UNIVERSITY OF EDUCATION, WINNEBA

INVESTIGATING THE IMPACT OF INQUIRY-ORIENTED INSTRUCTIONAL APPROACH (IO-IA) ON TEACHING AND LEARNING OF PARTIAL DIFFERENTIAL EQUATIONS TO UNDERGRADUATE STUDENTS OF UEW

MOHAMMED ALI

DOCTOR OF PHILOSOPHY

2019

UNIVERSITY OF EDUCATION, WINNEBA

INVESTIGATING THE IMPACT OF INQUIRY-ORIENTED INSTRUCTIONAL APPROACH (IO-IA) ON TEACHING AND LEARNING OF PARTIAL DIFFERENTIAL EQUATIONS TO UNDERGRADUATE STUDENTS OF UEW

ALI MOHAMMED

9150110010

A thesis in the Department of Mathematics Education, Faculty of Science Education, submitted to the School of Graduate Studies in partial fulfilment of the requirements for the award of the degree of

Doctor of Philosophy

(Mathematics Education)

in the University of Education, Winneba

DECEMBER, 2019

Declaration

STUDENT'S DECLARATION

I, Ali Mohammed, declare that the thesis, with the exception and references contained in published works which have all been identified and duly acknowledged, is entirely my own original work, and it has not been submitted, either in part or whole, for another degree elsewhere.

SIGNATURE: . . . .

DATE: . . . .

SUPERVISOR'S DECLARATION

We hereby declare that the preparation and presentation of this work was supervised in accordance with the guidelines for supervision of thesis as laid down by the University of Education, Winneba.

Prof. Samuel Kwesi Asiedu-Addo, PhD (Principal Supervisor)

Signature: . . . .

Date: . . . .

Alhaji Prof. Issifu Yidana, PhD (Co-Supervisor)

Signature: . . . .

Date: . . . .

Dedication

This study is dedicated to my late mother, *Augustina Dwabeng Serwaa* and late father Mohammed Ali for their tireless effort which brought me to the level which served as a springboard for me to enroll in this program. *May their souls rest in the bosom of the Lord Almighty*. This study is also dedicated to my two daughters, Akua Atuobua Frimpong and Afua Serwaa Frimpong, my son Kwadwo Adu Frimpong (aka Kwadwo Junior) and my dear wife Ernestina M. Kumi as well as my entire family.

Acknowledgments

I wish to extend my profound gratitude to the Almighty Jehovah who in His wisdom and abundant grace made it possible for me to go through the entire program successfully. Special thanks also go to my supervisors Prof. S. K. Asiedu-Addo and Alhaji Prof. Issifu Yidana for their unmeasurable inputs, advice, motivation, support and encouragement. I am indeed most grateful to them for the precious time they spent on me to make this work a success. I am also grateful to all the Heads of Mathematics Education Department (both previous and current) as well as the deans (both previous and current) whose immense programs and organization helped in allowing other professors, senior lecturers, lecturers as well as students for their inputs especially during faculty seminars in the study in other to make it better. I also thank all lecturers in the faculty of science for their criticisms that have helped in shaping this research. To the entire Mathematics Education Department, I say you were an inspiration. Your tireless effort has brought me this far. Finally, I like to thank my entire family for their support and encouragement which has led to the success of this program. To my dear wife and sweet kids, I say thank you for being there for me all the time, including times that I came home late due to work. I cannot also forget my friends most especially Seidu Mahama Boateng (aka Lalas), Adjei Buabeng just to mention a few for their numerous advice, encouragement and sometimes financial support in making this study a success. I say, may Jehovah bless you all.

Contents

List of Tables

List of Figures

List of Abbreviations

DBI	Design-Based Instructions
DEs	Differential Equations
GES	Ghana Education Service
IB-A	Inquiry-Based Approach
ICT	Information and Communication Technology
IO-DE	Inquiry-Oriented Differential Equations
IO-IA	Inquiry-Oriented Instructional Approach
NRC	National Research Council
NSE	National Science Education
ODEs	Ordinary Differential Equations
PBI	Project-Based Instructions
PBL	Project-Based Learning
PDEs	Partial Differential Equations
RME	Realistic Mathematics Education

Abstract

The study investigated the impact of inquiry-oriented instructional approach (IO-IA) on the teaching and learning of partial differential equations (PDEs) at the undergraduate level with the purpose of addressing challenges that students face in the learning of PDEs. A purposive sampling technique was used to select four hundred (400) students from the two cohort groups (2016/2017 and 2017/2018 academic year students) of mathematics students. The selected students in each year group were put into two groups: control and experimental. The study employed sequential explanatory design. Observation, test and semi-structured interview were the data collection instruments used during the study. The independent sample t-test for equality of means indicated that there was significant difference between the mean scores of the control and the experimental groups in all the phases in favour of the experimental group. The study revealed that instructional strategies and methods, students' ability to recall and apply pre-requisite knowledge in solving PDEs, computational ability and availability of ICT tools were the major factors affecting teaching and learning of PDEs. The study recommended the integration of IO-IA by teachers in PDEs. Other recommendations made by the researcher is the integration of ICT into the teaching and learning of PDEs, adaption of measures to help students improve their computational abilities and the inclusion of inquiry-activities in teaching and learning to improve retention among students. The researcher also recommended that institutions should provide ICT tools and software and also adapt small class size for effective integration of IO-IA in teaching and learning process.

Key Words: *Differential Equations, Elaboration, Engagement, Exploration, Explanation, Evaluation, Epistemology, Ontology, Inquiry-Oriented, Inquiry-Based, Mathematical Modelling, visualization, Instructional Design, Ordinary Differential Equations, Partial Differential Equations, Realistic Mathematics Education.*

Chapter 1

INTRODUCTION

1.1 Overview

This chapter consists of the background of the study, the statement of the problem, the purpose and objective of the study, the research questions, the significance of the study, the limitation and delimitation of the study as well as the organization of the study.

1.2 Background of the Study

In almost every country, mathematics occupies a central place in the school curriculum. Mathematics is said to be logical, reliable and a growing body of concepts. It makes use of specific language and skills to model, analyse and interpret the world. Mathematics which is described as a human activity involves creativity in the discovery of patterns of shape and number, the modelling of situations, the interpretation of data and the communication of ideas and concepts. One of the rationale of the mathematics syllabus of Ghana Education Service (2012) is that the acquisition of knowledge and skills in mathematics always help students to discover, adapt, modify and come out with innovations to face the changes around the globe and future challenges as well. At all levels, the study of mathematics is more than just the acquisition of basic concepts and skills. More importantly mathematics aims at finding general strategies for finding solutions to real-life situations. Among the general aims of the elective mathematics syllabus for Senior High Schools (SHS) in Ghana (Ghana Education Service, 2010) is to help students use mathematics as a tool for analysis, critical and effective thinking, not forgotten its usefulness in commerce, trade and technology. Among the numerous aspects of mathematics in the undergraduate studies in most tertiary institutions is Differential

Equations (Ordinary and Partial Differential Equations) which has real-life application in many fields.

The concept of differential equations are mostly used in solving applied problems in various disciplines ranging from Physics through Medicine and Economics. Differential equations have remarkable ability to predict the world around us (Pinchover & Rubinstein, 2005; Braun, 1983). Louis L'Amour, a famous inspirational writer ones said "The only thing that never changes is that everything changes" (L'Amour, n.d.). That is, one thing that will never change is the fact that the world is constantly changing. And anything that changes is a dynamical system and differential equation is used to explain these kinds of phenomena. Mathematically, rates of change are described by derivatives of functions (Arrowsmith & Place, 1990). Derivative of functions are used to describe the world around us, say the growth of a plant, the fluctuations of the stock market, the spread of diseases, or physical forces acting on an object. The way these functions inter-relate with other mathematical parameters is all the time described by differential equations. Among the most basic examples of differential equations is the Malthusian Law of population growth, $\frac{dP}{dt} = rP$ which shows the change in population (P) of any species with respect to changes in time (t) (Braun, 1983). This growth rate is equal to the product of r and the population, P, where r is the constant which depends on the species. In the field of medicine, differential equations are used in the modelling of spread of disease. In the field of engineering, it plays a vital role in determining the flow of electricity. In the field of chemistry, differential equations are used in modelling chemical reactions and compute half-life of radioactive substances. In the field of economics, differential equations is used to find optimum investment strategies and money flow/circulation while in the field of Physics, it is used to describe motion of waves, pendulum or chaotic systems. Differential equations classified into either ordinary or partial. In ordinary differential equations, only one independent variable exists in the equation but in partial differential equations, two or more independent variables are used to determine the dependent variables. For example, to determine the inflation of a country, several commodities come into play (Braun, 1983).

Partial Differential Equations (PDEs) are among core parts of many applied studies when it comes to the field of sciences, economics and technologies. In the field of applied sciences for example, the change in one dependent variable may be as a result of the changes in several variables, such phenomenon is described by partial differential equations. The key defining property of partial differential equations is that there is more than one independent variable $x, y, \ldots$, and there is a dependent variable, that is unknown function of these variables $u(x, y, ..)$ and its partial derivatives (Strauss, 2008; Coleman, 2013). Partial differential equations have numerous applications in our daily activities and inventions and so play a major role in applied studies. PDEs describes all kinds of physical phenomena and applications in all sorts of fields (Strauss, 2008). For example, the price of a particular commodity may depend on the demand and supply on that commodity. The change in population of any species may depend on the availability of food, the environment, and even the predators which feed on that species. Whenever the independent variables affecting a particular unknown variable is more than one, the derivative is said to be partial and hence partial differential equation is applied in this situation.

Notwithstanding the numerous applications of differential equations, students at the undergraduate level are faced with difficulties in understanding the concept of differential equations and find it difficult to apply such concepts. This may be due to the way the subject is being taught in our universities or the way students also learn. Rasmussen, et al (2006) stated that when students approach the study of differential equations as mechanisms that describe how functions evolve and change over time, they develop a conceptual understanding of key themes around the course. But in most cases, mathematics teachers sometimes use approaches that are structured around learning activities that emphasize the classification of equations into various types and the use of specific algorithms to solve them (Rasmussen et al., 2006). In most of the cases, mathematics teachers' instructional approach is focused on the use of rules and algorithms in finding solutions to problems which lead to lack of conceptual understanding.

Students have also associated the analysis of differential equations with the use of series of algorithms and procedures to find their solutions and to solve corresponding problems (Rasmussen & Kwon, 2007; Kwon, 2008, Camacho-Machin, et al., 2012). From the researchers observation, it was realized that students in differential equations class were learning analytical, graphical and numerical methods in a compartmentalized manner. In this way students do not establish proper connections or relate meanings around mathematical properties associated with the approaches they use. This was seen in the students' inability to interpret the meaning of solutions obtained after solving differential equations. For example, in Partial Differential Equations (PDE) class, most of the students could not interpret the meaning of initial condition $u_t(x,0) = f(x)$ for a given wave equation $u_{tt} = c^2 u_{xx}$ and its corresponding boundary condition $u_x(0,t) = 0$, $u(L,t) = g(x)$, $0 < x < L$. Camacho- Machin, et al (2012) on their study on "An Exploration of students' conceptual knowledge built in a first Ordinary Differential Equations course (part 1)" stated that, the focus of learning activities shape to some extent the students comprehension and use of mathematical concepts.

The researcher has been teaching ODEs and PDEs over the years and has realized that, students lack the concept of limits and derivative of functions. This lack of understanding these concepts is easily seen in their differential equations courses (Ordinary Differential Equations (ODEs) and Partial Differential Equations (PDEs)). Apart from their lack of understanding of the concepts taught in Algebra, Linear Algebra and Calculus, they also lack the ability to transfer what is being taught to new concepts. For example, even though students were taught determinant of matrices in their Algebra and Linear Algebra courses, transferring that to their ordinary differential equations (as Wronskian) and partial differential equations (as Jacobian) in solving system of linear differential equations become difficult. It mostly seems as if students have no knowledge on the concept of determinants of matrices. Another typical example is the concept of linearly dependence or independence of fundamental set of solutions of a differential equation. The following theorems as cited in Asiedu-Addo (2016) are similar to those treated in linear algebra course, yet students are not able to connect them.

Theorem: 1.1. *If the functions y_1 and y_2 are linearly dependent in an interval I and their derivatives y_1' and y_2' exist in that interval, then the Wronskian (W) of y_1 and y_2 is identically zero.*

Theorem: 1.2. *If the Wronskian, W, of the functions y_1 and y_2 is not identically equal to zero ($W \neq 0$) in an interval, then y_1 and y_2 are linearly independent in the interval.*

Students in their Algebra classes, were taught that, if the system of linear equations has no solution (does not meet at a point geometrically), the determinant of the coefficient matrix is zero. Also, the determinant of the coefficient matrix of a system of linear equation with infinitely many solutions is also zero. This concept simply means that, when a system of linear equations has no solution, the determinant of their coefficient matrix will be zero. But is is not a guarantee that, if the determinant of the coefficient matrix of system of linear equations is zero, the system has no solution. The system could also have infinitely many solutions. But students still have difficulties in understanding that the converse of the Theorem 1.2 is not always true. This lack of understanding of concept may be attributed to the mode of instructional delivery and hence the need to take positive steps towards mode of instructional delivery to enhance teaching and learning of partial differential equations in particular and mathematics in general (Barrow, 2006).

Before the commencement of the actual study, a diagnostic test on Ordinary Differential Equations (ODEs) was conducted on the selected sample. The test (see Appendix I(c)) was made up of three questions and students were supposed to answer all the questions. The solutions presented by the students indicated that students preferred answering questions which involve rules and formulae to those involving real-life applications. The first question was framed as a real-life story problem (compartmental model) and the second question was the same question but was an already formulated model with initial conditions supplied (see appendix I(c)). The test revealed that for the 2016/2017 academic year, 170 students out of 218 students representing 77.98% attempted questions 2 and 3 but not question 1. In the 2017/2018 academic year, 152 students out of 182 representing 83.51% also attempted questions 2 and 3 but not question 1. During review of the diagnostic test, it was realized that those who did not attempt question 1 in both groups did not even see that the two questions were similar and hence they did not answer question 1 which was an application question. From the discussion, it was revealed that over 50% of the students did not read question 1 at all while 33% read the question but did not take time to analyse it. This shows how students try as much as possible to avoid application questions. It also shows how students have difficulties in linking what they have learnt in mathematics to real-life situations. One of the ways to overcome such a situation is the need to introduce an effective instructional design.

The need for instructional design models in our education has been emphasized by many researchers such as Rasmussen, Kwon and Allen (2005), Holton (2001) and Richard (1991) in the last two decades. According to Assem, et al. (2018), inquiry-based learning is a panacea to discoveries and inventions in Ghanaian education settings. The low conceptual understanding of concepts learnt, according to Mensah-Wonkyi and Adu (2016), is due to the traditional methods used in teaching mathematics. This is because the traditional method does not encourage creativity and critical thinking in the Mathematics class (Mensah-Wonkyi & Adu, 2016). Some of these models include electronic- instruction (E-Instruction), project-based learning, community of practice and mentoring and inquiry-based learning. All these learning models are supposed to follow some sound behaviourist and cognitive design principles (Smith & Ragan, 1999; Khan, 2000; Schneider, 2006). According to Rasmussen, et al (2005), the main purpose of inquiry- based learning is concept learning. All these earlier writers have a common goal, ”to come out with an instructional design model to facilitate teaching and learning in our institutions”. Instruction according to Driscoll (2000) is the deliberate arrangement of learning conditions to promote the attainment of some intended goals. It is the planned arrangement of information and experiences which leads learners to acquire particular capabilities. Design on the other hand is an activity or process that are engaged by people inventions or creations (Smith & Ragan, 1999) and is related to planning.

Instructional design according to Smith and Ragan (1999) refers to the systematic and reflective process of translating principles of learning and instruction into plans for instructional materials, activities, information resources and evaluation. According to Reiser and Dempsey (2012) cited in Seel, et al (2017), instructional design is also defined as a systematic procedure in which educational and training programs are developed, aiming at a substantial improvement of learning. These definitions are mostly associated with the assumptions that certain instructional design models could serve as a frame of reference and regulation of course development and lessons. They aim at learning improvement and also influence motivation in the learners and their attitudes for the achievement of deeper understanding. Conceptual understanding of the subject matter being taught in our schools and universities have gone down giving rise to rote learning and memorization of rules/laws and formulas without proper understanding of the concept. These modes of instruction and learning have gone a long way to make our young graduates unproductive and thereby affecting development of our nation as a whole (Seel et al., 2017). There is therefore the need to develop or design a very effective instructional design to improve teaching and learning in our schools.

The goal of this study was to use the inquiry-oriented instructional approach to point out some constructive ideas in teaching and learning of Partial Differential Equations at the undergraduate level. The National Council for Teachers and the National Research Council both in United States of America and other early writers like Rasmussen, Kwon, Richard and so on (National Research Council, 1996, 2000; Richard, 1991; Rasmussen, et al., 2005; Rasmussen, et al., 2006; Rasmussen & Kwon, 2007; Kwon, 2003, 2005; Kwon, et al., 2005; Kwon, 2008) found out the need to recognize the characteristics of effective teaching in order for teaching and learning of mathematics to improve in our various faculties or departments. The study partly adapted the "inquiry-oriented Differential Equations (IO-DE) project approach proposed by Rasmussen and Kwon (2007) and together with modelling and visualization of differential equations proposed by Mrozek (2014) and Blum (1993).

In the traditional mathematics classroom, which is textbook-dominated approach, students listen and watch the demonstration of mathematical procedures by the teacher which is followed by textbooks exercise which serves as a practice of what is being demonstrated. According to Alsina (2002), this prevalent phenomenon of the traditional approach to instruction is having a link to some general existing 'myths' and practices in mathematics teaching at undergraduate level. Among the myths are:

1. Good researchers are always good teachers and hence the key criteria for selection teachers for appointments and promotions is based on high quality research.
2. The self-made-teacher tradition: This myth is based on the claim that one does not need or require any specific training to be excellent in university teaching but can achieve that through accumulated experiences, clear presentational skill and a good knowledge of the subject matter.
3. Reduction organization: That is, topics or subject matter must be linearly presented, definitions, theorems or/and proofs must be sequentially stated in their most general form.
4. The top-down approach: This myth is of the view that it is not effective to help learners by teaching from top to down and that learning must be a bottom-up process.
5. The perfect-theory presentation: This makes students believe that mathematics is complete after the theorem presentation and that proving such theories is just a deductive game. Also, errors, false trials and zigzag argument which is very crucial in human life have no place in Mathematical world. In other words, once a

theory is proved, it is finished. This presentation style does away with the aspect of Mathematical discoveries in 'human nature'. This makes learning of mathematics similar to dictatorship.

Unfortunately, these assumptions and phenomena have not helped much in the teaching and learning of mathematics.

The tremendous growth in educational research in the last two decades have shown that, there is a big gap between what is taught and what is learnt when traditional method of teaching are used during instructional delivery (Rasmussen & Kwon, 2007; Dummont et al., 2010; haines and Crouch, 2007; Halcomb and Hickman, 2015). The results by earlier researchers have shown that there is a statistical evidence of the limitations of the traditional teaching practice. For example, conceptualization of the concept of limits of functions, derivative and integral of functions have recently been difficult and the discrepancies between formal definition of limits and derivative of functions (Orton, 1980; Tall & Vinner, 1981). Also, students have difficulties in logical reasoning and proofs, connection of graphs with physical concepts and the real world and the difficulties in learning the basic notations of linear algebra are all due to the traditional teaching approach. There is therefore the need to guide the students to make their own inquiries during lesson delivery and hence the need for the introduction of inquiry-based teaching (Alibert & Thomas, 1991; Schoenfield, 1985; Selden & Selden, 1995; McDermott et al., 1987; Svec, 1995; Harel, 1989).

The use of the term inquiry includes the following three principles: deep engagement in mathematics, peer-to-peer interaction, instructor interest in and use of student thinking. Inquiry according to Uno (1999) may be referred to as a technique that encourages students to discover or construct information by themselves instead of having teachers directly reveal the information. It is a multifaceted activity that involves making observation; posing questions; examining books and other sources of information to see what is already known; planning investigations; reviewing what is already known in light of experimental evidence using tools to gather, analyse and interpret data, proposing answers, explanations and predictions; and communicating the results (National Research Council, 1996, 2000; Rasmuseen & Kwon, 2007, Kwon, 2003, 2005, 2008). The need for students to investigate challenging problems and engaged in mathematical activities in order for them to reinvent many key mathematical ideas is of great importance in teaching-learning process.

Mohammed, et al (2012) in their study revealed that teaching practices, teachers' attribution, classroom climate, students' attitude towards mathematics and students' anxiety were factors that influences students' achievements in mathematics. Among these factors, teaching practices was rated the highest and the results showed a significant positive relationship between students' achievement and teaching methods and practices. Murray (2013) in his study also revealed that prior academic achievement, self-efficacy, academic resources, self-regulation and learning styles were the major factors that influence students' mathematics achievement. Murray (2013) revealed that prior academic achievement, learning styles and academic resources were rated the highest among these factors. Demographic, instructional strategies and methods and individual factors (background knowledge, computational ability, confidence) were considered as factors affecting learning of mathematics and instructional studies was rated the highest factor according to Belhu (2017). Also, according to Gunaseelan and Pazhanivelu (2016), instructional strategies and methods, teacher competency in mathematics education and motivation were the three most influential factors affecting the mathematics achievement and must be considered in the design decision during instructional design. In the field of partial differential equations, factors like epistemological mathematics problem-solving beliefs, belief about usefulness of mathematics, self-regulated learning (SRL) strategies and goal orientations have great potential to enhance differential equations problem ability (Bibi et al., 2007). Other factors included entry behaviour, pass mark, facilities and resources, communication in mathematics (poor knowledge in English Language and poor communication of mathematical facts), absenteeism from lectures, drug and alcohol, career choice, peer influence and mathematical knowledge acquisition (Alfred et al., 2012).

