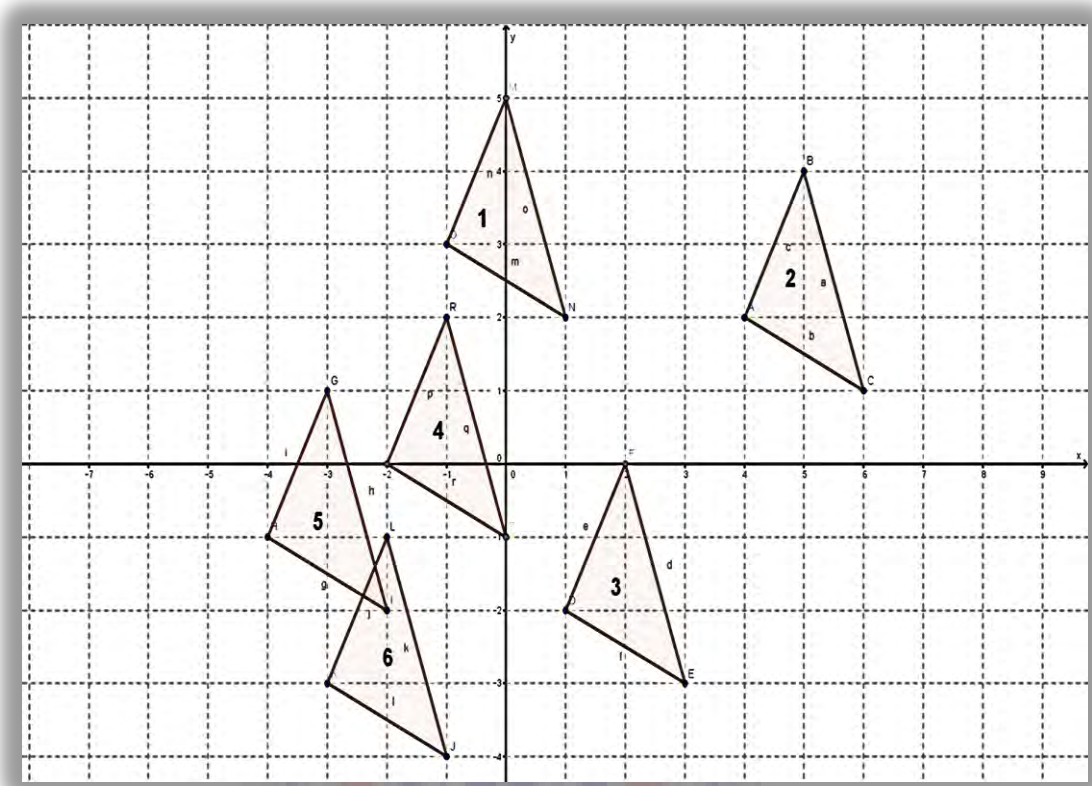

Figure 2: Translation of pre-image to image

5. Find the image of the point $V(3,7)$ under a translation by the vector $T\begin{pmatrix} 2 \\ -1 \end{pmatrix}$

……………………………………………………………………………………………

……………………………………………………………………………………………

…………………………………………………………………………

SECTION C – ROTATION

6. Describe fully each of the rotations in figure 3 below by stating the angle of rotation in each case.

a. Shape 1 onto shape 5……………………………………………………………

b. Shape 5 onto shape 2 ……………………………………………………………..