As mathematics educators, there is the need to investigates students' mathematical thinking and this can be achieved if students are guided to build mathematical models. Mathematics is responsible for the provision of opportunities for students to move beyond being passive recipients of knowledge to knowledge builders. We are also to ensure that our students are creative and innovative in finding solutions to problems. The role of mathematics educators in human progress is to equip students with the requisite knowledge, skills and dispositions for them to be able to solve the daunting problems of our age. We can only achieve these goals by making sure that our students move from intuitive understanding of concepts and natural curiosity to knowledge creation. That is, to move our students to space where ideas can be transformed into formalized understanding and further questioning. In order for us to achieve this task, there is the need for reformation in our pedagogical approach which will go a long way to reform the way students learn. Prominent among these pedagogical approaches are the inquiry-based teaching and learning approach. According to Barrow (2006), Dewey was the first to call for an inquiry-based teaching in science in 1910 when he reported that, too much emphasis is on learning of facts but not enough on critical thinking and conceptual understanding. These necessitated the need for the introduction of inquiry-oriented instructional approach (IO-IA) in the teaching and learning of partial differential equations (PDEs).

The IO-IA is an inquiry-based teaching and learning approach which was adapted from the inquiry-oriented differential equations (IO-DE) by Rasmussen (2001), Kwon (2005, 2008), Rasmussen and Kwon (2007), Rasmussen, et al. (2017) to address significant difficulties in the teaching of science and to address significant difficulties in the conceptualization of the limit processes underlying the notations of derivatives and integral (Orton, 1980), discrepancy between the form definitions students were able to quote and the criteria they used to check properties such as functions, continuity and derivative, students difficulties in learning the basic notations of linear algebra and differential equations (Rasmussen, 2001) and the need for prospective mathematics teachers to experience innovative teaching as learners of mathematics and science just to mention a few. These challenges and opportunities point to the need to explore innovative approaches to teaching and learning of mathematics (Rasmussen et al, 2017). One of such innovative approach is the IO-IA.

Inquiry-oriented instructional approach (IO-IA) is an inquiry-based teaching and learning approach which involves the following:

1. The teacher comes with a problem or design a prediction task in which the students are guided by a step-by-step approach to investigate it.
2. The students are guided to gather data or simulate the problem to come out with the relationship between the variables identified in the problem.
3. The students then formulate a model (equation) which relates the relevant variables using the data from the investigation.
4. The students are then guided to use either graphical approach, numerical approach, simulation, analytical or combination of these to solve the equation.
5. The students then interpret the solution and use it to make predictions.

The teacher's role in all these processes is that of a guide to help the students come out with their innovative ideas.

1.3 Statement of the Problem

Learning mathematics at all levels involves more than simply acquiring basic concepts and competencies. More specifically, it includes an understanding of the underlying mathematical reasoning, general problem-solving techniques, mathematical communication and the inculcation of positive attitudes towards an appreciation of mathematics as an essential and influential resource in everyday life (Rasmussen, 2001; Kwon et al., 2005). Despite the several real-life applications of partial differential equations (PDEs), it has been very difficult and complex to teach and learn PDEs. Among the several reasons are the following:

- The equations are complex and so it is very difficult to introduce students to real-life applications.
- Typical approaches to this field normally leads to finding analytical solutions to transport, heat, wave, Laplace, etc equations which themselves are complex and difficult to solve.
- Solution to these equations are functions and not numbers.
- Most of these equations do not have exact solution, or have no analytical solution and hence the need to apply numerical methods. However, numerical methods are difficult and requires sophisticated computer applications which can compute numerous iterations within a short time.

Partial Differential Equations are mostly seen by students as a bunch of functions or equations associated with rules and formulas as their only means of solving. It is therefore common to find approaches that are structured around learning activities that emphasize the classification of PDEs into various types and the use of specific algorithms in solving them as postulated by Rasmussen et al. (2006). This approach in teaching have limited the students' ability to think critically and analytically and have also limited their mathematical reasoning ability and problem-solving skills. There is therefore, the need to use an instructional approach that could encourage critical thinking and in-depth analysis among students. It is among these reasons that necessitated the need to look at the Inquiry-Oriented Instructional Approach (IO-IA) in teaching and learning of Partial Differential Equations in the undergraduate level of the Department of Mathematics Education at University of Education, Winneba (UEW). The IO-IA is an inquiry-based teaching approach. The focus of the IO-IA is to help students develop problem-solving skills and mathematical thinking or reasoning (to think critically and analytically), inculcate in them positive attitudes towards PDEs in particular and mathematics in general. The IO-IA is also focused on helping teachers to investigate student thinking or reasoning ability in the field of PDEs in particular and mathematics in general.

1.4 Purpose of the Study

The purpose of this study was to investigate the impact of the use of IO-IA on teaching and learning of PDEs with the aim of addressing challenges that undergraduate students face in their learning of PDEs.

1.5 Objectives of the Study

The objectives of the study were:

1. to investigate the factors that affect teaching and learning of Partial Differential Equations (PDEs) and the challenges associated with these factors in the undergraduate level.
2. to determine the impact of the use of Inquiry-Oriented Instructional Approach (IO-IA) on teaching and learning of PDEs.
3. to investigate the challenges associated with the use of the IO-IA in teaching and learning of PDEs in the undergraduate level.

1.6 Research Questions

The following research questions guided the study

1. What factors affect the teaching and learning of partial differential equations (PDEs) and the challenges that associated with these factors in the undergraduate level?
2. How does the use of inquiry-oriented instructional approach (IO-IA) impact teaching and learning of PDEs in the undergraduate level?

3. What are the challenges associated with the use of the Inquiry-Oriented Instructional Approach (IO-IA) in the teaching and learning of PDEs in the undergraduate level?

1.7 Hypothesis

The following hypothesis was tested to determine the impact of the IO-IA on teaching and learning of PDEs.

- H_o: There is no significant difference between the performance of the students who were taking through the IO-IA and the traditional method of teaching.
- H_a: There is a significant difference between the performance of the students who were taking through the IO-IA and the traditional method of teaching.

where H_o is null hypothesis and H_a is alternate hypothesis.

1.8 Significance of the Study

To the teachers, this study suggested an alternative instructional approach for teaching undergraduate PDEs in particular and mathematics in general. This would help teachers to design meaningful lessons for the students. This study would also help teachers to investigate students reasoning ability so that they would be able to design models that would help students to think critically and analytically and develop their problem-solving skills. The outcome of this study would again challenge teachers to adapt innovative ways of teaching PDEs in particular and mathematics in general. The IO-IA apart from helping the teachers to build models of student thinking, would also help teachers to learn new method of teaching and figure out the various questions or task which would be posed to students for them to learn mathematics effectively. The study would again help teachers to identify factors affecting teaching and learning of Partial Differential Equations in particular and mathematics in general. The study would also help teachers to assess the impact of inquiry-based teaching on students as well as challenges associated with the use of inquiry-based teaching. To the students, this study would help them to relate the study of PDEs in particular and mathematics in general to the real-life situations. The study would also help students through engagement of mathematical activities and investigation of challenging problems learn new mathematics. This would affect their belief about mathematics and the nature of mathematics in general. The study also contributes to the implementation of the educational policy requiring teachers and students to teach and learn with technology. The use of the 5E instructional model would also change the reserved-type and and students who are less interested in mathematics to share their ideas. The inquiry-oriented teaching and learning approach would help students reinvent many key mathematical ideas and methods for the analysis of solutions to the differential equations. In summary the use of the 5E model would help the teachers to design meaningful and purposeful lessons for the students. It would enable the teachers to assess students' knowledge before the start of the exploration activity which made the evaluation of the students' academic level more appropriate and enables teachers to personalize lessons according to student needs. The IO-IA would also introduce challenging task, which is situated in realistic situations to serve as the starting point for students' mathematical inquiry.

1.9 Delimitation of the Study

The study covers the entire level 400 students of the Department of mathematics Education in the University of Education, Winneba in the 2016/2017 and 2017/2018 academic years. The study was conducted for a period of two years. The first part of the study was conducted on level 400 students during 2016/2017 academic year. The second part was conducted on the level 400 students in the 2017/2018 academic year. The study was limited to these students because Partial Differential Equations is taught at level 400 in the Department of Mathematics Education in the university. The study focused on the formation of first-order PDEs, solutions to first-order PDEs and its applications as well as classification and application of second-order of PDEs and its applications on heat and wave phenomena.

The study could have been more appropriate if conducted on more than one university but because it involves teaching of students at regular time intervals on continuous basis, the research was limited to University of Education, where the researcher is teaching. The study should have been conducted for four years to cover not only partial differential equations but Differential Calculus, Integral Calculus, Ordinary Differential Equations (ODEs) and Vector Analysis which has very high connection to partial differential equations but could not since the study is time bound.

1.10 Limitation of the Study

In the study, the selection of the sample for the study was purposive and this have effect on the generalisability of the study even though the sample was 39.80% of the population. Also, the large number of students per class together with the number of classes per course is a major challenge in the application of the inquiry-oriented instructional approach (IO-IA). The large number of students makes it very difficult to give the needed attention to individual students. The challenges affected the impact of the IO-IA on the lesson delivery and therefore impacted negatively on the results of the study since IO-IA requires individual attention as well. The inadequate ICT tools and simulation software also hindered the effectiveness of the implementation of the IO-IA. Because the students involved were final year students, there was not enough time ascertain how the IO-IA could improve long-term retention of knowledge.

1.11 Organization of the Study

The study consisted of six chapters. Chapter 1 is the introduction which consist of the background of the study, the statement of the problem and the purpose of the study It also consist of the objectives of the study, the research questions, the scope of the study and the significance of the study. The rest are the delimitation of the study, the limitation of the study, the operational definition of terms and the organization of the study. Chapter 2 is the literature review. It consists of the conceptual framework, theoretical framework and related work or research which has been done by other researchers. Chapter 3, which is the methodology, consists of the research design, research paradigm, the population and sample, the sampling procedure/technique, data collection instruments, the pilot study and the implementation of the inquiry-oriented instructional approach (IO-IA). The fourth chapter looks at the presentation of the results from the study. The fifth chapter looks at the analysis and discussions of the results. Chapter 6, which is the last chapter looks at the summary of findings, recommendations and conclusion as well as suggested further studies.

1.12 Operational Definition of Terms

Below are the definitions of terms as they are being used in the study unless or otherwise being specified in the course of the study.

1. **Inquiry-Oriented Instructional Approach** (IO-IA) is an inquiry-based teaching and learning approach that allow the instructor to relating the subject matter being taught to the real world and emphasises on the use of investigations and simulation to model a given problem and used a combination analytical method, graphical method and/or numerical methods to find the solution to the model or equation.

2. **Comprehension** in this study refers to one's ability to read and understand the problem presented.

3. **Computational ability** which was described by Gunaseelan and Pazhanivelu (2016) as ***arithmetic ability*** consist of skills and techniques such as the manipulation of mathematical knowledge and concepts in the manner that transforms their meaning and implications.

4. **Mathematical modelling** as used in this study means modelling for the learning of mathematics unless or otherwise stated.

5. **Socio-economic factors** in this study refers to the financial constraints on students and their parents alike.

6. **Students' performance** in this study refers to grade performance. That is, their performance is measured base on the score they obtained in the confirmation test.

7. A **performance task** is any learning activity or assessment that asks students to perform to demonstrate their knowledge, understanding and proficiency.

8. **Method scores**: Scores obtain by the students from the methods they used in solving given problems.

9. **Concept scores**: Scores obtain by the students from their knowledge on the concept of the given problem.

10. **Total scores**: The total scores obtained by summing up the method and the concept scores.

Chapter 2

LITERATURE REVIEW

2.1 Overview

This chapter provides the conceptual and theoretical framework of the study. It also reviews related studies done by other researchers on pedagogy. It looks at learning as a whole and learning theories, instruction and the various instructional designs, inquiry-based approach in teaching and learning and modelling approach in teaching and learning of mathematics.

2.2 Conceptual Framework

The concept underlying the Inquiry-Oriented Instructional Approach (IO-IA) was adapted from the 5E instructional model which was proposed by Bybee and Landes (1990) to design science lesson and was also used by Duran and Duran (2004). The 5E instructional model consist of five elements which are Engagement, Exploration, Explanation, Elaboration and Evaluation (see Figure 2.1). The 5E instructional model cycle serves as a teaching model for the integration of science, technology and health in the teaching of science for life and living. Duran and Duran (2004) used the 5E instructional model during their project on the implementation of the inquiry-based teaching approach in the teaching of science because the 5E learning cycles emphasize on the explanation and investigation of phenomena, or of evidence to back up conditions and experimental design. Figure 2.1 shows the flowchart of the 5E instructional model. It must be emphasized that, one does not need to strictly follow the model in the order in which it is presented even though it is necessary if applicable. You can also move back when the need arises to make the instructional delivery and the link from one step to another more clearly to students.

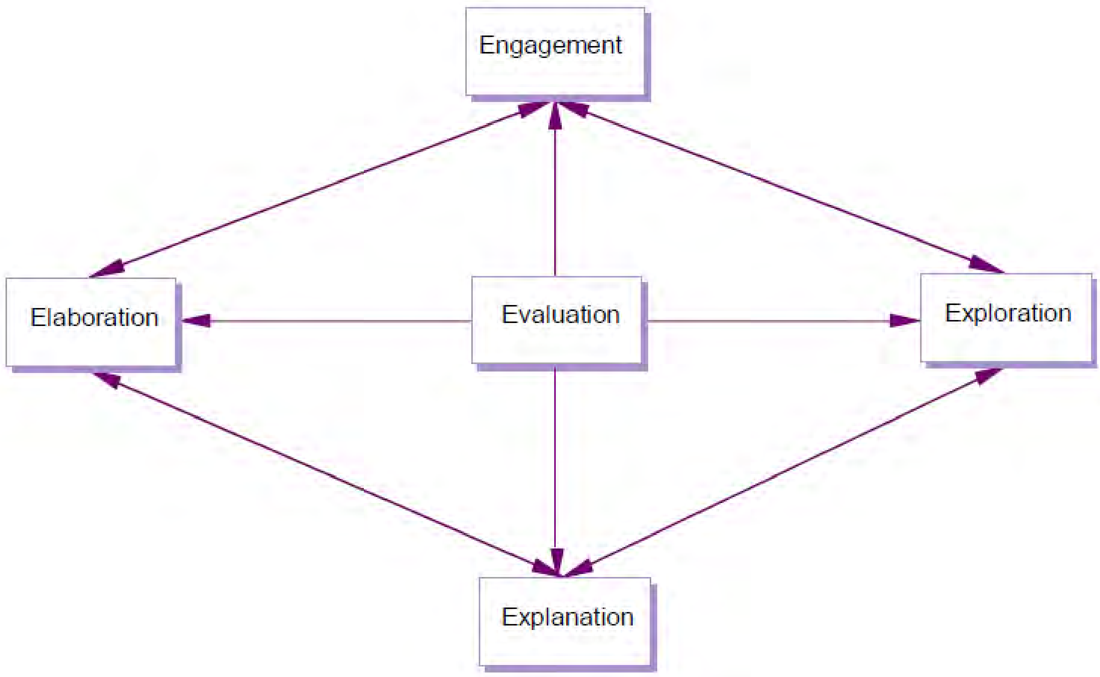

Figure 2.1: The 5E Instructional Model Cycle. Source: Duran and Duran (2004)

Engagement:- This is the first phase of the cycle. The instructor first assesses students' previous knowledge and/or identify possible misconceptions. This is a student-centred stage. It serves as a motivational period that aims at creating desire to learn more about the upcoming subject matter. It starts with brainstorming with the students on what they already know, popularly referred to as the relevant previous knowledge (RPK). Students are allowed to ask questions about the upcoming topic. Discrepant events, demonstrations, questioning, graphic organizers or computer simulations may be included to create interest in students or arouse curiosity of the students. Also, introducing students on the application of the upcoming topic may be used as a motivational tool to arouse the interest of the learners. But this phase does not serve as a lecture time used in defining terms and for the provision of explanations. That notwithstanding, the students are invited to raise their own questions about the process of scientific inquiry. They are encouraged to compare their ideas with their peers. What the teacher does is to assess the students' knowledge by taking note of what they understand or do not understand about the stated outcome of the lesson.

Exploration:- The engagement phase promotes mental focus on the new concept. In the exploration stage, students are provided with common concrete materials, computer simulation software and any tool that is necessary for explaining the new concept. It is a student-centred phase with incorporation of active exploration. Application of process skills such as observation, questioning, investigation, testing of predictions, hypothesizing and communication are encouraged among students. This is one of the most important phases in the learning cycle. This is because this phase of the learning cycle tends to incorporate the main inquiry-oriented activity or experience, which encourages students to develop their skills and concept. The teacher's role at this phase is that of a facilitator or consultant or guide. Students are encouraged to work in addition to cooperative learning environment without direct instruction from the teacher. However, depending on the subject matter, teachers' guidance and directives might come in but should not take the centre stage or control of the whole activity. This phase is unique in the sense that, students are given a "hands-on" experiences before any formal explanation of terms, definitions or concepts are discussed or explained by the teacher. So, in summary, students

- interact with materials and ideas through classroom and small-group discussions;
- consider different ways to solve a problem or frame a question;
- acquire a common set of experiences so that they can compare results and ideas with their classmates;
- observe, describe, record, compare, and share their ideas and experiences; and
- express their developing understanding of testable questions and scientific inquiry (Nadine & Laurie, 2013).

Explanation:- This stage is a "minds-on" phase which follows the exploration phase. It is more teacher-directed but is guided by the students' prior experiences during the exploration phase. The explanation phase set the stage for the students to describe their understanding and pose questions about the concepts they have been exploring. This phase might give way to new questions. But before the teacher's explanation, the students are given opportunity to express their own ideas and provide their own understanding. At the initial stage of the explanation phase, the teacher serves as a facilitator while the students are directed to describe and discuss their exploration learning experiences.

The teacher then, after students had finished with their inputs and explanations, introduces the scientific and technical information directly to the students. The students are allowed to make deductions from computer simulations and other teaching-learning materials they used during the exploration stage. At this phase, the teacher clarifies misconception of students which might have emerged during the exploration phase or engagement phase. The teacher at this stage can provide formal definitions, notes, labels, laws or rules, and so on where applicable. The teacher also explains each stage of students' activity during the exploration stage using appropriate terminologies or terms. It is expected that, the students should be able to explain the important concepts to the teacher and to their peers. At this stage according to Nadine and Laurie(2013), students during the lesson

- explain concepts and ideas (in their own words) about a potential implication of the topic;
- listen to and compare the explanations of others with their own;
- become involved in student-to-student discourse in which they explain their thinking to others and debate their ideas;
- revise their ideas;
- record their ideas and current understanding;
- use labels, terminology, and formal language; and
- compare their current thinking with what they previously thought.

Elaboration:- After students' exploration and explanation stage, it is expected that students should be able to apply what have been taught or what they explored. The activities in this phase of the learning cycle serves as the platform for the encouragement of the students in the application of the new concept which should give rise to reinforcement of new skills. Students are encouraged to check for the understanding with their colleagues or should be able to design new experiments or models based on the new skills or concepts they acquired from the exploration and the explanation phase. This phase is aimed at helping students in the development of deeper and broader understanding of the new concepts. Additional investigations may be conducted by students as well as development of new products and sharing of information and ideas. Application of their knowledge in other disciplines are also required at this phase. At this phase, integration of the Mathematical concepts learnt in other content areas and technology integration are expected. According to Nadine and Laurie(2013), students at this stage

- make conceptual connections between new and former experiences, connecting aspects of their departmental investigation with their concepts of scientific inquiry;
- connect ideas, solve problems, and apply their understanding to a new situation;
- use scientific terms and descriptions;
- draw reasonable conclusions from evidence and data;
- deepened their understanding of concepts and processes; and
- communicate their understanding to others.

Evaluation:- This phase deals with assessment stage of the lesson. Inquiry-based setting's assessment differs in so many ways from the traditional mathematics lessons. A combination of formal and informal assessment approaches is mostly appropriate. These include the use of non-traditional forms of assessment such as portfolios, performance-based assessment, concept maps, physical models or journal logs could be used as evidence of student learning. Assessment is viewed as ongoing process during inquiry-based lesson. Teachers are hereby requested to make observations of their students in the application of the new concepts and skills they acquired and look for evidence that points to behavioural change or modification in their way of thinking. Self and peer assessment are also to be conducted by students. Apart from these modes of assessment, summative assessment such as quizzes, examinations or writing assignments could also be used. Nadine and Laurie (2013) were of the view that lesson evaluation provides opportunities for students to

- demonstrate what they understand about scientific inquiry and how well they can apply their knowledge to carry out their own scientific investigation and to evaluate an investigation carried out by their classmate;
- share their current thinking with others;
- assess their own progress by comparing their current understanding with their prior knowledge; and
- ask questions that take them deeper into a concept.

It is important to note that, it is not always necessary to follow the order in which the model has been arranged. Some concepts may require explanation before exploration and so on. Also, the teacher can move forward and backward as many as possible along the cycle to make learning effective. The activities in this study were design using this model.

2.3 Theoretical Framework

The theory behind the study was based on cognitive psychology and constructivist-learning theory. According to Bada (2015), constructivism is a cognitive philosophy of learning that describes how individuals can acquire knowledge and learn. The constructivist-learning theorem is based on the view that, students are active thinkers and so, they construct their own understanding from interactions with phenomena, the environment and other individuals (Bruner, 1966). The theory suggests that from their experiences, people build knowledge and meaning (Bada, 2015). A constructivist view of learning recognizes that students need time to

- express their current thinking;
- interact with objects, organisms, substances, and equipment to develop a range of experiences on which to base their thinking;
- reflect on their thinking by writing and expressing themselves and comparing what they think with what others think; and
- make connections between among different subject matters and ideas and the students learning experiences and the real world.

Constructivism, according to Bada (2015), is a teaching and learning approach based on the premise that ”mental construction” results from cognition (learning). Constructivists believe that the context in which an idea is taught, as well as the believes and attitudes of students, affect learning (Bada, 2015). Tam (2000) was of the view that for students to be able to learn constructively, the learning environment must be conducive and convenient for students to construct their own ideas. Tam (2000) suggested four basic characteristics of constructivist learning environment that must be considered during its implementation. They include the following:

1. There will be knowledge sharing between teachers and students.
2. There will also be authority sharing among teachers and students.
3. The teacher’s role is that of a facilitator or guide.
4. Students must be in small number of heterogeneous groups.

Honebein (1996) cited in Bada (2015) summarizes seven pedagogical goals of constructivist learning environment as

1. To provide experience with the knowledge construction process so that students will be able to determine how they will learn.
2. To provide experience in and appreciation for multiple perspectives which will help in the evaluation of alternative solutions.

3. To embed learning in realistic contexts which will result in authentic tasks.

4. To encourage ownership and a voice in the learning process to make the teaching and learning student-centred.

5. To embed learning in social experience to promote collaborative learning.

6. To encourage the use of multiple modes of representation such as video, audio text, etc.

7. To encourage awareness of the knowledge construction process so that students will be able to reflect on their findings.

George (1991) enumerated six benefits of constructivist learning. They include

1. Children learn more than passive listeners, and enjoy learning more when they are actively involved.

2. Education works best when it is focused on thinking and understanding rather than rote memorization. Constructivism is focused on learning how to think and understand.

3. The learning of constructivism can be transferred. Students create organizing principles in constructivist classrooms that they can take with them to other learning settings.

4. Constructivism gives students control of what they know, because training is focused on questions and explorations from students, and students also also have a hand in designing the assessments. Constructivist assessment includes in their papers, research reports, physical models, and creative depictions the efforts of the students and personal investments. The presence of creative impulses enhances the capacity of students to convey knowledge in a variety of ways. Also, students are more likely to retain the new knowledge and transfer it to real life.

5. Constructivism encourages and inspires students by creating learning experiences in an authentic, real-world sense.

6. By creating a classroom environment that emphasizes collaboration and exchange of ideas, Constructivism promotes social and communication skills. Students need to learn how to clearly articulate their ideas and collaborate effectively on tasks through sharing in group projects. Therefore, students must exchange ideas and learn to "negotiate" with others and evaluate their contributions in a way that is socially acceptable. This is vital to the real world's success, as they will always be exposed to a variety of experiences in which they will have to cooperate and navigate among others' ideas.

In summary, the various studies stated above suggest that the instructor using the constructivist teaching-learning approach should

- Adapt curriculum to address students' suppositions
- Help negotiate goals and objectives with learners
- Pose problems of emerging relevance to students
- Emphasize hands-on, real-world experiences
- Seek and value students' points of view
- Social context of content.
- Provide multiple modes of representations / perspectives on content
- Create new understandings via coaching, moderating, suggesting
- Testing should be integrated with the task and not a separate activity
- Use errors to inform students of progress to understanding and changes in ideas. (Source: Honebein, 1996 & Christie, 2005)

The students on the other hand are to

- help themselves develop their own goals and assessments.

- create new understandings through coaching, moderating, suggesting and so on from the teacher.
- control learning (reflecting).
- be members of community of learners.
- collaborate among fellow students.
- learn in a social experience - appreciate different perspectives.
- take ownership and voice in learning process. (Source: Honebein, 1996 & Christie, 2005)

The study adapted the Inquiry - Oriented Differential Equations (IO-DE) project by Rasmussen and Kwon (2007) which is based on the constructivist-learning theory. The Inquiry - Oriented Differential Equation (IO - DE) project explored the prospects and possibilities for improving undergraduate differential equation. It is used in the identification of assumptions and the use of critical and logical thinking (Rasmussen & Kwon, 2007). Richard (1991) cited in Rasmussen and Kwon (2007), in his view on the philosophy of mathematics education, characterized inquiry learning as learning to speak and act mathematically through participation of students during mathematical discussion, posing of conjectures and solving of new or unfamiliar problems. According to Rasmussen and Kwon (2007), inquiry process encompasses both the teacher activity and the student activity. The IO-DE teachers routinely inquire into students mathematical thinking and reasoning which serves three important functions. First, it enables the teacher to construct models for their students to interpret and generate mathematical ideas. Secondly, it provides opportunity for teachers to learn something new about particular mathematical ideas in light of student thinking. And thirdly, it puts the students thinking by posing new questions and tasks (Rasmussen et al., 2005; Rasmussen et al, 2006; Rasmussen & Kwon, 2007).

For IO-DE students on the other hand, new mathematical concepts are learnt through inquiry as they are engaged in mathematical discussions, posing and following up on conjectures, explanation and justification of their thinking and finding of solutions to noble problems. The inquiry - oriented approach also empowers the learners to see themselves capable of reinventing mathematics and to see mathematics as human activity (Blanchard et al., 1998). Blanchard et al (1998) developed analytical, graphical and numerical approaches which represent three distinct methods for analysing solutions to differential equations. Based on Blanchard, et al (1998) approaches, three goals for inquiry-oriented teaching and learning was developed by Rasmussen and Kwon (2007). The first goal of the inquiry-oriented teaching and learning approach was to help students reinvent many key mathematical ideas and methods for the analysis of solutions to the differential equations. The second goal is the introduction of challenging task, situated in realistic situations to serve as the starting point for students' mathematical inquiry. And thirdly, the inquiry-oriented approach is aimed at creating balanced treatment of analytical, numerical and graphical approaches in order for these approaches to emerge more or less simultaneously for learners, rather than as three isolated separate methods (Rasmussen et al. 2005, 2006; Rasmussen & Kwon, 2007).

Considering the integrated leveraging of developments both within mathematics and mathematics education, the IO-DE project is a paradigmatic of an approach to innovation in undergraduate mathematics, which serves as a model for undergraduate course reforms (Kwon, 2005). According to Freudenthal (1991), the adaption of the instructional design theory of Realistic Mathematics Education (RME) to the undergraduate level is the cornerstone of the IO-DE project. This is because, the RME is centred on the design of instructional sequences that challenge learners to organize key subject matter at one level to produce new understanding at another level. This process which is referred to as mathematizing, graphs, algorithms and definitions become useful tools when they are built by students from bottom-up through a suitable guided reinvention process (Kwon, 2003; Rasmussen et al. 2005). Embedded in the core heuristics of guided reinvention and emergent models is the mathematization process. According to Freudenthal (1991) cited in Kwon (2003), guided reinvention speaks to the need to locate instructional staring points that are experimentally real to students and that take to accounts students' current mathematical ways of knowing. There is diverse characterization of inquiry-oriented approaches by different researchers which highlights important aspect of student activity. But the characterization only addresses part of the process of inquiry so there is the need to encompasses teacher activity (Kwon, 2008).

In order for teachers to improve learning of mathematics in the undergraduate level, there is the need for teachers to recognize and value characteristics of classroom learning environment that contribute to powerful student learning (Rasmussen & Kwon, 2007). According to Brok (2005) and Holton (2001) cited in Rasmussen and Kwon (2007), there is the need to develop effective and innovative approach in differential equations called the Inquiry-Oriented Differential Equations (IO-DE) project due to the combination of an increasing diverse student body faced by mathematics departments and also the declining number of mathematics major in the pre-university education. The IO-DE project according to Rasmussen and Kwon (2007), seeks to explore prospects and possibilities for improving undergraduate mathematics education.

Cognitive psychology is the area of psychology that focuses on internal mental processes like thinking, decision-making, problem-solving, language, attention, and memory (Lu & Dorsher, 2007). The core focus of cognitive psychology is on how people acquire, process, and store information. Cognitive psychology is the study of knowledge and how it is used by people (Glass et al., 1980). According to Greer (1981), it is possible to detect signs of increasing interest in mathematical thinking among cognitive psychologists although it still looks suspiciously as if they are driven by ease rather than interest in the topic itself. Greer (1981) advanced the following reasons to support the assertion:

1. By their nature, mathematical processes are very likely to be represented by information-processing models as they break down into operational sequences,

transformations, logical steps, etc. There is an insidious danger here of assuming that cognitive processes necessarily mirror the formal expressions of these sequences.

2. To cognitive psychologists, the role of imaging in thought is a current focus of interest; its position in mathematical thinking has long been recognized by anecdotal accounts.

3. The general notion of and translations of different representations of a particular problem is a shared interest, illustrated, for example, by studies of how mathematical representations of problems are abstracted from their verbal statements.

According to Greer (1991), problem-solving is perhaps the most obvious area of mutual interest for cognitive psychologists and mathematicians and was of the view that cognitive psychology can be tapped to the benefit of mathematics education. Lesh and Doerr (2003) were of the view that there is the need to move our field beyond the view of constructivist ideologies since all the goals of mathematics education do not meed to be achieved through personal construction processes. English (1995) study emphasize that not all the learning needs of mathematics students need to be independently invented by the students and so construction is not the only process that contribute to the development of constructs. English (1995) and Lesh and Doerr (2003) reiterated the need for unifying, non-ideological, scientific and eclectic approach such as modelling to research that allows for the consilience of knowledge across the disciplines. The views of the writers in both aspects indicated the need to base the theory behind this study on both cognitive psychology and constructive-learning theory.

2.4 Learning and Learning Theories

Learning according to Schneider (2006) is a complex multi-dimensional phenomenon consisting of different types and levels. Schneider (2006) was of the view that pedagogical theory and practice are strongly influenced by learning theory. Attitudes, factual information (memorization), concept (discrimination) and reasoning (inference and deduction) were listed among the learning types by Schneider(2006). Others are procedure learning, problem-solving and learning strategies (which involves how to learn). Schneider (2006) describes the levels of learning using Bloom's taxonomy of the cognitive dimension: knowledge, comprehension, application, analysis, synthesis and evaluation. According to Smith and Ragan (1999) and Savery (2007), learning is the development of new knowledge, skills or attitudes as an individual interacts with information and the environment to facilitate learning. Khan(2000) listed presentation, demonstration, tutorials, storytelling, role-playing, interaction, facilitation, debate, apprenticeship, generative development, exhibit, drill and practice, games, simulations, discussions, modelling, collaboration, filed trips, case studies and motivation as the pedagogical methods and strategies that improves learning. Verbal information, intellectual skills, cognitive strategies, motor skills and attitudes are the five major categories of learning identified by Gagne (1977). Gagne (1977) was of the view that a learner must be exposed to persuasive argument to be able to learn attitudes and mathematics educators must have a chance to practice new solutions to a class with particular problem in order for cognitive strategies to be learned. Gagne (1977) again stated that the events of instruction are derived from the information-processing model of learning and memory and that learning processes follow a stage-like progression from sensory registration to long-term storage and learner performance. Looking at the numerous challenges in teaching and learning, there is the need to implement an instructional design that will make student in the undergraduate universities able to think and inquire to find solutions to problems and apply them to new situations as well.

Behaviourists are of the view that, learning takes place when there is a behaviour change as a result of stimulus-response associations made by the learner (Zhou & Brown, 2014). Zhou and Brown (2014) stated that in order for learning to take place, the desired response must be rewarded since the key element to the theory of learning is the rewarded response (Parkay & Hass, 2000). Skinner (1972) cited in Shaffer (2000) was of the view that, habits that are developed by individuals are as the results of our unique operant learning experiences . These experiences are as a result of what we observe in our environment or surroundings. Modelling which is also known as observational learning, according to Bandura (1986), is the basis for a variety of child behaviour. Bandura (1986) was of the view that, the most common among the numerous clues that influences behaviour at any point in time is the action of others(Bandura, 1986). There is therefore the need to shape our learning behaviour in order to modify our behaviour to elicit better classroom performance from reluctant students (Zhou & Brown, 2014).

In Piaget's stages of cognitive development, Piaget understood that children create ideas and are not limited to receiving knowledge from parents and teachers (Wood et al., 2001) but actively constructed their own knowledge. The educational implication of Piaget's theory is adaptation of instruction to the learners' developmental level. The main role of the teacher is to provide a variety of experiences to facilitate learning and this can be better achieved through "discovery learning". Discovery learning provide the learners, the opportunity to explore and experiment to enhance their understanding in the concept being presented (Zhou & Brown, 2014). Piaget suggested provision of concrete props and visual aids, such as models and/or time line, the use of familiar examples for the facilitation of more complex ideas such as story problems, giving opportunities for the classification and grouping of information with increasing complexity and presentation of problems in logical sequence which allows for analytical thinking (Renner et al., 1976; Huitt & Hummel, 1998, 2003).

Bloom's Taxonomy also seeks to promote higher forms of thinking in education which include analysis and evaluation rather than just remembering facts which is normally seen as rote learning (Bloom et al., 1956). There are three domains of educational activities or learning which are cognitive (mental skills), affective (growth in feeling or emotional areas) and psychomotor skills (manual or physical skills). When learning take place, it is expected that the individual has acquired these three skills and that individual is said to have moved from passive absorption of information to active engagement with information (Cuban, 1993). The learning environment therefore, recognizes the learners as its core participants, encourages their active engagement and develops in them an understanding of their own activity as learners (Dummont et al). Gagne (1977) study identified five categories of learning outcomes which cut across traditional subjects of curriculum such as mathematics. These learning outcomes include verbal information, intellectual skills, cognitive strategies, motor skills and attitudes. What all these theories have in common is that they proposed instructional methods thought to provide the necessary learning conditions for a particular type of learning goal (Driscoll, 2000).

2.5 Factors Affecting Teaching/Learning of Mathematics

Several factors affect teaching and learning of mathematics. According to Belhu (2017), demographic, instructional strategies and methods and individual factors were the factors affecting teaching and learning of mathematics. Belhu (2017) rated instructional strategies and methods as the most influential factor as was followed by individual and demographic factors respectively. Others were inadequate teaching-learning materials (TLM) and facilities. Belhu (2017) considered language problem, income, computational ability, background knowledge and confidence to understand mathematical concepts as individual factors. Epistemological mathematics problem-solving beliefs, belief about usefulness of mathematics, self-regulated learning (SRL) strategies and goal orientations have great potential to enhance differential equation problem-solving ability (Bibi et al., 2017). Other factors included include entry behaviour, pass mark, facilities and resources, communication in mathematics (poor knowledge in English Language and poor communication of mathematical facts), absenteeism from lectures, drug and alcohol, career choice, peer influence and mathematical knowledge acquisition (Alfred et al., 2012).

Gunaseelan and Pazhanvivelu (2016) study revealed that instructional strategies and methods, teacher competency in mathematics education and motivation or concentration were the three most influential factors affecting mathematics achievements of students.

They were of the view that these factors should be considered when designing instructional approach. Socio-economic status, gender, prior mathematics achievement, parental support, peer influence and student's perception of good classroom assessment were the relevant factors affecting teaching and learning of mathematics (Kiwanuka et al., 2015). According to Mohammed et al (2012), teaching practices, teacher attribution, classroom climate, students attitude towards mathematics and students' anxiety are factors that influences students' achievement in mathematics. Among these factors, Mohammed et al (2012) rated teaching practices as the most influential factor. Prior academic achievement, self-efficacy, academic resources, self-regulation and learning styles were the factors that mostly influences mathematics achievement according to Murray (2013).

2.6 Instructions and Instructional Theories

Instruction is the intentional or deliberate facilitation of learning towards leaning goals (Smith and Ragan, 1999). It is also defined as the deliberate arrangement of learning conditions to promote the attainment of some intended goals (Driscoll, 2000). In general, instruction is the intentional arrangement of experiences, leading to learners' acquisition of particular capabilities which may vary qualitatively in form, from simple recall of facts or knowledge to cognitive strategies that allow a learner to find new problems with a field of study (Smith & Ragan, 1999). According to Smith and Ragan (1999), it is the duty of the instructional designer to develop materials and activities that are intended to prepare the learners to use. Smith and Ragan (1999) described education as all experiences in which people learn and was of the view that all instructions consist of experiences leading to learning. But they were quick to add that not all education is instruction because many experiences that lead to learning were not developed and implemented towards particular learning goals.

According to Dillon and Sternberg (1986), instructional psychologists are concerned with how best they can enhance learning and for that matter rely on the findings of psychological and instructional research to solve instructional problems and also make decision about instructional practice (Gagne and Dick, 1983). The primary concern of Gagne and Dick (1983) was instruction and how the knowledge on learning can be systematically relate to instructional design. For this reason, Gagne and Dick (1983) proposed a comprehensive and integrated theory of instruction that is based on cognitive information and Gagne's own observations of effective teachers in the classroom. The theory which is summarized in Figure 2.2 was made up of three components (Driscoll, 2000):

1. a taxonomy of learning outcomes
2. conditions of learning and
3. nine events of instructions

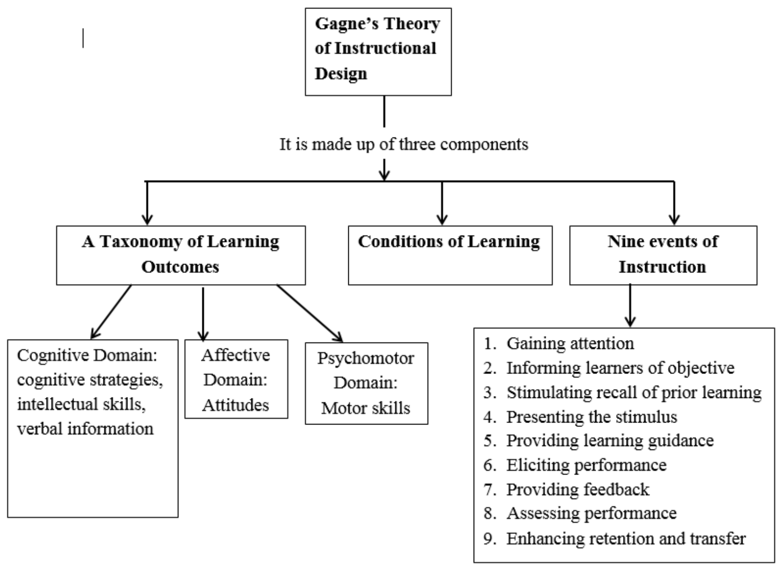

Figure 2.2: Gagne's Theory of Instruction. Source: Driscoll (2000)

The taxonomy of learning outcomes defines the types of capabilities that can be learned by humans. It includes the cognitive domain (consisting of cognitive strategies, intellectual skills and verbal information). The second component which is the condition of learning concerns with the association of internal and external learning conditions with the acquisition of each category of learning outcome. The third component is the nine events of instruction and it include:

1. Gaining attention: A stimulus change to alert the learner.

2. Informing learners of objective: A statement directing the learner to the goal of the instruction.

3. Stimulating recall of prior learning: Previous knowledge or experience in the form of questions or activity that will serve as a requirement to the new subject matter. That is, the relevant previous knowledge.

4. Presenting the stimulus: An activity or information used in presenting the content of the subject matter.

5. Provision of learning guidance: Promotion of encoding through the use of a cue or any strategy.

6. Eliciting performance: Given students opportunity to practice or demonstrate what have been learnt.

7. Providing feedback: Provision of information by the learners for correction to improve their performance.

8. Assessing performance: Students are given opportunities for the demonstration of what have been learnt.

9. Enhancing retention and transfer: Designing of activities and examples that prompts the learner so that he or she will recall what have been learnt at all time as well as transferring to another field.

For instructional designers to be able to integrate multiple goals into their instruction, Gagne and Merrill (1990) proposed an enterprise schema notion which defines the context learning and the reason for learning a particular set of goals (Driscoll, 2000). Land and Hannafin (2000) cited in Na (2012) describes (1) creating multiple experiences for knowledge construction and (2) creation of authentic and complex socio-cultural learning environments for the mediation of knowledge construction as the core concept of student-centred learning and instruction. Na (2012) was of the view that student-centred learning environments have been greatly influence by contemporary learning theories. There is therefore the need for continuous implementation in the various settings and testing of practical strategies like inquiry-based learning, situated learning, project-based learning, self-regulated learning and collaborative learning (Na, 2012; Land & Hannafin, 2000; Gagne & Merrill, 1990). Jonassen (1999) cited in Na (2012) stated that, enormous opportunities in creativity affects student-centred learning environments by the various educational technologies. For example, the creation of rich perceptual experiences on the computer-based learning environment for students and the mass information on the Internet have laid emphasis on the implementation of technology enhanced learning and instruction. There is therefore the need to design computer-aided instructions (CAI) for effective and easily accessible instructional materials (Na, 2012). It should not be forgotten that, in this century, students are smart in diverse ways and their smartness has led to different learning approaches. This has made the student-centred approach a very necessary learning style (Hudson, 2009).

2.7 Design

Design according to Smith and Regan (1999) is an activity or process that people engage in to improve the quality of their subsequent creations and is related to planning. The main difference between planning and design is that, when the expertise and the care with which planning is conducted reaches a certain point, it is then referred to as design (Smith & Ragan, 1999). It is a way of solving problems in time and space (Formia, 2012). Rowland (1993) on his study on instructional design defined design as a goal-directed process in which the goal is to conceive and realize some new thing. And the new thing that results from designing has practical utility. According to Rowland (1993), a basic task of designing is to convert information in the form of requirements into information in the form of specifications. Rowland (1993) was of the view that, design requires social interaction and involves problem-solving but not all problem-solving is designing. Problem-solving and problem understanding may be simultaneous or sequential process.

Design, which may also be defined as a science or a combination of science and art or neither science nor art also involves technical skills and creativity and rational and intuitive thought process. It is therefore considered as a learning process (Rowland, 1992, 1993; Rowland & Wilson, 1994).

2.8 Instructional Design

Instructional design refers to the systematic and reflective process of translating principles of learning and instruction into plans for instructional materials, activities, information, resources and evaluation (Smith & Ragan, 1999). It also refers primarily to operative processes of forming and composing effective learning environments in order to serve the educational needs of addresses and clients. It is a systematic procedure in which educational and training programs are developed and composed. It is aimed at improving learning outcomes so that the learner can be motivated and his attitudes influence in such a way that deeper understanding of the subject matter can be achieved (Reiser & Dempsey, 2007). Seel, et al (2017) were of the view that the central concepts of instructional design are learning and teaching and stated that, there is a strong relationship between learning and instruction. The role of instructor is to structure the learner's engagement with knowledge and practice high-level cognitive skills that enable them to make that knowledge their own but not to transmit the knowledge to passive recipient (Laurillard, 2008).