c. Shape 2 onto shape 4……………………………………………………………

d. Shape 1 onto shape 4……………………………………………………………

e. Shape 2 onto shape 3……………………………………………………………

f. Shape 1 onto shape 2……………………………………………………………

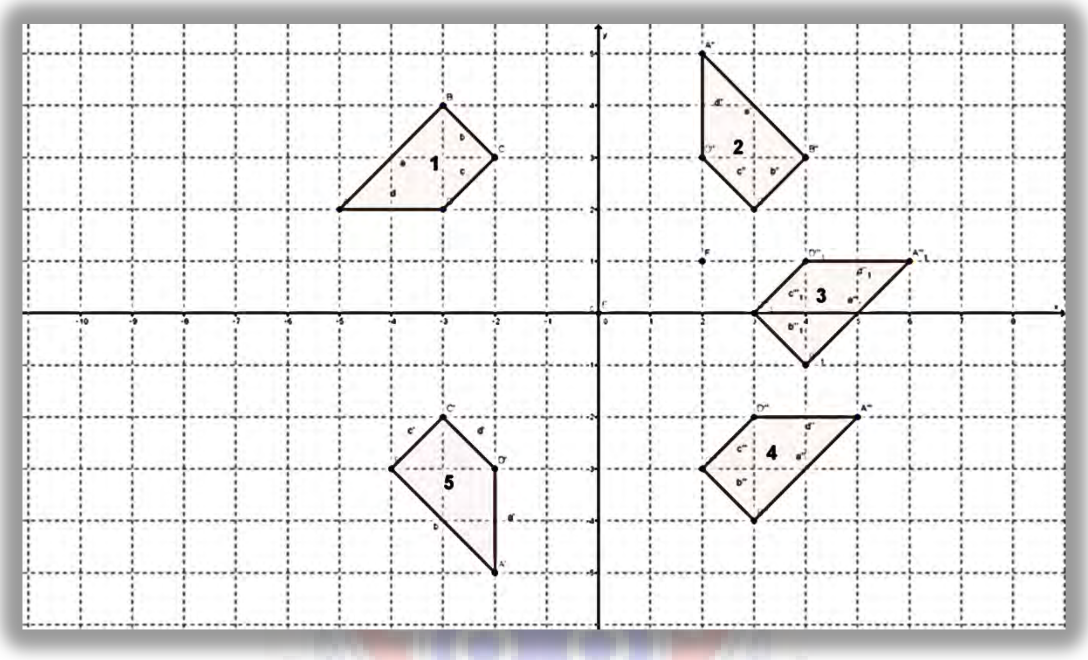

Figure 3: Rotation of pre-image to image

7. If P' is the image of $P(2,-4)$ under a clockwise rotation through 270^0 about the origin, find the coordinates of P'

………………………………………………………………………………

………………………………………………………………………………

………………………………………………………………………………

8. Find the image of the point $R(4,1)$ when it is rotated through -90^0 about the point $(-1,3)$

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SECTION D- GENERAL/COMPOSITION OF TRANSFORMATION

9. Use figure 4 to answer the following questions.

a. The single transformation that maps Shape 2 to Shape 4 is …………

b. The single transformation that maps Shape 4 to Shape 3 is…………

c. The single transformation that maps Shape 1 to Shape 2 is……………

d. Describe the transformation that will map Shape 3 to Shape 1…………

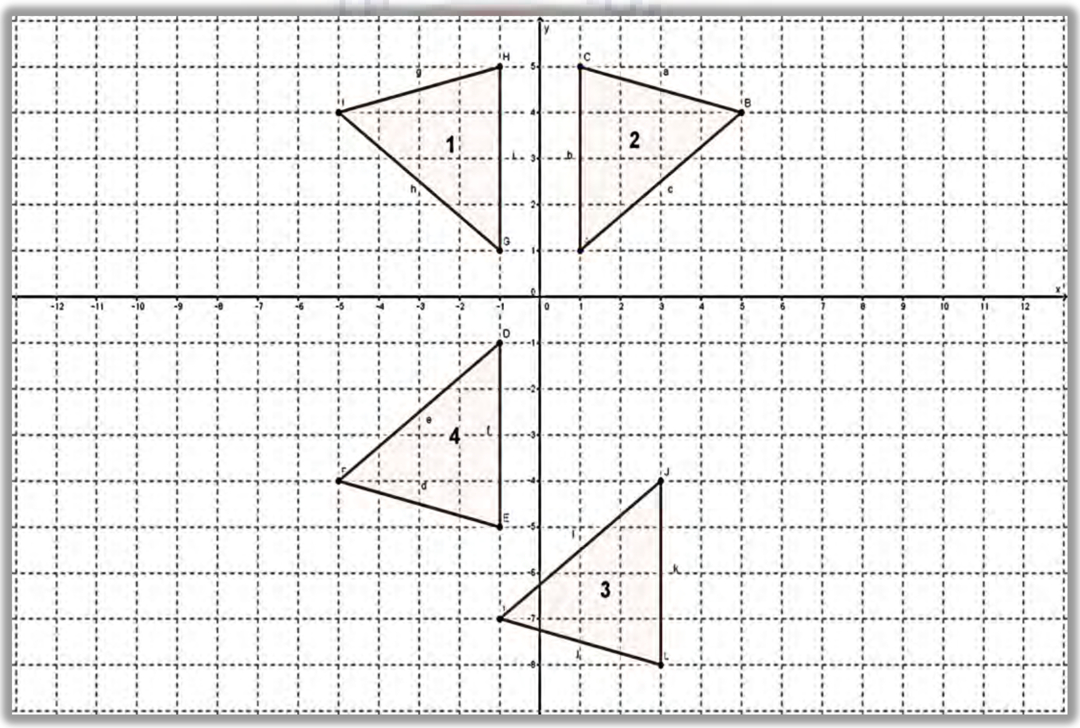

Figure 4: Multiple transformation

10. Find the image of the point $(-3,2)$ under the transformation given by

$\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} 3x \\ 3y+2 \end{pmatrix}$……………………………………………………………..

…………………………………………………………………………………

APPENDIX C

Marking Scheme (Pre-Test)

Q1

a) $x = 0$ A1

b) $x = -y$ A1

c) $x = y$ A1

d) $y = 0$ A1

e) $y = \frac{1}{2}$ A1

f) $x = 1$ A1

Q2

$$\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} x \\ 2a - y \end{pmatrix}$$

$$\begin{pmatrix} -4 \\ -3 \end{pmatrix} \rightarrow \begin{pmatrix} -4 \\ 2(1) - (-3) \end{pmatrix}$$

$$\begin{pmatrix} -4 \\ -3 \end{pmatrix} \rightarrow \begin{pmatrix} -4 \\ 5 \end{pmatrix}$$

$\therefore (-4, -3) \rightarrow (-4, 5)$ A1

Q3

$$\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} 2a - x \\ y \end{pmatrix}$$

$$\begin{pmatrix} -2 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 2(-3) - (-2) \\ 0 \end{pmatrix}$$

$$\begin{pmatrix}-2\\0\end{pmatrix}\rightarrow\begin{pmatrix}-4\\0\end{pmatrix}$$

$(-2,0)\rightarrow(-4,0)$ A1

Q4

a) $\begin{pmatrix}3\\-2\end{pmatrix}$ A1

b) $\begin{pmatrix}1\\-7\end{pmatrix}$ A1

c) $\begin{pmatrix}0\\-3\end{pmatrix}$ A1

d) $\begin{pmatrix}-3\\-1\end{pmatrix}$ A1

e) $\begin{pmatrix}-5\\7\end{pmatrix}$ A1

f) $\begin{pmatrix}-1\\-1\end{pmatrix}$ A1

Q5

$$\begin{pmatrix}3\\6\end{pmatrix}+\begin{pmatrix}-3\\-2\end{pmatrix}=\begin{pmatrix}0\\4\end{pmatrix}$$