Several alternative approaches of instruction were argued by several writers. These include learning design which is the description of the teaching-learning process that takes place in a unit of learning (e.g. a course, a lesson, etc) (Cooper et al., 2006). Instructional design and learning design are two sides of the same coin and there is practically no difference in case of procedures and steps in designing. And both instructional design and learning design are tools for the development of learning environment (Seel et al., 2017). The field of instructional technology encompasses analysis of learning and performance problems as well as the design, development, implementation, evaluation and management of instructional and non-instructional process and resources intended to improve learning and performance in a variety of settings, particularly educational institutions and the workplace (Reiser, 2001). According to Seel, et al (2017), instructional design is primarily concerned with the generation of detailed and precise prescriptions for the development, implementation, evaluation and maintenance of situations that aim at the initiation and facilitation of learning processes within subject areas. According to Seel, et al (2017), the major intention of instructional design is the development of learning environment on the basis of suitable theories of learning and teaching that ensures the quality of teaching and education interventions. It contains completely, the process of planning which starts with analysis of needs and objectives which goes along with instructional materials development up to the implementation and evaluation level of its effectiveness (Reigeluth, 1983). Reigeluth (1983) applied three strategies of instructional design:

1. Organizational strategies with both the gross and detailed planning of settings of teaching and learning in order to determine how a course of lesson should be arrange and sequenced.

2. Delivery strategies which is concerned with decision on how information can be transmitted to the target group of learners.

3. Execution strategies which is concerned with decision on methods to assist the learner to deal effectively with instructional materials.

Based on these strategies, Seel, et al (2017), defined instructional design as a dynamics process which aims at the development of effective instructional design which is related to planning of a small instructional unit within a short-time period. In the wider perspective, it is related to the planning of lengthy courses and the most comprehensive interpretation refers to as long-term implementation of educational programs and their evaluation.

2.9 Inquiry-Based Approach

According to Uno (1999), inquiry may be referred to as a technique that encourages students to discover or construct information by themselves instead of having the information directly reveal to them by teachers. Science for example was considered as body of facts that students are supposed to learn through direct instruction and memorization, the situation which requires change (Duran and Duran, 2004; Rutherford & Algren, 1990). Inquiry is multifaceted activity that involves making observations; posing questions; examining books and other sources of information to verify what is already known; planning investigation, reviewing what is already known in light of experimental evidence; gathering, analysis and interpretation of data using tools; proposing answers, explanations and predictions and communicating the results (Rutherford & Algren, 1990; National Research Council, 1996). It is also the art of developing challenging situations in which students are asked to observe and question phenomena; pose explanations of what they observe; devise and conduct experiments in which data are collected to support or contradict their theories; analyse; draw conclusions from experimental data; design and build models; or any combination of these (Friesen & Scott, 2013).

It requires more than simply answering questions or getting right answer but also includes investigation, exploration, search, quest, research, pursuit and study which is enhanced by involving community of learners (Kulklthau et al., 2007). Inquiry-based learning is an approach to teaching and learning that places student questions, ideas and observations at the centre of the learning experience (Scardamalia, 2002). The application of inquiry-based pedagogical practices may necessitate the transition from textbook based teaching approach to a more hands-on approach, where the students are central to the learning episodes (Duran & Duran). Duran and Duran (2004) on their study on ”The 5E Instructional Model” argue that inquiry needs to be a central strategy of all science curricula. They used the 5E learning cycle approach in the classroom for the facilitation of the inquiry practices. The learning cycle according to Duran and Duran (2004) focuses on constructivist

principles and emphasize the explanation and investigation of phenomena, the use of evidence to back up conclusion and experimental design. They concluded that, the incorporation of learning cycles in the classroom aids teachers in the pursuit of the development of effective inquiry-based science lessons. After the study, they concluded that the 5E instructional model according to the responses they obtained from the teachers who put it into practices yielded the following results:
The use of 5E model

- help the teachers to design meaningful and purposeful lessons for the students.
- enables the teachers to assess students' knowledge before the start of the exploration activity which made the evaluation of the students' academic level more appropriate.
- enables teachers to personalize lessons according to student needs.
- changes students who were more reserved and less interested to share their opinions and ideas.

According to Etheredge and Rudnitsky (2003), inquiry-based approach is more beneficial to students and that young children can as well learn through inquiry process. Irrespective of the importance of the inquiry-based approach, many educators are either unaware of the ways of designing it or uncomfortable with its usage in the classroom (Abraham, 1997).

According to National Science Education (NSE) standard cited in cited in National Research Council (NRC) (1996) in the United States of American, inquiry-based needs to be the central strategy of all science curriculum. They further stated that, the pedagogical methods by which students are taught influences students' learning. The report of NRC (1996) and project 2061: Science for all Americans (Rutherford & Algren, 1990) revealed that science teaching should actively engage students, deemphasize rote learning or memorization of facts and incorporate cooperative learning, hence the need to inculcate inquiry-based approach. There are three approaches to inquiry-based learning: project-based learning, problem-based learning and designed-based instruction (Barron & Darling-Hammond, 2008).

2.9.1 Project-Based Learning (PBL)

In project-based learning (PBL), learning is organized around the creation of a presentation or a product that is usually shown to an audience. It includes the creation of an original play, a video, or an aquarium design judged by local architects (Barron & Darling-Hammond, 2008). According to Thomas (2000) cited in Friesen (2013), PBL projects involve complex tasks, based on challenging questions or problems that involve students in design, problem-solving, decision making, or investigative activities. It gives students the opportunity to work relatively autonomously over extended periods of time and also culminate in realistic products or presentations. Thomas (2000) presented the following characteristics of a PBL. Project-Based Learning projects

1. are focused on questions or problems that ”drive” students to encounter (and struggle with) the central concepts and principles of a discipline.
2. involve students in a constructive investigation.
3. are student-driven to some significant degree.
4. are realistic, not school-like.

2.9.2 Problem-Based Learning

Several studies have adapted problem-based learning. The problem-based learning model has recently been adapted for a range of subjects including social studies, science, and mathematics (Stepien and Gallagher, 1993). Although a number of approaches to learning have adopted this title, Barrows (1996) argued that its core characteristics include a student-centred approach to learning, and learning that occurs in small groups under the guidance of a tutor who acts as a facilitator or guide. Additionally, students engage in authentic problems before they receive any preparation or study, and may have to find information on their own to solve the problem. Other authors argue that assessment and evaluation is mostly concerned with how the students applied their knowledge to solve the problem, rather than assessing one correct answer (Serger et al., 1999). The problem-based learning is the bedrock of the IO-IA and is upon which the 5E instructional model was based.

2.9.3 Design-Based Instruction (DBI)

According to Barron and Darling-Hammond (2008) cited in Friesen and Scott (2013), students are asked to design and create artefacts that requires them to apply principles and knowledge they drawn from a particular discipline when using design-based learning approach. The design-based learning approach is mostly used in the field of art and technology, engineering and architectural fields where students are guided to come out with their own ideas which leads to the creation of prototypes. In design-based approach, students are encouraged to work in small groups by taking on specific task.

2.10 Factors Affecting Inquiry-Based Teaching and Learning

Jingoo and Tuula (2016) categorize the factors affecting the implementation of inquiry-based teaching into two: teacher-related factors and school-related factors. The teacher-related factors include

1. teachers' confidence in teaching science,
2. determination on the part of the teachers to explore a new way of teaching,
3. teachers' collaboration on improving science teaching and
4. teachers' pedagogical skills.

The school-related factors on the other hand are

1. class size and

2. school resources which include accessibility to ICT tools, TLMs, computer applications and the likes.

Another contributing factor is the technological support for inquiry-based teaching and learning Learning according to Blumenfield et al. (1991). New opportunities to support inquiry-based learning are offered by computing and networking technologies. Blumenfield et al. (1991) in their study on technology as a support for project-based science learning, identified six contributions technology is capable of making to learning processes:

- enhancing interest and motivation;
- providing access to information;
- allowing active, manipulable representations;
- structuring the process with tactical and strategic support;
- diagnosing and correcting errors;
- managing complexity and aiding production.

They were of the view that all of the fundamental properties of computing technologies offer benefits for inquiry-based Learning - the ability to store and manipulate large quantities of information, the ability to present and permit interaction with information in a variety of visual and audio formats, the ability to perform complex computations, the support for communication and expression, and the ability to respond rapidly and individually to users.

Motivation is also an important factor of high failure in mathematics. It is the internal and external factors that stimulate desire and energy in students to be continually interested and committed to learn something. The process of motivation stems from stimulation which is turns followed by an emotional reaction that lead to a specific behaviour response (Bed, 2017).

2.11 Challenges Associated with Inquiry-Based Teaching

Harris and Rooks (2010) considered management interactions in inquiry-based science learning as a major challenge in the implementation of an inquiry-based teaching approach. Several researchers (Schauble et al., 1995; Krajcik et al., 1998) found out that students have difficulties in the conduction of systematic scientific investigations. According to Edelson et al. (1999), data gathering, analysis, interpretation and communication are all challenges militating against content-area knowledge. Their study revealed five of the most significant challenges associated with successful implementation of inquiry-based teaching and learning. They are:

1. **Motivation**: According to Edelson et al (1999), students must be sufficiently motivated in order for them to be engaged in inquiry-based learning. They were of the view that motivation must be as a result of interest in the investigation and its implementations.

2. **Accessibility of investigation techniques**: Students must know how to perform task that is required of their investigations, understand the goals of their practice and be able to interpret their results.

3. **Background knowledge**: Provision of opportunities for learners to develop and apply scientific understanding is very paramount when designing inquiry-based teaching and learning. Lack this knowledge on the part of the students will lead to their inability to complete a meaningful investigation.

4. **Management of extended activities**: Students must be able to organize and manage complex, extended activities to achieve the ultimate goal of open-ended inquiry.

5. **The practical constraints of the learning context**: "The technologies and activities of inquiry-based learning must fit within the practical constraints of the learning environment, such as the restriction imposed by available resources and fixed schedules".

2.12 Inquiry-Oriented Mathematics Instruction in University Education

Rowe (1973) defined inquiry-oriented instruction as teacher practices that foster students use of personal experience and schooled knowledge for generating new information, new problem-solving approaches or new solutions that were not heretofore a part of the learning environment. According to Kwon (2005), research on the relationship between different teaching methods and students' understanding of mathematics at the university level is essential for cumulative improvement in mathematics.

Kwon (2005), Ebert-May et al (1997) and Rasmussen et al (2006) stated that the reports of some researchers about the gap between what is taught and what is learned in mathematics in traditional modes of teaching indicated that, students most of the time have difficulties in conceptualizing what is being taught in the classroom. Kwon (2005) emphasizes on the need for faculties to recognize the characteristics of effective teaching for the improvement of teaching and student learning. Kwon (2005) was of the view that, because many instructors were not introduced to this knowledge-based teaching during their graduate or post-doctoral years and have not acquired this perspective, they may struggle through teaching assignments. And they often re-develop techniques and approaches that have been tested and disseminated by others. In the use of traditional method of teaching in the mathematics classroom, that is, textbook-dominated approach, students' participation in the lesson delivery is through listening to and watching the demonstrations of the teacher's mathematical procedures. And practicing is only through the completion of textbook exercises.

Alsina (2002) related this prevailing phenomenon of traditional approach to the believe in many researchers that, a good researcher is always a good teacher and claim that university teaching does not require any specific training but accumulated experiences does it all(Alsina, 2002). Other believes include linear presentation of instructions and must take bottom-up process as well as perfect theory presentation. Many studies by early researchers have shown statistical evidence of the limitations of the traditional teaching practices (Orton, 1980; Harel, 1989; Alsina, 2002). Orton (1980) for example indicated that students have difficulty in conceptualizing the limits processes underlying the notions of derivative and integral. Discrepancy between the formal definitions and the criteria used by the students in checking the properties of functions such as continuity and the derivative was obvious (Tall & Vinner, 1981). Harel (1989) studies show evidence of students' difficulties in learning the basic notations of linear algebra and again there is difficulties in connecting graphs with physical concepts and the real world (McDermott et al., 1987). Students again have difficulties in logical reasoning and proofs (Alibert & Thomas, 1991; Schoenfield, 1985; Seldon & Seldon, 1995).

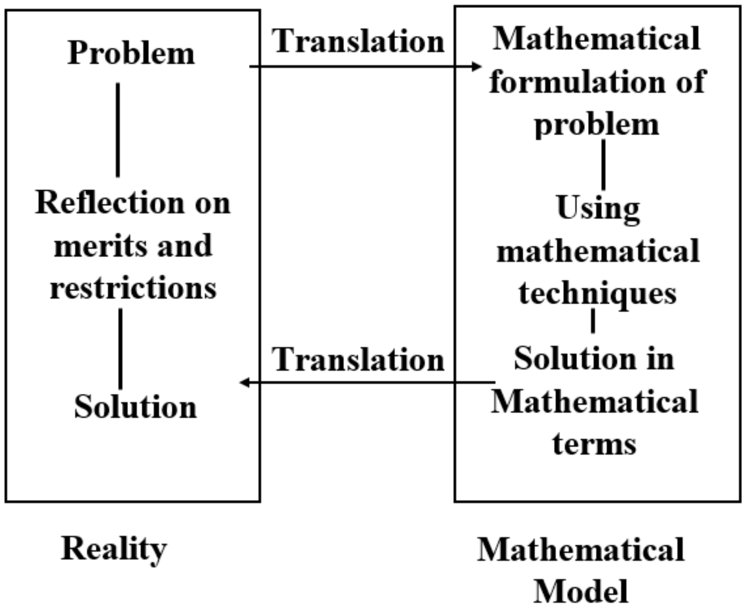

Figure 2.3: A Model for Mathematical Inquiry Process (Source: Wubbles, et al (1997))

Wubbles, et al (1997) also conducted a study on the roles of instruction in a community of inquiry. According to Wubbles, et al (1997), inquiry process is characterized by the consecutive steps of translating a real world problem into a mathematical problem, the analysis and structuring of such a problem, the creation of mathematical solution, the translation of this solution to the real world and the reflection on the merits and restrictions of the solution. This can be followed by a next cycle in which the translation of the problem is refined, generalized or otherwise changed. Wubbles, et al (1997) suggested the model in figure 2.3 as the articulation of the roles of teacher and students in the inquiry community.

Kwon (2005) also designed a model which suggested the articulation of the roles of teacher and students in the inquiry community as shown in figure 2.4. In the model, realistic task, shared responsibility for learning and teaching, authentic investigation, inter-dependency of groups, negotiation of understanding and public sharing is the critical element of the community of inquiry. The professor/instructor and students are the key players in this community of inquiry.

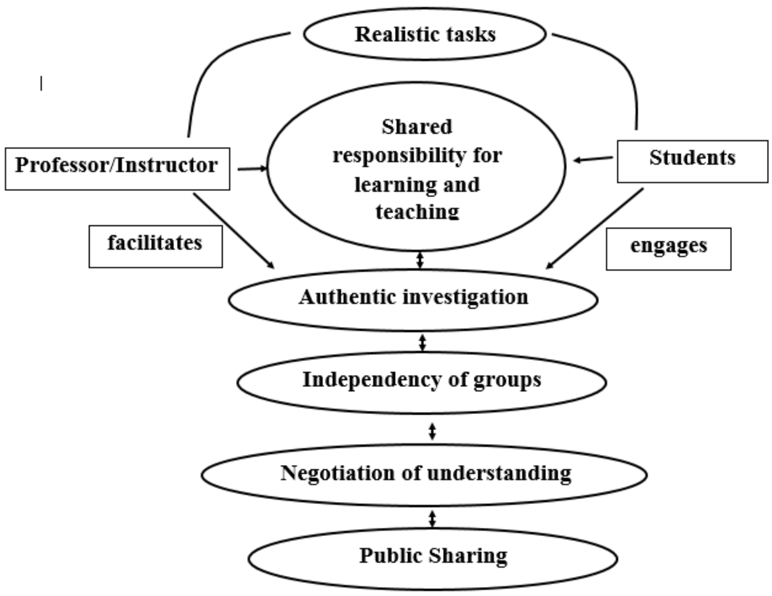

Figure 2.4: Roles of instructor in a community of inquiry (Source: Kwon (2005))

2.13 Strands of Mathematics Proficiency

A strand is a consistent thread running through a course offer irrespective of its subject According to National Research Council (2000) report, mathematics proficiency has five strands which are:

1. *Conceptual understanding*:- This is concerned with comprehension of Mathematical concepts, operations and relations.

2. *Procedural fluency*:- This is concerned with skill in carrying out procedures, flexibility, accurately, efficiently and appropriately.

3. *Strategic Competence*:- The ability of the learners to formulate, represent and solve Mathematical problems.

4. *Adaptive reasoning*:- This looks at the capacity for logical thought, reflection, explanation and justification.

5. *Productive dispensation*:- This is concerned with the students' habitual inclination to see mathematics as sensible, useful and worthwhile, which is coupled with a belief in diligence and one's own efficacy.

The study focused on these strands in all the activities to improve the students' mathematics proficiency.

2.14 Modelling Approach

There are two main approaches in modelling, namely ”modelling for the learning of mathematics” and ”learning mathematics for modelling”. The earlier involves the use of modelling approach in the teaching and learning of mathematics in the classroom while the later is about the application of mathematics in modelling or formulation of functions that can be used to solve problems. This study focused on the earlier one, ”modelling for the learning of mathematics”. Mathematical modelling as used in this study means modelling for the learning of mathematics unless or otherwise stated.

According to Blum (1993), the teaching of mathematics is intended to help students to understand so that they will be able to cope with real-world solutions and problems. This is what Blum (1993) termed pragmatic argument. In the case of formative arguments, Blum (1993) stated that, to be concerned with mathematics means, students should acquire general qualifications (such as the ability to tackle problems) or attitudes (such as openness towards new situations). In the case of cultural argument, Blum (1993) was of the view that students should be taught mathematical topics as a source for reflection, or in order to generate a comprehensive and balanced picture of mathematics as a science and part of human history and culture. And finally, on Blum's psychological arguments, mathematical contents can be motivated or consolidated by suitable modelling examples which may contribute towards deeper understanding and longer retention of mathematical topics or they may improve students' attitudes toward mathematics (Blum, 1993).

According to Lesh and Doerr (2003a) cited in Erbaş, et al (2014), Mathematical modelling in educational settings has considered a way of improving students' ability to solve problems in real-life. It consists of both conceptual system in learners' minds and the external notation system of ideas, representations, rule, materials, etc. A model is used to interpret and understand complex nature systems (Lesh & Doerr, 2003a). Model is an attempt to construct an analogy between an unfamiliar system and a previously known or familiar system (Lehrer & Schouble, 2003). Models which are based on thinking and also emphasizes on developments are those that make sense of real-life situations (Lehrer & Schouble, 2003).

Even though the terms mathematical model and modelling are usually reserved for concrete materials in elementary education, the use of concrete materials is useful for helping children to develop abstract mathematical thinking (Lesh & Lehrer, 2003; Erbaş et al., 2014). Dienes (1960) cited in Erbaş, et al (2014) is of the view that mathematical modelling is used to refer to a more comprehensive and dynamic process than just the use of concrete materials. Haines and Crouch (2007) cited in Erbaş (2014) characterize mathematical modelling as a cyclical process in which real-life problems are translated into mathematical language, solve within a symbolic system.

Mathematical modelling is a process in which real-life situations and relations in these situations are expressed by using mathematics (Verschaffel et al., 2002). Lesh and Doerr (2003a), described mathematical modelling as a process in which existing conceptual systems and models are used to create and develop new models in new context. Thus, a model is a product and modelling is a process of creating a physical, symbolic or abstract model of a situation. Different mathematical modelling approaches with different theoretical perspectives have been proposed in mathematics education and so no single view is agreed upon among educationists (Erbaş et al., 2014). Erbaş, et al (2014) considered Kaiser (2006) and Sriraman et al (2006) classification system for the presentation of modelling approaches as the leading perspective. The perspective are classified as (i) realistic or applied modelling which is a modelling as a purpose of teaching mathematics, (ii) contextual modelling which is a modelling as a means to teach mathematics, (iii) educational modelling, (iv) socio-critical modelling, (v) epistemological modelling or theoretical modelling and (vi) cognitive modelling. In mathematical modelling, students experience the stages of developing, reviewing and revising important mathematical ideas and structures during the modelling process as compared to the traditional problem-solving approach in which students are expected to use taught structures such as formulas, algorithms, strategies and mathematical ideas (Lesh & Lehrer, 2003; Erbaş et al., 2014). Teaching of linear second-order Ordinary Differential Equations (ODE) according to Paraskaki (2003) cited in Mrozek (2014) is done in a very procedural manner where the emphasis lies in identifying the type of equation and then apply a number of well-established steps that yield to the solution (Paraskakis, 2003; Mrozek, 2014).

2.15 The Concept of Partial Differential Equations (PDEs)

Henri Poincaŕe stated: ”If you wish to foresee the future of mathematics, our proper course is to study the history and present condition of the science. However, varied may be the imagination of man, nature is a thousand times richer. Each of the theories of physics presents (partial differential) equations under a new aspect. Without the theories, we should not know partial differential equations” (Myint-U & Debnath, 2007).

The study of surfaces in geometry together with a wide variety of problems in mechanics gave rise to partial differential equations. A number of famous Mathematicians like Gottfried, Leibniz, Bernoulli and so on were actively involved in the investigation of numerous problems presented by PDEs during the 19th century. The main reason was that PDEs both express many fundamental laws of nature and frequently arise in the mathematical analysis of diverse problems in science and engineering. The next phase of the evolution of linear partial differential equations was characterized by efforts to develop the general theory and various methods of solution of linear equations. Partial differential equations are essential to the theory of surfaces on the one hand and to the solution of physical problems on the other. These two areas of mathematics can be seen as linked by the bridge of the calculus of variations. With the discovery of the basic concepts and properties of distributions, the modern theory of linear partial differential equations is now well established. The subject plays a central role in modern mathematics, especially in physics, geometry, and analysis (Myint-U & Debnath, 2007).

The key defining property of partial differential equations (PDEs) is that there is more than one independent variable $x, y, ...$ and there is a dependent variable that is unknown function of these variables $u(x, y, ..)$. For this reason and many more, partial differential equation has numerous applications in our daily activities and inventions and so play a major role in applied studies. PDEs describes all kinds of physical phenomena and applications in all sorts of fields (Strauss, 2008).

PDEs can be classified according to its homogeneity, order, linearity and so on. Some of the classifications are also peculiar to the order of the equation. For example, second-class order PDEs has other classifications which the first-order PDEs do not have. The first-order PDEs can be classified as linear, semi-linear, quasi-linear or fully non-linear while the second-order differential equations are classified as hyperbolic, parabolic or elliptic depending on the discriminant of the characteristic equations (Asiedu-Addo & Ali, 2019). There are several examples of PDEs but notable among them are what Coleman (2013) called "The Big Three PDEs": The heat or diffusion Equation, the wave and the Laplace equation. Heat is a form of energy which results from the motion of molecules at a macroscopic level and it appears to flow from warmer to cooler regions. The heat equation is a second-order partial differential equation used to determine for example, the temperature of a rod at a point x at a time t. The heat equation is given by

$$u_t = \alpha^2 u_{xx}$$

where $\alpha^2 = \kappa / c\rho$ is called the thermal diffusivity, c is the specific heat capacity (the quantity of heat energy required to raise the temperature of a unit quantity of material by 1 degree of temperature), ρ is the density of the material and κ is the thermal conductivity of the material. The wave equation is also a second-order linear partial differential equation for the description of waves - as they occur in classical physics - such as mechanical waves (e.g. water waves, sound waves and seismic waves) or light waves. It arises in fields like acoustics, electromagnetics, and fluid dynamics. The wave equation is given by

$$u_{tt} = c^2 u_{xx}$$

where c is the wave speed. Laplace equations are second-order partial differential equations with broad utility in electrostatics, mechanical engineering and theoretical physics. It came as a result of the steady state of heat or wave equations. One-dimensional Laplace equation is given by $u_{xx} = 0$ while a two-dimensional Laplace equation is given by

$$u_{xx} + u_{yy} = 0$$

These three equations has significant application in real-life situations today. Although many mathematicians in the late 18th and early 19th centuries investigated the problem of heat conduction, Joseph Fourier (1768 - 1830) soon was able to derive the heat equation and solved it using trigonometric series (Coleman, 2013).

The study of the vibrating string was prompted by the writings of French composer and music theorist Jean-Philippe Rameau (1683– 1764). In 1727, John Bernoulli (1667–1748) approximated a continuous string by a massless string loaded with a finite number of discrete masses. Although Bernoulli was believed to have "taken the limit," the wave equation as we know it did not appear until the 1760s, in the works of Euler and d'Alembert. Laplace's equation, or the potential equation, actually appeared first in 1752 in a paper by Euler. The paper dealt with the motion of fluids and was influenced by Daniel Bernoulli's seminal work Hydrodynamics, which appeared in 1738 and in which he coined the term "potential function." Pierre-Simon de Laplace (1749–1827) got his name attached to the equation through his re-derivation and use of it in connection with the problem of gravitational attraction. In particular, Laplace's main goal was to prove that the solar system is stable, a problem that has returned to the forefront with recent advances in dynamical systems and the study of chaos systems. Of course, these are but a few of the highlights of the rich and varied history of the Big Three PDEs (Coleman, 2013).

2.16 Summary of the Literature Review

The study focused on the use of inquiry-oriented instructional approach (IO-IA) in teaching and learning of Partial Differential Equations (PDEs). The concept on which the study was based is the 5E instructional learning cycle model proposed by Bybee and Landes (1990). The 5Es consist of the engagement, exploration, explanation, elaboration and evaluation phase of the learning cycle. The theory behind the study was based on cognitive psychology and constructivist-learning theory. The study adapted a constructivist-learning approach called the inquiry-oriented differential equation (IO-DE) projects by Rasmussen et al (2006) on their study on the inquiry-oriented approach to teaching of differential equations. The IO-DE project capitalizes on advances within mathematics and mathematics education, including the instructional design theory of Realistic mathematics Education and the social negotiation of meaning.

According to the studies conducted by some of the earlier writers, several factors influence teaching and learning of mathematics. Instructional strategies and methods/teaching practices, learning styles, teacher competency in mathematics education, teacher's attribution and teaching-learning materials and facilities were among the factors that influences teaching and learning of mathematics (Belhu, 2017; Gunaseelan & Pazhanivelu, 2016; Murray, 2013; Mohammed et al., 2012). Other factors are students' perception of good classroom assessment, motivation or concentration, prior mathematics achievement, peer influence, parental support, students' attitude towards mathematics, students' anxiety, students' attitude towards mathematics and academic resources (Kiwanuka et al., 2015, Gunaseelan & Pazhanivelu, 2016; Murray, 2013; Mohammed et al., 2012). Individual factors like socio-economic status, gender, parental support, language problems, computational ability, background knowledge and confidence to understand mathematical concepts also influences teaching and learning of mathematics (Belhu, 2017; Kiwanuka et al., 2015).

Chapter 3

RESEARCH METHODOLOGY

3.1 Overview

This chapter consists of the research paradigm, research design, population, sampling and sampling techniques, data collection instruments, reliability and validity of the data collection instrument, data collection procedures and pilot studies. It also consists of the implementation of the treatment, hypothesis and data analysis procedures.

3.2 Research Paradigm

According to Sayyed and Abdullah (2013), the term paradigm was first introduced by Kuhn who defined paradigm as an integrated cluster of substantive concepts, variables and problems attached with corresponding methodological approaches and tools. According to Guba and Lincoln (1994), cited in Sayyed and Abdullah (2013), paradigm is a basic system or worldview that guides the investigator. According to James (2012), a paradigm consists of ontology, epistemology, methodology and methods.

Ontology is the study of 'being' and is concerned with 'what is', that is, the nature of existence and structure of reality as such (Crotty, 1998, 2003; McQueen and McQueen, 2010). According to Wand and Weber (1993) cited in Antwi and Hamza (2015), ontology refers to a branch of philosophy concerned with articulating the nature and structure of the world. And it specifies the form and nature of reality and what can be known about it.

Epistemology on the other hand is concerned with the nature and forms of knowledge (Cohen et al., 2007). Epistemological assumptions are concerned with how knowledge can be created, acquired and communicated, that is, what it means to know (Guba & Lincoln, 1994; Richard, 2003). It is the way of looking at the world and making sense of it (Crotty, 1998). Trochim (2000) cited in Krauss (2005) defines epistemology as the philosophy of knowledge or how we come to know. Krauss (2000) was of the view that epistemology is intimately related to ontology and methodology since ontology involves the philosophy of reality and epistemology addresses how we come to know of these realities while methodology identifies the particular practices used to attain knowledge of it (Krauss, 2005).

This study is more of epistemology than ontology because the study sought to establish the relationship between the students and the subject matter and guides them to know what they are supposed to know and the key concepts that the students are supposed to know. Again, positivism which is an epistemological position focuses on the importance of objectivity and evidence in searching for the truth and the world is unaffected by the researcher. According to Snap and Spencer (2003), facts and values are very distinct in positivism and it is possible to conduct objective and value-free inquiry which is the focus of the inquiry-oriented instructional approach (IO-IA). Also, since positivism epistemology hold the position that meaningful realities which exist in any kind of people's consciousness, always resides in the objects to be discovered according to Crotty (1998), the IO-IA is more of epistemology than ontology.

3.3 Research Design

Design in research is not just a work plan but detailed description of what the researcher has to do to complete the study (Yin, 1989). It is the study conducting blueprint that maximizes control over factors that might interfere with the validity of the findings. Designing a study helps the researcher to plan and implement the study in such a way as to help the researcher obtain intended results, thereby increasing the chances of obtaining information that might be associated with the real situation (Burns & Grove, 2001).

The function of research design according to Yin (1989), is to ensure that the evidence obtained enables us to answer the initial question as unambiguously as possible. According to Yin (1989), research design does not deal with logistical problems but with logical problems and it is different from the method by which data are collected. In the study, both quantitative and qualitative data were collected so the study employed one of the mixed-methods approach. Creswell et al. (2003) suggested four decisions to consider when approaching mixed methods. They are (1) the implementation sequence of the data collection, (2) the method that takes priority during data collection and analysis, (3) what the integration stage of the findings will be involved and (4) whether theoretical perspective will be used or not.

Based on the suggesting made by Creswell et al (2003), the study employed a mixed-method approach known as sequential explanatory design. According to Creswell, et al (2003) cited in Nataliya, et al (2006) , the mixed-method sequential explanatory design consists of two distinct phases: quantitative followed by qualitative. Tahshakkori and Teddie (2003), furthering Creswell, et al (2003) study, identified three different approaches to mixed methods: concurrent, sequential and conversion. The IO-IA undertook a sequential approach where quantitative method was then followed by the qualitative method. The qualitative data findings were used to set the quantitative data in context. In the quantitative data phase, the researcher conducted a test. The students' solution were scored and analysis. After the test, 5 students were randomly selected and interviewed. The data collected from the interview together with some observations that were made during the implementation of the intervention were then put together to form the qualitative data for the study. The entire data collection for both the quantitative and qualitative took two academic years. The data collection was done in phases and in each phase, the quantitative data was first collected followed by qualitative.

3.4 Population

The population of the study was made up of the entire undergraduate mathematics education students in the Department of Mathematics Education in the University of Education, Winneba (UEW) for the two academic years: 2016/2017 and 2017/2018. Table 3.1 gives the details of the population.

Table 3.1 Population of the Study

Academic Year	Male	% Males	Female	% Females	Total
2016/2017	754	91.95%	66	8.05%	820
2017/2018	729	92.51%	59	7.49%	788

Students of the Department of mathematics Education in UEW were selected because the study was on partial differential equations and the students in the department offer the course which is also the researcher's area of interest. Also, the research design involves a module that the students went through and so there is the need to have access to the students for effective implementation. Since the researcher is a lecturer in the Department of Mathematics Education in UEW, it became appropriate for the students in the department to be selected for the study.

In the 2016/2017 academic year, the population of the students in the department was 820 comprising of 754 males (91.95%) and 66 females (8.05%). The 2017/2018 year group has a population of 788 with 729 (92.51%) males and 59 (7.49%) females. The department is generally a male dominated department.

3.5 Sample

The sample was made up of the entire level 400 students of the Department of Mathematics Education, in University of Education, Winneba (UEW) for 2016/2017 and 2017/2018 academic years. There were 218 level 400 students in 2016/2017 academic year of which 191 (87.61%) were males and 27 (12.39%) females (see Table 3.2).

Table 3.2 Sample for the Study

Academic Year	Male		Female		Total
	N	%	N	%	
2016/3017	191	87.61%	27	12.39%	218
2017/2018	164	90.11%	18	9.89%	182
Total	355	88.75%	55	11.25%	400

There were 182 level 400 students in the 2017/2018 academic year of which 164 students (90.11%) were males and 18 students (9.89%) were females. So, in all, 400 students of which 355 students (88.75%) were males and 55 students (11.25%) were females were used for the study in the two academic years. The purpose of the study was explained to the students and the opinion of the students in terms of their participation of the study was discussed and agreed before the commencement of the study. Table 3.3 indicates the sample distribution for the study.

Table 3.3 Sample Grouping for the Study

Academic Year	Group 1			Group 2			Total		
	M	**F**	**T**	**M**	**F**	**T**	**M**	**F**	**T**
2016/2017	96	13	109	95	14	109	191	27	218
2017/2018	82	9	91	82	9	91	164	18	182

Keys: M:- Male, F:- Female, T:- Total

In each of the two academic years, the students were put into two groups: Group 1 and Group 2. Table 3.3 gives the distribution of the students in the two groups. The grouping was done using the diagnostic test. The students were sorted according to their performance in the diagnostic test in the descending order using their scores as a measure and was done on gender basis. For the male students, the students with odd number positions were put in group 1 and those with even number positions were put in group 2. This was done to make sure that, the academically good students will not cluster in one group. For the female students, those with even number positions were put into group 1 and those with odd number positions were put into group 2. This was done so that there will be a proportional representation of males and females in each group. In the 2016/2017 year group, group 1 was made the experimental group and group 2 was made the control group. In the 2017/2018 year group, group 1 was made the control group and group 2 was made the experimental group.

Table 3.4 indicates how the allocation was done.

Table 3.4 Group Allocations

Academic Year	Phase	Total No.	Control Group	Experimental Group
2016/2017	1	218	Group 2	Group 1
2016/2017	2	218	Group 2	Group 1
2016/2017	3	218	Group 2	Group 1
2017/2018	1	182	Group 1	Group 2
2017/2018	2	182	Group 1	Group 2
2017/2018	3	182	Group 1	Group 2

The study was conducted in three phases in each academic year. In the first phase (Phase 1), the students were taken through concepts and definition of partial differential equations, classification of first-order differential equations and the formation of first-order differential equations. In the second phase (phase 2), the students were taken through the general and particular solution of first-order PDEs and the geometrical interpretation of the solutions. In the third and final phase (phase 3), the students were taken through the second-order PDEs and their applications. The applications were limited to heat and wave equations.

After each phase of the study in both academic years, 5 students were randomly selected from the experimental group for the interview. They were interviewed to solicit their view on the innovations and how it influenced their conceptual understanding of PDEs. During the process of the random selection, when a student who has already been interviewed is selected again, that student is dropped and another is selected in his or her place. So, in all, 15 students were interviewed in each academic year making it a total of 30 students for the study.

3.6 Sampling Technique

Two sampling procedures were used during the study: purposive sampling technique followed by random sampling technique. Purposive sampling technique was used to select the level 400 students. The level 400 students were selected because partial differential equations are taught at that level and so they constitute the community of interest. That is, the Partial Differential Equation course is always taken at level 400 in the Mathematics Education Department of UEW. The study was conducted in three phases for each of the academic year. After every phase of the study, random sampling procedure was used to select 5 students from the experimental group who were then interviewed.

3.7 Data Collection Instruments

Three data collection instruments were used in the study: observation, test and interview.

3.7.1 Observation

Observation according to Angrosino (2005) is a method of data collection in which researchers observe within a specific research field. Participant observation involves the observer being a member of the setting in which they are collecting data. The observation was done in both the control group and the experimental groups during the implementation of the IO-IA and at the time of marking of the confirmation test. The observation was used to determine the factors that affect teaching and learning of PDEs as well as the challenges associated with the implementation of the IO-IA. The researcher employed the help of a different observer to observe the processes during the implementation of the IO-IA. There was limited interference from the researcher during the period but few questions were asked by the researcher to ascertain their reason of using a certain procedure or method as well as what informed their decision to do so. Data collected using this instrument was used to answer research questions 1 and 3.

3.7.2 Test

Two types of tests were conducted: diagnostic test and confirmation test.

Diagnostic Test

A diagnostic test was administered to all students in the sample (level 400 students). The test was on Ordinary Differential Equations (ODEs), which was an immediate pre-requisite for Partial Differential Equations (PDEs). The diagnostic test was conducted once in each of the two academic years. It helped the researcher to identify the factors that affect the teaching and learning of PDEs. It also helped the researcher to identify the challenges the students have on differential calculus, integral calculus and ordinary differential equations (ODEs), which could affect the teaching and learning of PDEs.

The diagnostic test was also used as a tool to put the students into two groups so that students who are academically good are all not put in one group. Based on the results of the diagnostic test, students were sorted according to their scores. Those with odd number positions were put in one group and those with even number positions were put in the other group. This process was informed by the diagnostic test conducted during the pilot study and the confirmation test conducted during the pilot study. After each of the diagnostic tests in the two academic years, the researcher had an interaction with the students after which a review on calculus and ODEs was done to the selected students for the study so that it minimized the negative impact it might have had on the implementation of the treatment. The data collected from this instrument was used together with the data from the observations to answer research questions one and three.

Confirmation Test

In the study, confirmation tests were administered to 218 level 400 students in the 2016/2017 academic year and 182 level 400 students in the 2017/2018 academic year after each phase of the study. Each year group consisted of two groups: the experimental and the control groups. The results from the test were used to investigate the impact of the use of IO-IA on teaching and learning of PDEs. The data collected using this method were used to answer research question 2.

3.7.3 Interview

Semi-structured interviews were conducted after each phase of the implementation of the inquiry-oriented instructional approach (IO-IA). They were used to solicit students' impression about the approach used during the lesson deliveries (see appendix VI for the interview guide). It was used to identify the challenges associated with the implementation of the IO-IA in teaching and learning of PDEs. The data gathered with this instrument was used to answer research question 1 and 3.

3.8 Reliability and Validity

Reliability and validity are used for enhancing the accuracy of the assessment and evaluation of research works. Reliability according to Blumberg et al (2005) refers to a measurement that supplies consistent results with equal values. It measures consistency, precision, repeatability and trustworthiness of a research (Cohen & Morrison, 2011; Chakrabartty, 2013). Validity on the other hand is the extent to which any measuring instrument measures what it is intended to measure (Blumberg et al., 2005; Thatcher, 2010; Robinson, 2011). Reliability and validity have different meaning under different types of research (Tavakol & Dennick, 2011): quantitative and qualitative research. According to Sinkovics et al (2008), qualitative research should be geared towards trustworthiness while reliability, validity, generalisability and objectivity are fundamental concerns for quantitative research. In quantitative research, reliability refers to the consistency, stability and repeatability of results. According to Twycross and Shields (2004) and Thatcher (2010) reliability is divided into three parts: stability, homogeneity and equivalence. Stability according to Twycross and Shields (2004) is when a researcher obtains the same result in repeated administrations or when the same test tools are used on the same sample size more than once. Homogeneity on the other hand is a measure of the internal consistency of the scales. It is assessed using item-to-total correlation, split-half reliability, Kuder-Richardson coefficient and Cronbach's alpha.

3.8.1 Reliability of the Test Items

In this study, the split-half reliability model was used because, the questions were answered once and was not repeated in all the cases. In split-half reliability, a test for a single knowledge area is split into two parts and then both parts given to one group of students at the same time. The test was divided into two equivalent halves and the correlation between the scores of the two halves was calculated which gave the half-test reliability (Cohen, et al., 2011). Split-half reliability measures the degree of internal consistency by checking one half of the results of a set of scaled items against the other half. In calculating the reliability coefficient using the split-half model, the questions were split into odd and even question and the Spearman-Brown coefficient for equal length was computed (Cohen, et al., 2011; Furr, 2012). After computation, the Spearman-Brown coefficient for equal length was found to be 0.907 which indicated the test was highly reliable and because the coefficient is high, the test can be said to be homogeneous according to Cohen et al. (2011) (see Appendix VI(a) for the test results).

3.8.2 Validity of the Test

In order for the test instrument to be valid, care was taken to make sure that the test instrument is first reliable since validity requires that an instrument is first reliable (Kimberlin & Winterstein, 2008). Then the researcher made sure that, all possible questions were asked about the content or skill in accordance with Creswell (2003) and Cohen et al. (2011) view on validity (content validity). Content validity addresses how well the items were developed to operationalize a construct and provided an adequate and representative sample of all the items that might measure the construct of interest. Because there is no statistical test to determine whether a measure adequately covers a content area or adequately represents a construct, content validity usually depends on the judgment of experts in the field. For this reason, the test was examined by mathematics teachers/lectures who were researchers as well. Also, care was taking to make sure the test measures the students conceptual understanding and ability to apply the concept learnt. The inputs made by the assessors helped the researcher to re-arrange the test items and modify some of them to measure the effectiveness of the IO-IA.

3.8.3 Reliability and Validity of the Interview

According to Cannel and Kahn (1968) cited in Cohen et al. (2007), in interviews, inferences about validity are made too often on the basis of face validity (Cannell and Kahn 1968), that is, whether the questions asked look as if they are measuring what they claim to measure.

To address the issue of validity and reliability in relation to interview, Farr (1982) was of the view that, there should be a theory behind every interview encounter. The theory should incorporate psychological as well as social factors. Kuzmanic (2009) distinguished the process of addressing the validity of a research interview into four: the production of data, data presentation and interpretation, process of preparation and transcription of verbal data and analysis and interpretation.

From the preparation of the interview guide through to transcription and data interpretation, care was taken to make sure that the interview was reliable and valid. In order to verify the reliability of the interview guide, the wording was considered since wording is particularly important factor in attitudinal questions than factual questions (Oppenheim, 1992). The interview was then conducted on two groups comprising of five students separately and their responses were compared and the way they understood the questions too were also compared. The interview guide was then shown to experience researchers for validation and their inputs lead to the modification of some of the questions in the interview guide. During the interview, the researcher avoid biasness on the responses of the participants as was indicated by Cohen et al. (2007). Their contributions helped the researcher to modify the questions. The data gathered from the interview was used to answer research questions 1 and 3.

3.9 Data Collection Procedures

Different procedures were used in the data collection process for the observation, interview and the test. But apart from the test and the interview, the researcher also documented the challenges faced during the implementation of the IO-IA which was also used together with the interview to answer research question 3.

3.9.1 Data Collection Procedure for the Observation

During the marking of the scripts (both the diagnostic and confirmation test), students' solutions were observed and analysed on the processes and procedures they used in answering questions. Their challenges on individual basis were noted down for analysis. Observations were also done during the instructional delivery period during the teacher's interaction with the students and during group discussions. The way students argued among themselves were recorded for further analysis and discussion.

3.9.2 Data Collection Procedure for the Test

In each academic year, three activities were done and so three tests were administered to both the control group and the experimental group. Each of the tests looked at the analytical approach in solving the problem, the graphical representation and interpretation of the solution where necessary. The first test looked at how students solved first-order partial differential equation problems analytically, the interpretation of the results and the graphical representation of the solution where possible. The second test looked at the application of first-order PDEs in a real-life situation. And finally, the third test looked at second-order PDEs and its application to heat and wave. During scoring of the test, the emphasis was on methods and procedures used in solving the given problem, the interpretation of the results and the graphical representation of the solution. See appendix I (d to e) for the test items. The solutions presented were marked and scored and the results compared using independent sample t-test. The tests were scored on three areas: the method or approach in which the students employed in finding the solution to the test and the scores for that were termed "method scores", the students proved of understanding of the concept and the scores were termed "concept scores". The sum of these two scores, method and concept scores, were termed considered as their total performance and were termed "total scores". The scores from the test items were used so answer research question 2.