$(3,6)=(0,4)$ A1

Q6

a) Anticlockwise rotation through 90^0 about the origin/ clockwise rotation through 270^0 about the origin. B1

b) Clockwise rotation through 90^0 about the origin/ anticlockwise rotation through 270^0 about the origin. B1

c) Clockwise/ anticlockwise rotation through 180^0 about the origin. B1

d) Clockwise/ anticlockwise rotation through 180^0 about the origin. B1

e) Clockwise rotation through 90^0 about the origin/ anticlockwise rotation through 270^0 about the origin. B1

f) Anticlockwise rotation through 90^0 about the point $(-3,1)$. B1

Q7

$$\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} -y \\ x \end{pmatrix}$$

$$\begin{pmatrix} -2 \\ 4 \end{pmatrix} \rightarrow \begin{pmatrix} -4 \\ -2 \end{pmatrix}$$

$(-2,4) \rightarrow (-4,-2)$ A1

Q8

$$\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} -(y-b)+a \\ (x-a)+b \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 4 \end{pmatrix} \rightarrow \begin{pmatrix} -(4-1)+(-1) \\ (1-(-1)+1 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 4 \end{pmatrix} \rightarrow \begin{pmatrix} -4 \\ 3 \end{pmatrix}$$

$(1,4) \rightarrow (-4,3)$ A1

Q9

a) Rotation through 180^0 about the origin. B1

b) Translation by the vector $\begin{pmatrix} -8 \\ 8 \end{pmatrix}$. B1

c) Reflection in the line $y = 4$. B1

d) Rotation through 180^0 about the origin followed by a translation by the vector

$\begin{pmatrix} -8 \\ 8 \end{pmatrix}$ B2

Q10

$$\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} 3x \\ 3y+2 \end{pmatrix}$$

$$\begin{pmatrix} 3 \\ -2 \end{pmatrix} \rightarrow \begin{pmatrix} 3(3) \\ 3(-2)+2 \end{pmatrix}$$ M1

$$\begin{pmatrix} 3 \\ -2 \end{pmatrix} \rightarrow \begin{pmatrix} 9 \\ -4 \end{pmatrix}$$

$(3,-2) \rightarrow (9,-4)$ A1

TOTAL SCORE =30

APPENDIX D

Marking Scheme (Post-Test)

Q1

(a) $x = 2$ A1

(b) $x = y$ A1

(c) $y = 0$ A1

(d) $y = -1$ A1

(e) $x = -y$ A1

(f) $x = 0$ A1

Q2

$$\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} x \\ 2a - y \end{pmatrix}$$

$$\begin{pmatrix} -2 \\ -6 \end{pmatrix} \to \begin{pmatrix} -2 \\ 2(1) - (-6) \end{pmatrix}$$

$$\begin{pmatrix} -2 \\ -6 \end{pmatrix} \to \begin{pmatrix} -2 \\ 8 \end{pmatrix}$$

$\therefore (-2,-6) \to (-2,8)$ A1

Q3

$$\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} 2a - x \\ y \end{pmatrix}$$

$$\begin{pmatrix} -3 \\ 1 \end{pmatrix} \to \begin{pmatrix} 2(-2) - (-3) \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} -3 \\ 1 \end{pmatrix} \to \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$(-3,0) \to (-1,1)$ A1

Q4

(a) $\begin{pmatrix} -3 \\ -4 \end{pmatrix}$ A1

(b) $\begin{pmatrix} -1 \\ -3 \end{pmatrix}$ A1

(c) $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ A1

(d) $\begin{pmatrix} 4 \\ 1 \end{pmatrix}$ A1

(e) $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ A1

(f) $\begin{pmatrix} -3 \\ -4 \end{pmatrix}$ A1

Q5

$$\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} x+a \\ y+b \end{pmatrix}$$

$$\begin{pmatrix} 3 \\ 7 \end{pmatrix} + \begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 5 \\ 6 \end{pmatrix}$$