3.9.3 Data Collection Procedure for the Interview

After each test, 5 students were randomly selected for an interview and their responses were recorded and transcribed and finally analysed. The interview was used to gather information on some of the factors affecting teaching and learning of PDEs as well as the challenges associated with these factors. The information gathered during the interview was used to answer research questions 1 and 3.

3.10 Pilot Study

Eighty level 400 students of 2015/2016 academic year were randomly selected for the pilot study. Out of the 80 students, 40 students constituted the control group and the remaining 40 constituted the experimental group. These groups of students were not part of the actual implementation of the treatment for this study. The students in the experimental group were taken through first-order partial differential equations and its applications using inquiry-oriented instructional approach (IO-IA). The remaining 40 students were allowed to go through the same topics using the traditional method of teaching. They were taken through three phases. The first phase was on the introduction of partial differential equations (PDEs) which comprised of basic concepts and definition of PDEs, formation of PDEs, solutions to PDEs, linear operators and principle of superposition. The second phase was on first-order partial differential equations and it covered classification of first-order PDEs, formation of first-order PDEs and solutions to PDEs, methods of solving first-order PDEs (directional derivatives, methods of characteristics and change of coordinates). The third phase was on application of first-order PDEs.

Before the commencement of the pilot study, a diagnostic test was conducted on ODEs, which is a pre-requisite to the PDEs. The purpose of the diagnostic test was to assess the students' performance on ODEs. After the diagnostic test, 40 students out of the 80 students were randomly selected to form the experimental group. The 40 students were then taught using the IO-IA using the 5E instructional model. At the engagement phase, students' prior knowledge was accessed and linked to the new subject matter. Students' curiosity was aroused and interest generated through brainstorming on the importance and application of the first-order PDEs. At the exploration phase, which is where the IO-IA was implemented, students were directed to interact with the materials in their small groups. They were directed to look for ways of solving the problem in order to identify and realized different skills which then led to the acquisition of common experiences. They were directed to observe, describe, record, compare and share their ideas and experiences. They were given activities which led them to the discovery of ways of finding solution to the given real-life application problems. The students were then asked to present their findings to the whole class.

After the exploration phase, the instruction entered the explanation phase. The right terms and terminologies were explained and formal definition and theorems were then given to students. New terminologies and scientific terms were then introduced and explained. At the elaboration phase, new but similar situations were explored. The students were directed to look for different situations where what they have learnt could be applied. The students were then evaluated after the elaboration phase.

After the first phase of the implementation of the study, a test was conducted on both the control group and the experimental group (see Appendix (Ib) for the test). The results were analysed using independent sample t-test (see Appendix V(c) for the analysis). From the analysis, the mean score of the experimental group was 7.975 and that of the control group was 7.225. Even though the means score for the experimental group was slightly higher than that of the control group, the p-value (sig. value) for Levene's Test for Equal variance of 0.629 (see Appendix V(c)(i) Tables 6.9 and 6.10) indicated that the variability in the scores of the control group and the experimental group was about the same. That is, the variability in the two groups is not significantly different and so the researcher assumed equal variances in our t-test for equality of means. The p-value (Sig (2-tailed)) for equality of means on the assumption that the variances are equal was 0.053 ($t = -1.962$). This means that there was no significant difference in the mean values of the control and the experimental group. But the researcher's expectation was to achieve a significant difference in the mean-scores of the two groups. So' the diagnostic test results of the two groups were analysed (see Appendix V(b)(i) Tables 6.5 and 6.6). The p-value (significant value) for Levene's test for equality of variance of 0.326 indicated that there was no significant difference in the variability and so the p-value (Sig. value) for 2-tailed for t-test for equality of means was found to be 0.001. P-value of 0.001 indicated a significant difference between the mean scores of the control group and the experimental group. But the mean score for the control group was 7.45 which was higher than the mean score for the experimental group which was 6.025. This indicated that, during the grouping of the students, the highly performed students were accidentally placed in the control group. For this reason, the students were sorted according to their performance from the highest to the lowest. Those at odd number positions were used to form the control group and those at even number positions were used to form the experimental group. Based on this re-grouping, their diagnostic test was analysed again (see Appendix V(b)(ii) Tables 6.7 and 6.8). In this case the p-value for t-test for equality of means was found to be 0.894 indicating that, the mean values for the two groups (control and experimental) were not significantly different. The pilot study then entered the second phase.

After the second phase of the pilot study, a second test was administered for both the control group and the experimental group. The mean score for the control and the experimental were found to be 6.200 and 7.725 respectively and the p-value for t-test for equality of means was found to be 0.000 which was less than the α-value of 0.05 (see Appendix V(c) Tables 6.11 and 6.12). This indicated that there was a significant difference between the mean scores of the control group and the experimental group in favour of the experimental group. It indicated that, the IO-IA model impacted positively on the teaching and learning of PDEs. The results after the second phase informed the researcher the need to have equity in terms of performance when putting the students into the two groups: control and experimental.

The third and final phase of the implementation of the IO-IA model was conducted and a test was administered after. The analysis (see Appendix V(c) Table 6.13 and 6.14) indicated the mean score of 4.60 for the control group and 7.45 for the experimental group. And the p-value (2-tailed) for t-test for equality of means was 0.000 indicating a significant difference between the mean scores of the two groups. It was therefore established that the IO-IA model have significant and positive impact on the performance on teaching and learning of PDEs as far as the pilot study was concerned.

The pilot study informed the researcher of the following:

1. The grouping of the students into control and experimental groups should be done in such a way that the academically good students should not be placed in one group. For this reason, the grouping was done by ranking the students according to their scores and students with the even number positions formed one group and those with the odd number positions formed the other group in the main study.

2. It was realized in the third test that the mean difference between the control and the experimental group was bigger as compared to tests 1 and 2. That is, as the study progresses in terms of difficulty level, the performance of the students in the experimental group got better which may be attributed to the application of the

IO-IA. For this reason, in the main study the control and the experimental groups was done on rotational basis between the two groups for the study. This was an ethical decision which was to make sure that all students benefit from the IO-IA model. It was important because the course, PDEs, was a graded course in that semester in the academic years and so any deficiency in the instructional delivery would affect their actual performance.

3. It was also realized that, some students wanted to switch groups, that is, move from the control group to the experimental groups especially. To avoid such actions which would negatively impact on the main study, the rotation of the control and the experimental groups between the two groups helped to minimize the practice.

Apart from the observations above, the pilot study helped the researcher to identify some of the challenges associated with the implementation of the IO-IA model as well as factors like instructional strategies, computational ability, teaching-learning materials and so on affecting teaching and learning of PDEs.

3.11 Scoring of Test Items and Interview Recordings Process

In the main study, the test items were marked and scored by the researcher. The researcher was interested in the methods or approaches used, the explanation of the concepts, the reason for the concept used, the interpretation of the solutions and the graphical representation of the solutions. So, care was taken to mark and score so that several aspects of students conceptual understanding was ascertained. The Interview was recorded and transcribed by the researcher. Five students were interviewed after each stage of the implementation of the IO-IA and so 30 students were interviewed for the whole studies, that is, 15 per academic year (see Appendix VI for the interview guide).

3.12 Data Analysis Procedures

In analysing the test scores, both descriptive and inferential analysis were done on the results from the test conducted on both groups at each stage. The analysis was done phase-wise since there was rotation of groups at every phase of the instructions in terms of the control and the experimental group (see Table 3.4). For this reason, the analysis could not have been done at the end of the entire treatment but a total analysis was done at the end of each phase. The generalization of the conclusion was drawn on how consistent a particular group in terms of the control or the experimental performs at the end of each phase performed. At each phase, an independent sample t-test was conducted to test the difference in means of the two categorical groups. After that, the effect size was calculated to determine the size of the difference.

In order for the independent sample to be valid, the following assumptions underlying the independent sample t-test were tested:

1. The dependent variable should be measured on a continuous scale (that is, it is measured at the interval or ratio level).
2. The independent variable should consist of two categorical, independent groups.
3. There should be independent observations, which means that there should be no relationship between the observations in each group or between the groups themselves.
4. There should be no significant outliers.
5. The dependent variable should be approximately normally distributed for each group of the independent variable.
6. The variances should be homogeneous.

The effect size was also computed to determine the size of the difference between the means of the experimental group and the control group using Cohen's d. An effect size is the difference between two means (e.g. treatment minus control) divided by the standard deviation of the two conditions (Cohen, 1990; Fritz et al., 2012). Effect size allow researchers to move away from the simple identification of statistical significance toward a more generally interpretable, quantitative description of the size of an effect (Fritz et al., 2012).

The interview was also conducted phase-wise. That is, after every phase of the intervention, an interview was conducted on randomly selected five students. With the permission of the participants, the original interview was audio-recorded. In the analysis of the interview, the following steps were taken:

1. The recorded audio was transcribed.

2. The transcripts were read and notes about first impression were made. The transcripts were then re-read one after the other and line by line.

3. Relevant pieces such as words, phrases, sentences or sections were labelled. The labels were about actions, activities, concepts, differences, opinions, processes or any relevant information. This is referred to as coding/indexing. To determine how relevant a certain information is, the researcher considered the following:

   (a) Is the information repeated in several places or respondents?

   (b) Does it surprise the researcher?

   (c) Was it insisted by the interviewee?

   (d) Have the researcher read about it on any article or published report?

   (e) Does it reminds the researcher a certain theory or concept?

   (f) Is it superficial or conceptualization of an underlying pattern?

4. The researcher made sure there was no bias in the coding process. The opinion of the participants was accepted and every relevant statement or response by the participants were recorded without the researcher's personal interpretation.

The researcher then decided which of the codes was very important and categories were created to put several codes together in themes. Important codes were then grouped and the data conceptualized. The categories were then labelled and the relevant ones were considered. Their connections with each other were found and the results were put into tables and discussed. The interview data were collected at the end of each phase in the two academic years.

In the case of the observations, the challenges observed were noted at every phase of the study but were put together and added to those observed during the pilot study. This was also done phase-wise as was done on the test and interview.

3.13 Implementation of the Inquiry-Oriented Instructional Approach (IO-IA)

3.13.1 Background of IO-IA in Lesson Delivery

Students are involved in more than listening and reading during lesson delivery. They develop skills, analyse and evaluate evidence, experience and discuss, and talk to their peers about their own understanding of the subject matter. Students work collaboratively with others to solve problems and plan investigations. When active, collaborative learning is directed toward scientific inquiry, students succeed in making their own discoveries. They ask questions, observe, analyse, explain, draw conclusions, and ask new questions. These inquiry-based experiences include both those that involve students in direct experimentation and those in which students develop explanations through critical and logical thinking.

The study as stated earlier, was conducted in two academic years: 2016/2017 academic year and 2017/2018 academic year. Before the implementation of the treatment on each academic year, the students were put into two groups: group 1 and group 2. The students were made to understand the purpose of the study and were informed that, at each phase of the implementation, one group would be considered the control group while the other would be the experimental group but will be done on rotational basis. That is, on the first phase of the treatment, one group was made the control group while other was made the experimental group. On the second phase, the group which was made the control group was then made the experimental group and vice-versa. This arrangement was put in place to reduce or minimize the rate at which students change groups and as such ensure the independence of the two groups. Another purpose for the rotation of groups was an ethical decision. The researcher made sure that every student benefited from the model since PDEs was their course of study for that semester. It also helped the researcher to establish that the performances of the students were improved due to the Inquiry-Oriented Instruction Approach (IO-IA) used. The study was conducted on two topics:

1. First-Order Partial Differential Equations and their applications.
2. Second-Order Linear Partial Differential Equations and their applications.

The same topics were repeated for both academic years. This was done to establish the consistency of the IO-IA. The topic for the first and second stage of the implementation was ”First-Order Partial Differential Equations” and was limited to equations involving two independent variables. That of the third stage of the implementation was Second-order partial differential equations. The implementation of the treatment was done phase-wise because of the rotation of the groups in terms of control and experimental group. Teaching Partial Differential Equations (PDEs) comes with great difficulties and as such inclusion of interactive visualization went a long way to explain the concept to the students.

3.13.2 Stage 1 of the Implementation

Topics: *First Order Partial Differential Equations and their classification.*

Applications: *GeoGebra, MATLAB/Octave*

In the first phase, group 1 was made the control group and group 2 was made the experimental group. Group 1 during the first phase was taught by the traditional method while students in group 2 (the experimental group) were instructed using the inquiry-oriented instruction approach (IO-IA). All activities and lesson plans for the experimental groups were implemented by considering stages of 5E learning cycle model proposed by Bybee and Landes (1990).

Engagement Phase

The objectives of this phase were to

- access the students' prior knowledge and connect the current subject matter to their past knowledge
- arouse the students' curiosity and generate their interest in PDEs
- determine how students can make scientific inquiry
- encourage the students to raise their own questions about the process of scientific inquiry
- encourage students to share and compare their ideas with their peers
- set the parameters of the students' focus
- enable the instructor to assess what students do or do not understand about the stated outcome of the lesson

The researcher's main focus was for the students to essentially deduce several key mathematical ideas and methods for analysing and solving first-order partial differential equations. For this reason and many more, functions of several variables were reviewed. According to Joseph (2008), solution of linear, constant-coefficient PDEs requires the basics of multi-variable calculus. Students used computer applications to graph and compare functions of one and two variables. For example, the graphs of $g(x) = x^2$ and $f(x,y) = x^2 + y^2$ were compared and discussed among students (see figure 3.1). In summary, the students were able to explain that, the graph of the function $f(x,y) = x^2 + y^2$ was a solid (cone) while that of the function $g(x) = x^2$ is a line (parabola).

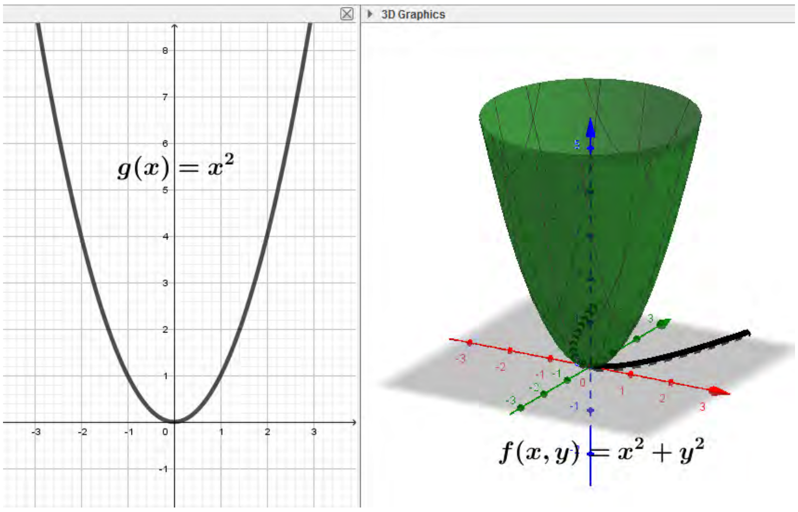

Figure 3.1: Graphs of $g(x) = x^2$ and $f(x,y) = x^2 + y^2$

Also, the graphs of $f(x) = |x|$ and $g(x,y) = |x+y|$ was compared and discussed among students (see figure 3.2). The students were able to conclude that functions to single variables are lines while those of two variables are solids or surfaces.

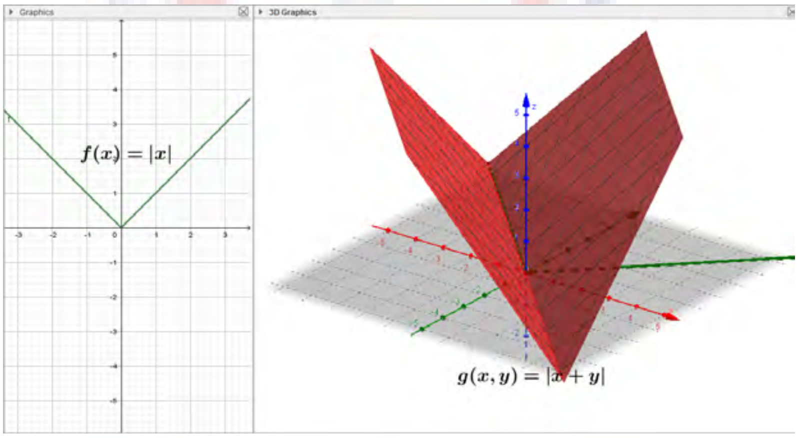

Figure 3.2: Graph of $f(x) = |x|$ and $g(x,y) = |x+y|$

In order to further generate the interest of the students, the importance, meaning and application of differentials were discussed with them. This was followed by revision of derivatives and integrals of functions of two variables and their geometrical interpretation. Sketches/graphs, videos and simulations were used for demonstration using computer applications. This was followed by revision of Ordinary Differential Equations (ODEs). Classification of ODEs followed by geometrical interpretation of solutions of ODEs were reviewed and was linked to PDEs. The various applications of ODEs were also discussed.

Students' prior knowledge on partial derivatives of functions involving two variables were reviewed. The geometrical interpretation of partial derivatives was reviewed and was eventually linked to partial differential equations (PDEs). This served as the students' entry point to partial differential equations. After several discussions and deliberations the students concluded that the function $z = f(x,y)$ geometrically represents the surface S (the graph of f) and the partial derivatives $f_x(x,y)$ and $f_y(x,y)$ at a point (a,b) can be interpreted as the slopes of the tangent lines at the point $P(a,b,c)$ to the traces C_1 and C_2 of S in the planes $y = b$ and $x = a$ as shown below in Figure 3.3.

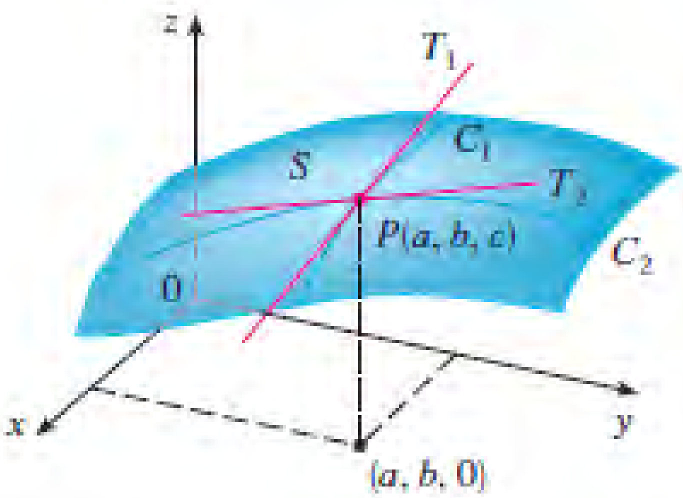

Figure 3.3: Partial Derivatives of f at (a,b)

The students discussed the main difference between ODEs and PDEs. After the revision of classification of ODEs according to the order and linearity, students were given a list of ordinary differential equations to classify and the ideas were linked to the classification of partial differential equations.

Exploration Phase

The main purpose of the exploration phase was to guide the students to experience key concepts, discover new ideas or skills, probe, inquire and question experiences, examine their thinking and establish relationship and understanding. This phase incorporated the inquiry-oriented instructional approach (IO-IA) in the lesson delivery. At the engagement phase, students were made to

a. interact with materials/applications and ideas through classroom and small group discussions.

b. consider different ways of solving problems in order to discover new skills.

c. acquire common set of experiences so that they can compare their results and ideas with their peers which lead to probing, inquiring and questioning of their experiences.

d. observe, describe, record, compare and share their ideas and experiences so that they can examine their thinking.

e. express their understanding of testable questions and scientific inquiry which help them to establish relationships among variables and understanding of concepts.

The exploration activities were in line with Gagne (1977) studies on learning which says ”learner must be exposed to persuasive arguments in order to be able to learn attitudes and must have a chance to practice new solutions”. The lessons were delivered as follows:

Activity 1: Formation of First-Order PDEs

The following geometrical problem questions were raised by the instructor. The purpose was to introduce the students to the methods of forming partial differential equations.

1. Show that the tangent plane to the graph $u = u(x,y)$ at any arbitrary point $(a,b,u(a,b))$ which passes through the origin is given by the PDE $au_x + bu_y - u = 0$.

2. Show that the set of all spheres with centres on the z-axis is characterized by the first-order PDE $yz_x - xz_x = 0$.

3. Form a first-order PDE for all surfaces described by an equation of the form $u = f(x^2 + y^2)$ where f is an arbitrary function.

The first question served as a guide which helped the students to ascertain that, the equation of tangent plane is characterized by first-order partial differential equations.

Students used their previous knowledge on tangent plane to a surface at a point $(a,b,u(a,b))$ and is given by

$$u_x(a,b)(x-a)+u_y(a,b)(y-b)-(u-u(a,b))=0$$

Students solution is summarized as follows:

Since the plane passes through the origin $(0,0,0)$, the equation of the tangent plane is given by

$$-u_x(a,b)a-u_y(a,b)b+u(a,b)=0$$

This gives

$$au_x+bu_y-u=0 \tag{3.1}$$

Equation (3.1) is a first-order partial differential equation.

The second question was used to introduce the students to the formation of PDEs by elimination of arbitrary constants. For the second question, the solution in summary were presented as follows:

Since on the z-axis, the values of x and y are zeros and any point on the z-axis is of the form $(0,0,c)$ where c is a real number on the z-axis, the equation of the sphere is given by

$$x^2+y^2+(z-c)^2=r^2 \tag{3.2}$$

where r is the radius and r and c are arbitrary constants and they represent the set of all spheres whose centres lie on the z-axis. Taking the derivative of equation (3.2) with respect to x, we have

$$2x+2(z-c)\frac{\partial z}{\partial x}=2(x+(z-c)z_x)=0 \tag{3.3}$$

Differentiating with respect to y, we have

$$2y+2(z-c)z_y=0 \tag{3.4}$$

Eliminating c from both equations gives

$$yz_x - xz_y = 0 \tag{3.5}$$

which is also a first order PDE.

The third question was used to introduce the students to the formation of PDEs by elimination of arbitrary functions. For the third question, the students assigned $w = x^2 + y^2$ so that $u = f(w)$, then $u_x = 2xf'(w)$ and $u_y = 2yf'(w)$, where $f'(w) = \frac{df}{du}$. Making $f'(w)$ the subject in both partial derivatives and eliminating $f'(w)$, they have

$$\frac{u_x}{2x} = \frac{u_y}{2y}$$

$$2yu_x = 2xu_y$$

and this gave

$$yu_x - xu_y = 0 \tag{3.6}$$

which again is a first-order PDE similar to equation (3.5). The instructor then summarized the finding as follows: First-order PDEs can be formed either by eliminating arbitrary constants or arbitrary functions. After students have gone through these activities in the formation of PDEs, prediction task was then given. This task was given to students to verify how application of conservation principles often yields a first-order PDEs.

Activity 2: The Prediction Task (The Traffic Flow)

The prediction task was based on theory of traffic flow invented by Sir James Lighthill and G. B. Witham in Manchester in 1955. See appendix II(a) for details of the prediction task at page 215. In the task, we imagine a number of cars on a particular stretch of road and proceeded as follow:

Let x be the distance along the road. Note that, the road may not necessarily be a straight line. Given that the traffic density $\rho(x,t)$ on the road is defined as the number of cars on the road per unit distance at a point x and time t, the traffic density function was deduced.

Figure 3.4: Cars on Road

The instructor through inquiry-oriented questions and with provision of minimal explanation and insight on some theorems of calculus, guided the students in the task of mathematization. The students were able to model the traffic density function which is a first-order partial differential equation which is given by

$$\rho_t + f'(\rho)\rho_x = 0$$

where ρ is the traffic density, t is the time and x is distance along the road. The students therefore were able to establish the application of differential equation traffic flow. Students were put in groups and were tasked to look at the transport equation. Students were guided to understand the concept of first-order PDEs using their previous knowledge on first-order ODEs.

Explanation Phase

At this phase, students previous experiences were connected to the new subject matter and the conceptual sense of the main ideas of the module was made. New terminologies and scientific terms were introduced and explained. In the classification of first-order PDEs, further explanation and classifications were discussed with the students. The students were first allowed to categorized a list of first order PDEs into linear and non-linear equations. They were then asked to determine where the non-linearity of the non-linear PDEs and group them according to that. The students were finally able to put the non-linear PDEs into three categories:

i. Those in which the non-linearity is on the unknown functions but not on the derivatives of the unknown functions,

ii. those whose non-linearity is not on the derivative of the unknown functions but it is either due to the fact that there is a product of the unknown function and its derivative and

iii. finally those in which the non-linearity is on the derivative of the unknown function.

Examples of some of the first-order PDEs which were put in category (i) are follows:

$$xu_x + xyu_y = u^2 - x^2$$
$$(x^2-1)^2u_x + (y-1)^2u_y = \sin u$$
$$u_t + xu_x + e^u = 0$$

Samples of the first-order PDEs which were put in category (ii) are as follows:

$$x(y^2+u)u_x - y(x^2+u)u_y = (x^2-y^2)u$$
$$uu_x + u_y + u^2 = 0$$
$$(y^3-u^2)u_y - xyu_x = xu$$

And finally, samples of the first-order PDEs which were put in category (iii) are as follows:

$$u_x^2 + u_y^2 = 3$$
$$u_xu_y + xu_y - 1 = 0$$
$$\ln u_x + u_y = e^u$$

After this, the technical terms for the various categories which the students put those equations were then supplied and explained by the instructor. The following explanations were given:

1. The set of non-linear equations that were placed under category (i) above are said to be semi-linear. They are equations of the form

$$a(x,y)u_x + b(x,y)u_y = f(x,y,u)$$

   In this equation, the coefficients $a(x,y)$ and $b(x,y)$ are independent of the unknown function u but the function f is dependent on the unknown function u.

2. The set of non-linear equations that were placed under category (ii) above are said to be quasilinear. They are equations of the form

$$a(x,y,u)u_x + b(x,y,u)u_y = f(x,y,u)$$

   In this equation, the coefficients a and b are dependent on the unknown function u but the function f may or may not be dependent on the unknown function u.

3. The set of non-linear equations that were placed under category (iii) above are said to be fully non-linear. They are fully non-linear because they are not linear in the highest derivative, which is the first derivative.

Computer application and web resources for simulation of differential equations were introduced for students to get a better understanding of the new concept. Applications like, GeoGebra and MATLAB were used to further explain the solutions to differential equations. Students used GeoGebra to plot the solutions of the partial differential equations.

Elaboration or Extended Phase

At this phase, what is learnt is applied to a new similar situation, explored concepts are extended and explain and new understanding is communicated using the formal language or terms.

Evaluation Phase

This phase was used to assess student's understanding of the concept of first-order PDEs. The assessment includes self-assessment, peer assessment and teacher-made test. Students were allowed to demonstrate understanding of the concept of the new concept by observation or open-ended response. The assessment required students to show evidence of their understanding so they were instructed to provide reasons for every step they take. See appendix I(d) for teacher's assessment test.

3.13.3 Stage 2 of the Implementation

Topics: *Finding solution (general and particular) to first-order PDEs and their geometrical interpretation.*

At this stage, group 1 was made the experimental group and group 2 was made the control group. Before, the commencement of this lesson, the lesson on introduction of differential equations and first-order partial differential equations was reviewed. This was done so that, the group 1 students who went through the same topics but with traditional teaching will be at the same page with the group 2, who were the experimental group during stage one.

Engagement Phase

At this phase, students' prior knowledge on first-order partial differential equations were reviewed as well as the classification of these equations. Student's prior knowledge on vectors was also reviewed. This was then followed by a review on their knowledge on directional derivatives. Hands-on activities on verification on formation of PDEs from functions of two variables were given.

Exploration Phase

At this phase, students were given a hands-on activity (see Activity 1 of Appendix II(b)). The objective of the activity was to introduce them to geometric method of finding solution to first-order partial differential equations with constant coefficients. The activity helped the students to understand that the quantity $au_x + bu_y$ is actually a directional derivative of the function u in the direction of the vector $\mathbf{V} = a\mathbf{i} + b\mathbf{j}$. Students through the activity were able to established that the partial derivative u_x is a directional derivative in the direction of the unit vector $\mathbf{i}$ and the partial derivative u_y is a directional derivative in the direction of the unit vector $\mathbf{j}$ and so, the quantity $u_x + u_y$ is the directional derivative of the resultant vector in the direction of the unit vectors $\mathbf{i}$ and $\mathbf{j}$. And so, the quantity $au_x + bu_y$ is a directional derivative in the direction of the vector $\mathbf{V} = a\mathbf{i} + b\mathbf{j}$.

The second activity (see Activity 2 of Appendix II(b)) was used as a drill for student to verify the solutions to simple PDEs. So students were asked to find partial derivatives of some given functions of two variables. They were then asked to reverse the process in order to find the functions. Different initial conditions were given to get different particular solutions. These helped the students to establish the relationship between partial differential equations and their solutions and also determine the nature of general solutions as compared to particular solutions. Geogebra was used to animate the effect of different initial conditions on the graphs of the solutions of the PDEs.

Activity 3 was designed for the students to discuss and explain the geometrical interpretation of first-order linear partial differential equations with constant coefficient and to established the solution of a first-order PDE of the form $au_x + bu_y = 0$. The students were able to established that the equation $u_x = 0$ means the rate of change of the function u in the direction of the unit vector $\mathbf{i} = (1, 0)$ which is parallel to the x-axis is zero. Similarly, students were able to established that, the rate of change of the function u in the direction of the unit vector $\mathbf{j} = (0, 1)$ which is parallel to the y-axis is zero. They were able to write the mathematical statement for both equations using the dot product as

$$u_x = (1,0) \cdot (u_x, u_y) = 0$$
$$u_y = (0,1) \cdot (u_x, u_y) = 0$$

In the final analysis, even though students were able to predict that the solution of the equation $au_x + bu_y = 0$ is a constant, they had some challenges in coming out with the general solution. But through the guidance of the instructor and revision of their prior knowledge on vectors which were orthogonal to other vectors, they were able to deduce that the vector $(b, -a)$ is orthogonal to the vector **V**, the resultant vector shown in Figure 6.1 at page 218. It was established that the lines parallel to the vector **V** have the equation $bx - ay = constant$ and these lines are called characteristic lines. And since the solution is constant in each line, and the solution $u(x,y)$ depends on $bx - ay$ only, the solution $u(x,y)$ of the equation $au_x + bu_y = 0$ is given by

$$u(x,y) = f(bx - ay)$$

where f is any arbitrary function of one variable. The solution $u(x,y) = f(bx - ay)$ is called the general solution of the partial differential equation of the form $au_x + bu_y = 0$. Students then solved the problems in activity 4 of appendix II(b).

The students were again taken through activities, which were in the form of questions, for finding solutions to quasilinear first order differential equations. They were guided to investigate the geometrical content of a first-order quasilinear partial differential equation and hence deduce the general solution (see Appendix II(d)). Students were again taken through the method of finding solution to first-order PDEs with variable coefficients using the method of characteristics.

Explanation Phase

The instructor, through the students' prior knowledge on vector algebra and gradient vectors and directional derivatives, guided the students in to deduce the characteristic equation for finding the solution to the given first-order partial differential equation. For details of the instructor's explanation, go to the Instructor's Explanation section of Activity 1 in appendix II(c). The entire explanation was not given at the end of the activity but were given as the need arose during the entire activities. The explanation was used to connect prior knowledge and background to the new discoveries. They were also used to communicate new understandings as well as the connection of informal language or terminologies to the formal language or terminology.

Elaboration Phase

At this phase, students were guided by the instructor to apply the various techniques acquired in new concepts. For example, the results involving the characteristic equation in solving quasilinear PDEs was extended to linear and semi-linear PDEs including those with constant coefficients and variable coefficients. Students were encouraged to use terms and definitions appropriately in their interpretation of solutions to differential equations. More problems were given to students to solve.

Evaluation Phase

At this stage, students were guided to explore the new concepts and skills in solving different problems. Students went through graphing of solutions and give geometrical interpretation of solutions to some given PDEs. Students were then assess with a written test. For the test questions see appendix I(e).

3.13.4 Stage 3 of the Implementation

Topic: Second-order PDE and its applications.

Tools: GeoGebra, MATLAB and Microsoft Excel

Experimental Group: Group 2

Control Group: Group 1

Before the commencement of this phase, a review of the methods of finding solutions to the first-order PDE and their geometrical interpretation was done. This put the experimental group who were the control group in the previous stage to be on the same level with the control group (group 1) who were the experimental group. This was done to make sure the students have all the necessary previous knowledge needed for the next topic.

There are several methods of solving second-order differential equations and it all depends on the type of PDE. The most common approach to the solution of second-order linear PDEs according to Myers, et al (2008) is the technique of separation of variables. This stage of the study covered the derivation of the heat and wave equation and the solution to these equations. It has been noted from several writers of PDEs like Strauss(2008) and Myint-U and Debnath (2007) that separation of variables leads immediately to boundary-value problems and Fourier series. Therefore, it is the instructors' decision to either start with separation of variables followed by Fourier Series or vice-versa when taking the standard approach to PDEs. This most of the times, possess match difficulties during teaching and learning. The series of formulas in the solution process in PDEs alone dumps the interest of the students in learning such important area of mathematics. The researcher therefore, at this stage, used modelling approach in the derivation of heat and wave equations and combined both analytical and numerical approach in finding solution to these equations. The lesson as usual began at the engagement phase using the 5E-learning cycle approach.

Engagement Phase

Students first brainstormed on the importance of second-order differential equations using the various applications of the equations. The classical second-order equations commonly known as the ”Big Three PDEs” (the heat equation, the wave equation and the Laplace equation) was used to generate the interest of the students because of their numerous applications in real-life. Students previous knowledge on both first-order PDEs as well as the second-order ordinary differential equations were also reviewed. Students reviewed their prior knowledge on formation of second-order PDEs from functions of two variables. Students were given functions like $u(x,y) = \sin xy$ to form one-dimensional and two-dimensional equations.

Exploration Phase

In this phase, students explored the derivation of heat and wave equations and looked at the use of Laplace transform in finding analytical solutions to PDEs. Numerical approach using Microsoft Excel was also implemented and graphs were drawn using GeoGebra. At some point, the modelling served as a motivating factor of the students while serving as the application of the second-order linear PDEs. In some cases, ample time was given to students to discover the strategies by themselves while some cases instructions, directions or guidance was included.

Two modelling activities were undertaken by the students: Heat conduction in a thin rod (see Appendix III(b)) and Wave equation in a string (see Appendix III(c)). In the case of solution to second-order linear PDEs with constant coefficients, students were introduced to Laplace transforms and were guided on the use of the Laplace transform in finding analytical solution to second-order PDEs. In addition to that, numerical approach using Microsoft Excel to find solution to the heat equation was implemented (see Appendix III(d)). Graphs of heat distribution in thin rod was plotted from the results obtained from Microsoft Excel at different times to investigate the heat distribution. For the students to acknowledge the behaviour of the general solutions as the constant is varied, GeoGebra was used to crate animations for the students to explore the behaviour of the graph of the solution. Throughout the activities in the modelling process, students were allowed to feel the uncertainty of the moment in which small and cautious steps of which may or may not be right must be taken. This was done on group bases for the students to be able to share ideas and test their assumptions as well as criticize them as well.

The instructor's responsibility was that of a facilitator. Even though laboratory to carry out an experiment for students to have the real-life feel of the activity was not available, all the necessary teaching/learning materials was provided with the necessary guidance. Students were provided with pre-requisite information and the relevant previous knowledge. Some of the extensions of the models under consideration were given to the students as homework.

Students in the cause of the model were asked good-inquiry oriented questions relevant to the study such as ”what ... if?” questions. Students were guided to flow from physical phenomena to equation formulation, to numerical solutions and then to analytic solutions. In this process, students gained ownership and clear understanding of concepts which led to the formulation of the models/equations. Solution strategies and interpretations were guided by the instructor. Graphical solutions were further used to compare the analytical solutions (see Appendix III(d) and III(e)).

Explanation Phase

At this phase, students were encouraged to explain their findings and models. They were also guided to classify the resulting equations and the two strategies were adapted in the solutions of these equations. Terms and concepts associated with their findings were explained by the instructor. Definitions and theorems were also provided to the students and questions were raised to further clarify the students understanding of concepts. Graphical representation of solutions was used to further explain the concept. Numerical and analytical solutions were compared and discussed.

Elaboration Phase

Students at this phase were guided to use the concept gained to solve other second-order PDEs including linear PDEs with variable coefficients. Students were also encouraged to use terms and definitions previously acquired. The concepts acquired were further applied to new problems and students were encouraged to apply both numerical and analytical solutions followed by graphical representations.

Evaluation Phase

Apart from observation of students' behaviour, contributions and explanation of concepts, students were given problems involving applications of second-order PDEs to solve to access their level of understanding (see Appendix I(f) for the test).

3.13.5 Summary of Implementation of IO-IA

At the engagement phase, students' interest was generated, their prior knowledge was assessed which established connection between their past knowledge and the partial differential equations. The parameters of their focus were also set and these ideas were framed which prepared the students for the new concepts.

At the exploration phase, which is the phase where the IO-IA was actually implemented, students were guided to experience new concepts and discovered new skills. Their experiences were probed, inquired and questioned. Their reasoning ability were examined and relationships between variables which were in play were established. Students reasonable responses to inquiry questions were accepted and modify where necessary.

The explanation phase was started with connection of prior knowledge to new discoveries. New understanding and concepts were communicated. Informal and formal languages (terms, definition and theorems) were connected. Questions for clarification and justification of concepts and ideas were asked by the instructor and all reasonable responses with modifications/corrections were accepted.

At the elaboration stage, application of new concepts gained were applied to knew or similar situations. Concepts being explored were extended and explained by students. Students were encouraged to use new terms and definitions previously acquired. In all cases, observations, explanations and solutions were recorded for further interrogation and usage.

Finally, concepts learnt were assessed. The assessment included self, peer and teacher assessment. Understanding of new concepts were demonstrated by observation or open-ended responses. The evaluation was done phase-wise. At the end of each phase of the model implementation, the students were assessed. The confirmation test tested two areas: method used in solving the problem and application of concept.

Chapter 4

RESULTS/FINDINGS

4.1 Overview

This chapter presents the data obtained from the study, the analysis of the data and the results and findings from the study. Each research question was analysed and discussed in order to identify the factors affecting teaching and learning of PDEs, to investigate the impact on the use of the IO-IA in the teaching and learning of PDEs and to identify the challenges associated with the use of the IO-IA in the undergraduate level.

4.2 Demographic Characteristics of Respondents

Two level 400 groups in two consecutive academic years (2016/2017 and 2017/2018) were used as a sample for the study. Table 4.1 indicates the number of students per academic year.

Table 4.1 Sample for the Study

	Male		**Female**		**Total**	
Academic Year	**N**	%	**N**	%	**T**	%
2016/2017	191	87.61	27	12.39	218	54.5
2017/2018	164	90.11	18	9.89	182	45.5
Total	355	88.75	45	11.25	400	100.0

As indicated in Table 4.1, a total of 400 students comprising of 218 students in the 2016/2017 academic year and 182 students in the 2017/2018 academic year. Out of the 400 students, 355 students (88.75%) were males while 45 students (11.25%) were females. In the 2016/2017 academic year, out of a total of 218 students, 191 students

(87.61%) were males while 27 students (12.39%) were females. In the 2017/2018 academic year, out of a total of 182 students, 164 students (90.11%) were males while 18 students (9.89%) were females.

Even though Gunaseelan and Paxhanivelu (2016), Belhu (2017) and Kiwanuka, et al (2015) considered gender as a factor affecting teaching and learning of mathematics, the results from the study revealed otherwise. When the students were asked about the effect of gender on teaching and learning, some of the responses were as follows:

Respondent 1 (Female)

Interviewer:*Does your gender have influence on your mathematics learning ability?*

Respondent: *No sir, gender is not the issue at all. After all, somehow we the ladies have been performing better than some of our male counterparts.*

Respondent 2 (Male)

Interviewer: *Does your gender have influence on your mathematics learning ability?*

Respondent: *No, sir. Some of the female students are even better academically than we the males. So, gender does not even come in this situation.*

4.3 Factors Affecting Teaching and Learning of PDEs

Research Question 1: What factors affect teaching and learning of partial differential equations (PDEs) and the challenges associated with these factors in the undergraduate level?

Research question one sought to find the factors that affect teaching and learning of PDEs in the undergraduate level. Observations, interactions with the participants and the interviews conducted after the treatment was used to identify the factors affecting teaching and learning of PDEs. The diagnostic test conducted also helped the researcher to identify the factors that affect teaching and learning of PDEs. The factors identified were categorized into four: pedagogical factors, conceptual factors, technological factors and modelling factors. The pedagogical factors include instructional strategies and methods, motivation and socio-economic factors. The conceptual factors include the students' ability to recall some pre-requisite knowledge and computational ability. The modelling factors include comprehension, computational ability and computer applications to mathematics and the technological factors include the availability of ICT tools and computer application for mathematics. Sample of the interview and their responses on the pedagogical factors are as follows:

Interviewer: *Are you affected by the instructional strategies and methods I employed during your lessons on PDEs?*

Respondent: *Yes. The way you teachers teach have affected our way of understanding what you are teaching us a lot. The way you go about your teaching and the kind of strategies you use in your presentation has a lot to do with our understanding of the topic. Sometimes, some of the things are too abstract and we can't get the head and tail of it.*

Interviewer: *Comparing the traditional teaching method you were going through to this instructional approach, which of them will you prefer and why?*

Respondent: *You taught us ODEs, if the way you are teaching now was the way you taught us ODEs, I believe I could have scored 'A' straightaway. This method even though is slow is far better than the traditional method. Here we get to know the application of what you teach us and this even motivates to get ready to learn. The way the teaching was done and the activities we went through to get the equations out of the real-life situation gave me a clear understanding of partial differential equations. Most especially, the explanation that followed after we struggled to come with the equations gave me a clear picture of what is happening. I wish all lessons will be taught through this approach.*

Interviewer: *What are some of the challenges you faced if the instructional strategy used by the teacher is not good enough?*

Respondent: *We find it difficult to understand what the lecturer is teaching and so it demotivates us. Because of that, we most of the time perform poorly during quizzes and exams and when it comes to application of concepts in real-life situations, it gets worse.*

Interviewer: *Does motivation affects teaching and learning of PDEs?*

Respondent: *We were well motivated when we realized the importance of some of these partial differential equations in our day to day activities. I never realized that most of the things I do every day has partial differential equations embedded in it. It highly motivated me during the course of the study and have really influence my studies. The videos and the simulations even motivated us even more. At least we got to know that we were not studying these complex and difficult courses for nothing. We realized its application is within us. Our mobile phones, how the FM stations determine how long their radio stations will cover and so on.*

Interviewer: *How were you motivated?*

Respondent: *It began with when you began showing us the importance and application of the PDEs in real-life application. And the videos you show us about all these applications really served as a stimulation and for me in particular, it emotionally affected me. I must say that, your cordial relationship before, during and after lectures also motivated us a lot. In some cases, in some of lessons, the presence of the lecturer along demotivates you.*

Interviewer: *Can you think of any other factor that affects your understanding of lessons you take in the lecture hall?*

Respondents: *Hmm, sir, one of the things nobody seems to look at is how we come to school.*

Interviewer: *What do you mean by that?*

Respondent: *Sir, sometimes we come to school with empty stomach. Our parents do not* *have enough money for us as 'chop' money. So, we sometimes loose concentration in the lecture hall and this affects our studies. As others are thinking of what you are teaching, we are thinking how we are going to eat. I hope the university is like secondary school where we go for dining. This economic hardship is killing us.*

Interviewer: *So, you mean financial constraints is also a factor?*
Respondent: *Yes sir.*

Sample of the interview and their responses on the conceptual factors are as follows:

Interviewer: *Does your ability to recall and apply pre-requisite lessons like calculus and ODEs affect the way you understand the PDEs lessons?*
Respondent: *I believe one of our problems is that, we have forgotten almost all that we learnt in the previous semesters. We've forgotten most of the things [courses] we learnt in ODEs, calculus 1 [differential calculus] and calculus II [integral calculus]. We have also forgotten most of the things [courses] we learnt in algebra, trigonometry and introductory analysis.*

Interviewer: *What is or are the causes of your forgetfulness?*
Respondent: *All this is because we learnt them long time ago. We also left school for internship for 4 months and we did not have time to go back to our books. I think if we finish all the lectures before we go for our internship program [internship is moved to second semesters instead of first semester in level 400], we will be able to remember most of the things we have forgotten. I think it will be a good idea if PDEs follow ODEs in the next semester.*

Interviewer: *Are your lessons in PDEs affected by your computational ability? By computational ability I meant your ability to formulate mathematical formulas and algebraically manipulate the equations or functions in order to be able to solve them.*

***Respondent**: We have challenges in re-writing the functions in the way that will make it easy to solve. We sometimes do not know how to manipulate the functions or re-write them in equivalent form for their integration to be possible. That is why I most of the time gets these integrations wrong. For example, the quasilinear differential equation, was very difficult to solve because we could not realize that we can manipulate it algebraically to remove some of the variables to make it simple like that.*

Sample of the interview and their responses on the Technological factors are as follows:

***Interviewer**: Is availability of ICT tools a factor that affects you during teaching and learning of PDEs?*
***Respondent**: It is quite obvious that, the use of computers makes the lesson easy to understand. The graphs and the animations make the concepts clear. But because of the inadequate number of computers, we cannot even practice some of the things we know already and we are unable to practice what you are teaching now. This makes it difficult when it comes to the use of computers to investigate solutions of functions. I hope there is enough computers for us to practice. It makes some of us who do not have laptops always sit and watch those with laptops. Also, programmable calculator you were demonstrating is also not available. If the spoiled one you talk about, I mean the TI 83, are even working, it will have helped some of us. This is actually not helping us at all.*

Some of the students when asked about their view on the integration of computer tools in teaching and learning stated:

***Interviewer**: What about computer applications to mathematics? Do you consider that also as a factor affecting teaching and learning of PDEs?*
***Respondent**: The computer simulations gave me clear understanding that the equations I always see as a bunch of functions have real-life applications and are applied in our daily lives and are used in these emerging technologies. In fact, it helped me to*

understand the concept very well. My only problem is that even though I took a course on computer tools for mathematics (Octave, GeoGebra and Microsoft Excel), I have forgotten how to apply or use these software to simulate or draw graphs of functions and hence my inability to apply them in this situation. The simulation videos and the videos about the application of these concepts also motivated us. Sometimes, when you don't know the use or importance of what you are learning, it becomes annoying but I think the simulations and the videos served their purposes.