$(3,7) = (5,6)$ A1

Q6

(a) Anticlockwise 90^0 / clockwise 270^0 rotation about the origin B1

(b) Anticlockwise/clockwise rotation about the origin through 180^0 B1

(c) Clockwise 90^0 / anticlockwise 270^0 rotation about the origin B1

(d) Anticlockwise/clockwise rotation about the origin through 180^0 B1

(e) Clockwise 90^0 rotation about the point (2, 1) B1

(f) Clockwise 90^0 / anticlockwise 270^0 rotation about the origin B1

Q7

$$\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} -y \\ x \end{pmatrix}$$

$$\begin{pmatrix} 2 \\ -4 \end{pmatrix} \rightarrow \begin{pmatrix} 4 \\ 2 \end{pmatrix}$$

$(2,-4) \rightarrow (4,2)$ A1

Q8

$$\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} y \\ -x \end{pmatrix}$$

$$\begin{pmatrix} 4 \\ 1 \end{pmatrix} \xrightarrow{-(-1,3)} \begin{pmatrix} 5 \\ -2 \end{pmatrix}$$

$$\begin{pmatrix} 5 \\ -2 \end{pmatrix} \rightarrow \begin{pmatrix} -2 \\ -5 \end{pmatrix}$$

$$\begin{pmatrix} -2 \\ -5 \end{pmatrix} \xrightarrow{+(-1,3)} \begin{pmatrix} -3 \\ -2 \end{pmatrix}$$ A1

Q9

(a) Rotation about the origin through $\pm 180^0$ B1

(b) Translation by the vector $\begin{pmatrix} -4 \\ 3 \end{pmatrix}$ B1

(c) Reflection in the line $x = 0$ / reflection in the $y-axis$. B1

(d) Translation by $\begin{pmatrix} -4 \\ 3 \end{pmatrix}$ followed by reflection in the line $y = 0$ .B2

Q 10

$\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} 3(-3) \\ 3(2)+2 \end{pmatrix} \rightarrow \begin{pmatrix} -9 \\ 8 \end{pmatrix}$ M1A1

TOTAL SCORE = 30

APPENDIX E

Teachers Questionnaire

Dear colleague teachers,

This questionnaire seeks your views on how effective you find the use of GeoGebra in teaching rigid motion. Your candid responses will enable Mathematics teachers make informed choices when designing lessons on rigid motion and other concepts in Mathematics. This is expected to lead to an improved teaching and learning of Mathematics in our schools. The filling of this questionnaire will take you approximately one (1) hour to complete.

Counting on your maximum cooperation while assuring you that all your responses will be treated as strictly confidential.

Instruction: You are kindly requested to tick (√), circle or supply short response(s) where necessary in spaces provided for the various questions in the questionnaire.

A Teachers' Background Information

1. Sex Male [ ] Female [ ]
2. Age 25-30 [ ] 31-36 [ ] 37-42[ ] above 42[ ]
3. Years of teaching experience.

   1-3 [ ] 4-6 [ ] 7-9 [ ] above 9 years [ ]

4. Which university did you attend?..
5. What programme did you offer? ……………………………………………….
6. What was your major area of study? …..
7. What was your minor area of study, if any?...
8. What area of mathematics do you teach?

Core mathematics [ ] Elective mathematics [ ]

Both core and elective mathematics [ ]

B Teachers background knowledge in rigid motion

9. How many years have you been teaching in your current school?

   0-2 years [ ] 3-6 years [ ] above 6 years [ ]

10. How many times have you taught rigid motion in your current school?

    Once [ ] Twice [ ] More than twice [ ] Never [ ]

11. How many times have you ever taught rigid motion since you started teaching?

    Once [ ] Twice [ ] More than twice [ ] Never [ ]

12. How have you been teaching rigid motion over the years?

    Giving rules and formulae to students [ ]

    Guiding students to use graph sheets practically to discover formulae [ ]

    Guiding students to use computer software to discover formulae [ ]

13. In your view, which approach to the teaching of rigid motion are students comfortable with?

    Giving rules and formulae to students [ ]

    Guiding students to use graph sheets practically to discover formulae [ ]

    Guiding students to use computer software to discover formulae [ ]

14. Which of the following is/are more difficult to teach practically? *Choose as many as apply.*

    Reflection [ ] Rotation [ ] Translation [ ]

C Effectiveness of use of GeoGebra to teach rigid motion

For each of the statements below, please indicate the extent of your agreement or disagreement by placing tick (√) in the appropriate column to show your impression about how effective you find the use of GeoGebra to teach rigid motion.

	Strongly Agree	Agree	Undecided	Disagree	Strongly Disagree
(15) GeoGebra is user friendly.					
(16) GeoGebra is easy and intuitive to use.					
(17) Teaching reflection with GeoGebra enhances students‛ understanding.					
(18) Teaching rotation with GeoGebra enhances students‛ understanding					
(19) Teaching translation with GeoGebra enhances students‛ understanding					
(20) Teaching rigid motion with GeoGebra is the best practical approach.					
(21) Students easily deduced formulae for rigid motion after using GeoGebra.					
(22) Students‛ attitude towards the GeoGebra designed lessons was positive and encouraging.					
(23) Students were able to answer questions on rigid motion correctly after taking them through GeoGebra lessons.					
(24) Students‛ interest in learning rigid motion has suddenly soared up.					
(25) It is potentially helpful to use GeoGebra to teach senior high school students.					
(26) The use of GeoGebra to teach rigid motion is very effective.					

(27) In your view how do you find the use of GeoGebra to teach rigid motion?

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THANK YOU FOR YOUR KINDNESS

APPENDIX F

Observation Guide

	Difficulties observed			
Sections	Group 1	Group 2	Group 3	Group 4
1	Counting units to and away from mirror line		Counting units to and away from mirror line	Counting units to and away from mirror line
2	Drawing the lines $x = y$ $x = -y$ and getting perpendicular distances to and from mirror lines	Drawing the lines $x = y$ $x = -y$ and getting perpendicular distances to and from mirror lines	Drawing the lines $x = y$ $x = -y$ and getting perpendicular distances to and from mirror lines	Drawing the lines $x = y$ $x = -y$ and getting perpendicular distances to and from mirror lines
3	Interpretation of translation vectors		Interpretation of translation vectors	
4	Interpretation of clockwise and anticlockwise rotation	Interpretation of clockwise and anticlockwise rotation	Interpretation of clockwise and anticlockwise rotation	Interpretation of clockwise and anticlockwise rotation
5	Measuring of angles of rotation with protractor			Measuring of angles of rotation with protractor
6	Rotation about a point other than the origin	Rotation about a point other than the origin	Rotation about a point other than the origin	Rotation about a point other than the origin

APPENDIX G

Activities on translation, reflection and rotation.

Activity 1

Translation

(a) Using "Polygon" tool, make a polygon. Can be regular or not. The polygon is completed when you click on the original point.

(b) Create a "Vector between two points" along which translation will occur.

(c) Click "Translate object by vector", select the polygon, then the vector along which it will be translated.

(d) Play with the vector length/direction or the polygon using the selection tool.

Activity 2

Reflections

(a) Using "Polygon" tool, make a polygon. Can be regular or not. The polygon is completed when you click on the original point.

(b) Construct a line of reflection using the "Line" tool.

(c) Click "Reflect object" tool, select object to be reflected, and the line about which it is to be reflected.

(d) Use the "Selection" tool to highlight multiple objects. Also, draw multiple lines about which to reflect. Repeat the previous steps as in figure 5.

Activity 3

Rotations

(a) Using "Polygon" tool, make a polygon. Can be regular or not. The polygon is completed when you click on the original point.

(b) Select "Rotate" tool, choose a point of rotation, and an angle.

(c) Play with the point of rotation (inside/outside the polygon, vertices) and the angle.