Sample of the interview and their responses on the modelling factors are as follows:

Interviewer*: Is comprehension a factor that affects your ability to understand what you are being taught in PDEs? By comprehension, I meant your ability to read and understand the problem or tasks given to you.*

Respondent*: Hmmmm sir, that is one of my biggest problem. Sometimes, by the time I finish reading the whole question, I have already forgotten what I read initially. Sir, that is why we most of the time run away from application questions. But with this approach you took us through I have realized how I have to break the statements into pieces and take them one by one.*

Interviewer*: So, it means this semester, you are prepared for application question?*

Respondent*: Yes, this time I will. I have realized it is not because I am not good at English language but because I don't take my time to read through. This time, I will. In fact, the assignment you gave us, I answered everything by myself before we even met as a group.*

The results from the interview after coding and re-coding is presented in Table 4.2. In all, eight factors were identified and are presented in Table (4.2) and their associated challenges are presented in Table 4.3. The

Table 4.2 Factors Affecting Teaching and Learning of PDEs

	Responses			
	IF		NIF	
Are you affected by the following factors?	**N**	%	**N**	%
1. Instructional Strategies and methods	26	86.7	4	13.3
2. Students' ability to recall and apply pre-requisite knowledge	24	80.0	6	20.0
3. Computational ability	21	70.0	9	30.0
4. Availability of I.C.T. tools	20	66.7	10	33.3
5. Comprehension	19	63.3	11	36.7
6. Motivation	16	53.3	14	46.7
7. Computer applications for mathematics	15	50.0	15	50.0
8. Social-Economic factors (financial constraints)	12	40.0	18	60.0

Keys: ***IF*** - In favour, ***NIF*** - Not in favour

Table 4.3 Challenges Associated with Factors Affecting Teaching and Learning of PDEs

Factor No.	Challenge(s) Associated with Factor
1	a. Lack of conceptual understanding of PDEs. b. Inability of students to apply concept learnt. c. Short-term memory on the side of students d. Poor acquisition of knowledge. d. Poor students' performance on PDEs. f. De-motivation on the side of students.
2	Difficulties in solving partial differential equations.
3	a. Lack of problem-solving skills b. Ineffective self-instructions
4	a. Inadequate ICT tools b. Poor state of available ICT tools c. Lack of hands-on activity d. Difficulties in using the ICT tools
5	a. Lack of understanding of the problem b. Student's inability to interpret and analyse the problem. c. Lack of self-efficacy.
6	a. Low interest in mathematics. b. Discouragement. c. Poor performance d. Low self-efficacy in Mathematics.
7	a. Inadequate classroom practices. b. Graphical representation of solutions.
8	a. Lack of concentration b. De-motivation c. poor performance

According to the responses from the interview, instructional strategies and methods was the most influential factor affecting teaching and learning of PDEs and it followed by students' ability to apply pre-requisite knowledge to new subject matter. Other factors include computational ability, Availability of I. C. T. tools, comprehension, motivation, students' ability to use computer tools for mathematics and socio-economic factors (financial constraints) on the part of the students.

The instructional strategies and methods which is a pedagogical factor includes instructional approaches and methods used during lesson delivery as well as teaching/ learning materials (TLMs). From the results in Table 4.2, instructional strategies and methods is the most influential factor among the 8 factors identified. Twenty six out of thirty respondents (86.7%) considered instructional strategies and methods as a factor affecting teaching and learning of partial differential equations.

The second most influential factor according to the results in Table 4.2 is students' inability to apply pre-requisite knowledge due to forgetfulness. This factor falls under the conceptual factor. Out of the 30 respondents, 24 (80%) considered this as one of the factors that influences teaching and learning of PDEs. The pre-requisite knowledge includes students' previous knowledge on subject areas like ordinary differential equations, differential and integral calculus, linear algebra, vectors and mechanics and so on.

Computational ability falls under conceptual factors. Out of the 30 respondents, 21 (70%) of the respondents considered computational ability as a contributing factor to teaching and learning of PDEs. It is the third most contributing factor according to the results in learning PDEs. It refers to the students ability to apply some known concepts in other areas like calculus, ordinary differential equations, linear algebra and the likes to solve the given problem. The availability of I.C.T. tools is categorized as a technological factor. From the results, 20 respondents (66.7%) of the respondents considered the availability of I.C.T. tools as one of the factors affecting teaching and learning of PDEs.

Comprehension was categorized as modelling factor. This is because, students' ability to understand the task is the key to their ability to complete the task successfully. Comprehension in this study refers to one's ability to read and understand the problem presented. Nineteen respondents (63.3%) of the respondents considered comprehension as one of the factors affecting teaching and learning of PDEs. Motivation was categorized as a pedagogical factor because it forms part of a teacher's introduction stage of a lesson. A good motivation prepares the students to open their minds for learning to take place. Out of the 30 respondents, 16 (53.3%) of the respondents considered motivation as one of the factors affecting teaching and learning of PDEs.

Computer application to mathematics was categorized as a technological factor. Fifteen respondents out the 30 (50%) of the respondents considered the use of computer application for mathematics as one of the contributing factors. The socio-economic factors were classified under pedagogical factors. The socio-economic factors in this study refers to the financial constraints on students and their parents alike. In this study, only 40% of the respondents considered socio-economic issue as a factor for learning PDEs.

The challenges associated with instructional strategies and methods are, lack of conceptual understanding of PDEs on the part of the students, the inability of students to apply concept learnt to other areas, short-term memory on the side of students, poor knowledge acquisition, poor students' performance on PDEs also de-motivates students to learn. Students inability to recall and apply pre-requisite knowledge posed a major challenge to them when they were required to apply them in solving the differential equations that emerged out of the models as well as other partial differential equations which were assigned to them. Most of the students were not able to integrate some of the functions.

Low interest in mathematics, discouragement, poor performance, and low self-efficacy in mathematics were some of the challenges associated with lack of motivation in students. Lack of concentration on the part of the students, de-motivation and poor performance are some of the challenges associated with mathematics students who have financial constraints (socio-economic constraints). The challenges students face due to their computational ability is lack of problem-solving skills and ineffective self-instructions. Inadequate ICT tools, poor state of available ICT tools, lack of hands-on activity and difficulties in using the ICT tools were some of the challenges associated with availability of ICT tools. Lack of relevant computer applications to mathematics possess some challenges such as inadequate classroom practices and graphical representation of solutions. The challenges associated with comprehension is lack of understanding of the problem which hinders the students ability to interpret and analyse the problem and lack of self-efficacy.

In summary, the study revealed the following as the factors that affect teaching and learning of PDEs.

- Instructional strategies and methods
- Students' ability to recall and apply pre-requisite knowledge
- Students' computational ability
- Availability of ICT tools
- Comprehension
- Motivation
- Computer applications for mathematics and
- Socio-economic factors (financial constraints)

4.4 Impact of the use of IO-IA on Teaching and Learning of PDEs

Research Question 2: How does the use of inquiry-oriented instructional approach (IO-IA) impact on teaching and learning of PDEs in the undergraduate level?

Research question 2 sought to find out the impact of the use of IO-IA on teaching and learning of PDEs. In order to answer this research question, testing and observation of students' performance in specific task in their learning process during the inquiry process were made. Students performance was measured on both the methods of solving PDEs and the conceptual understanding of the PDEs. The sum of the scores obtained from these two aspects forms their total performance. The performance of the control group which was taught using the traditional or normal classroom teaching and the experimental group which was taught using the IO-IA was compared using an independent sample t-test. The independent sample t-test tested the following hypothesis:

$$H_o : \mu_{c_i} = \mu_{e_i} \text{ for } i = 1,2,3$$

$$H_a : \mu_{c_i} \neq \mu_{e_i} \text{ for } i = 1,2,3$$

H_0: Null hypothesis

H_a: Alternate hypothesis

μ_{c_i}: The population mean of the control group for the ith phase.

μ_{e_i}: The population mean of the experimental group for the ith phase.

The test was carried out with 95% confidence interval. That is an alpha-value of 0.05. In order for this to be valid, the following assumptions underlying independent sample t-test were tested:

1. The dependent variable should be measured on continuous scale.
2. The independent variables should consist of two categorical groups.
3. The observations must be independent.
4. There should be no significant outliers.
5. The dependent variables should be normally distributed.
6. The variance should be homogeneous.

4.4.1 Test for Assumptions Underlying the Independent Sample T-Test

Preamble

In order to effectively use the results from the analysis of the independent sample t-test, the data must meet the assumptions underlying the independent sample t-test for the equality of means. A total of 218 students with 109 students in each of the two groups (control and experimental group) were used for the study in the 2016/2017 academic year while a total of 182 students with 91 students in each of the groups were used for the study in 2017/2018 academic year. Three aspects were tested after the treatment in each of the academic years.: the method the students used in solving the problem, the conceptual understanding of the concepts of PDEs and the overall performance of the students. The scores obtained from their methods were termed the ”method scores”, that obtained from the concepts were termed ”concept scores” and the scores assessing their total performance which was the sum of the method and concept scores were termed ”total scores”. Note that to test the significance of an outlier, the mean and standard deviation method was used.

For all the three phases in both academic years (2016/2017 and 2017/2018), the following three assumptions were met:

- The dependent variable was measured on continuous scale, (the data were the scores obtained from test scores).
- The independent variables should consist of two categorical groups. In each academic year two independent groups (control and experimental) were used for the study. The two groups were independent because no student belongs to more than one group. Steps were also taken to ensure no student swap groups or join the other group.
- The observations were independent. Another instructor was tasked to handle the control group while the researcher handled the experimental groups during the

treatment. The observations were also done by the two instructors and were independently done.

Test for Assumptions on Phase 1 Test Results

During the first phase of the treatment, group 1 was made the control group and group 2 was made the experimental group in both academic years (2016/2017 and 2017/2018). The following assumptions were verified:

- **Outliers**: Box plots were plotted for the test scores and the results are shown in Figures 4.1 to 4.6.

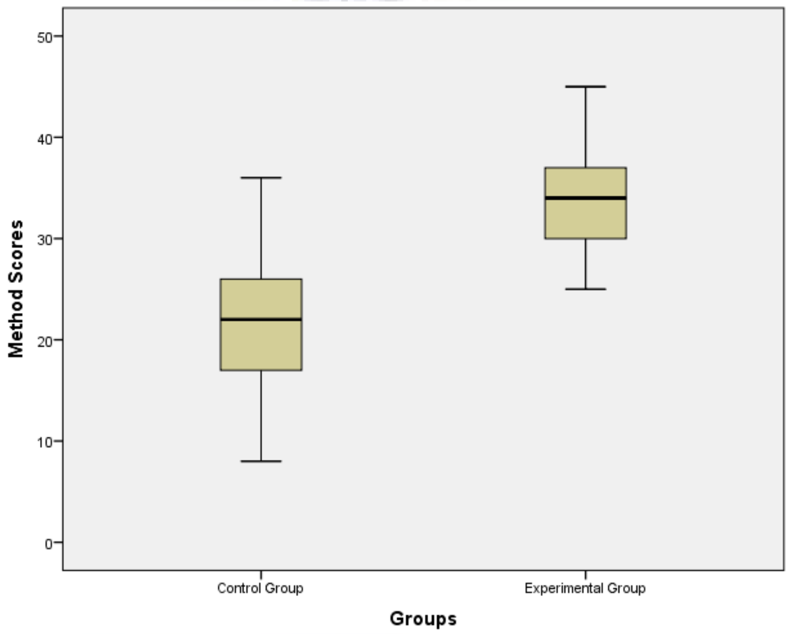

Figure 4.1: Box Plot for Method Scores for Phase 1 (2016/2017)

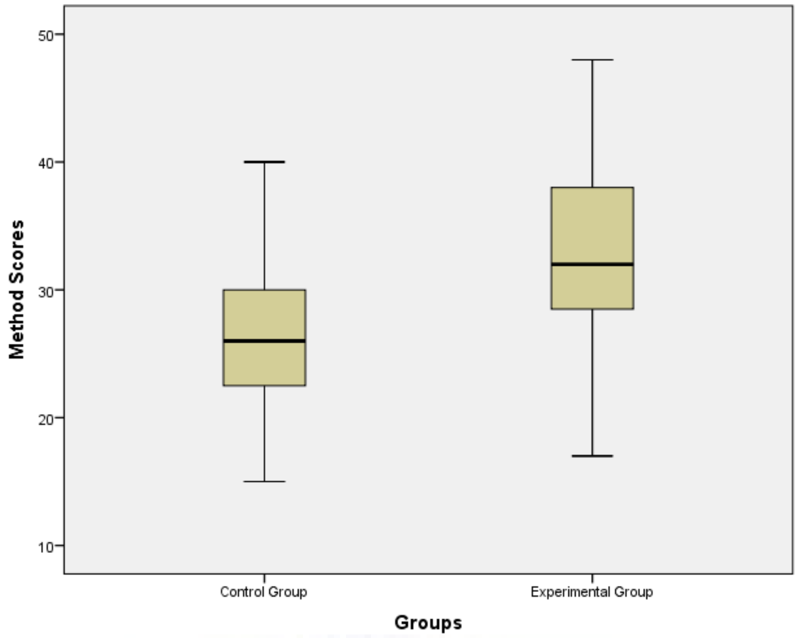

Figure 4.2: Box Plot for Method Scores for Phase 1 (2017/2018)

From the box plots for method scores for phase 1 in both academic years, there were no significant outliers. This indicates that the method scores for both academic years met the assumption for outliers as shown in Figures 4.1 and 4.2. Similarly to the method scores, there were no significant outliers for the concept scores for the 2016/2017 academic year group as shown in Figure 4.3. However, there was one outlier for the concept scores of the experimental group for the 2017/2018 academic year group as indicated in Figure 4.4. But the z-score of the outlier is -1.83 indicating that it is 1.8 standard deviations below the mean. This means it is within 3 standard deviations from the mean and so it is not a significant outlier according to the criteria defined by Paul et al. (2015). In the case of the total scores, which is the sum of the method scores and concept scores, there was one outlier for the control group for the 2016/2017 academic year group with a z-score of 2.7. This means that the score is within 3 standard deviations from the mean and hence it is not a significant outlier. For the 2017/2018 year group, there was also one outlier for the experimental group (see Figure 4.6) with a z-score of -3.0 which is 3 standard deviations from the mean so the outlier was not significant (Paul et al., 2015). So, there was no significant outliers in the scores obtained for both academic years.

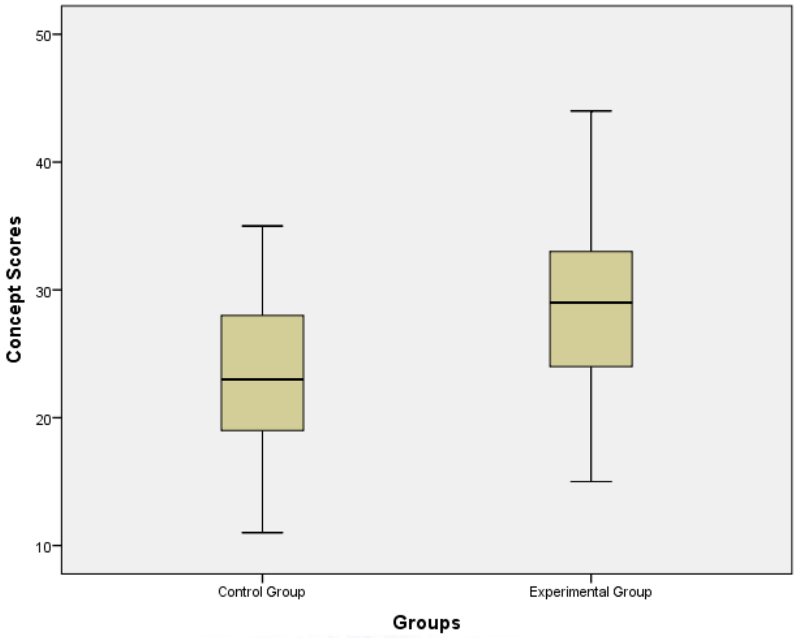

Figure 4.3: Box Plot for Concept Scores for Phase 1 (2016/2017)

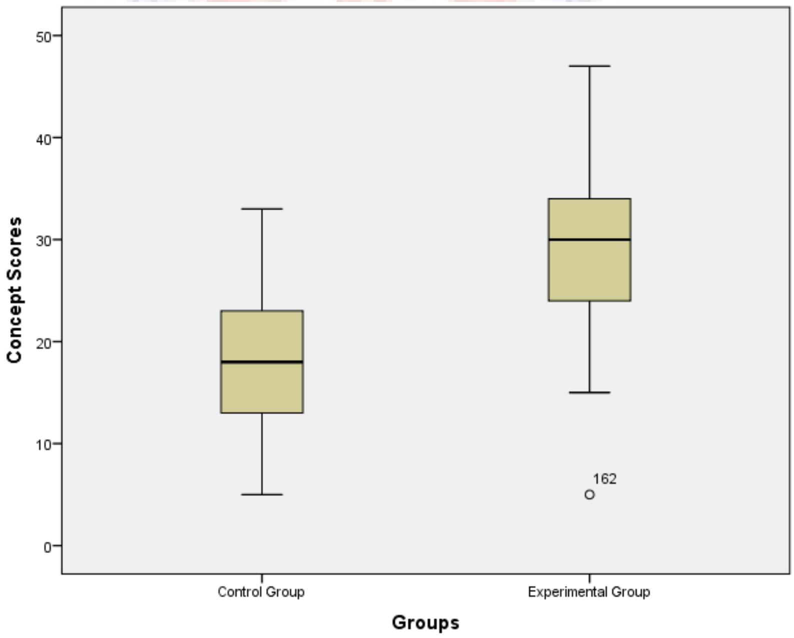

Figure 4.4: Box Plot for Concept Scores for Phase 1 (2017/2018)

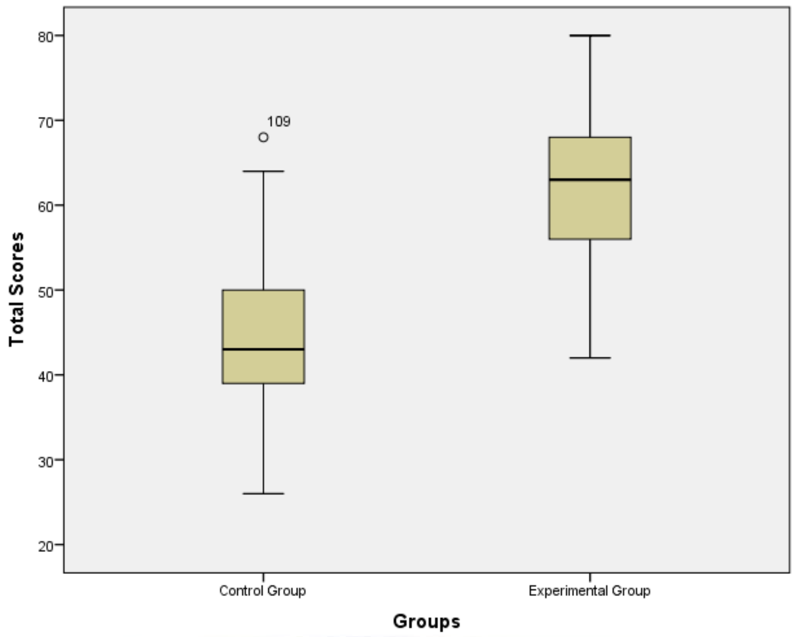

Figure 4.5: Box Plot for Total Scores for Phase 1 (2016/2017)

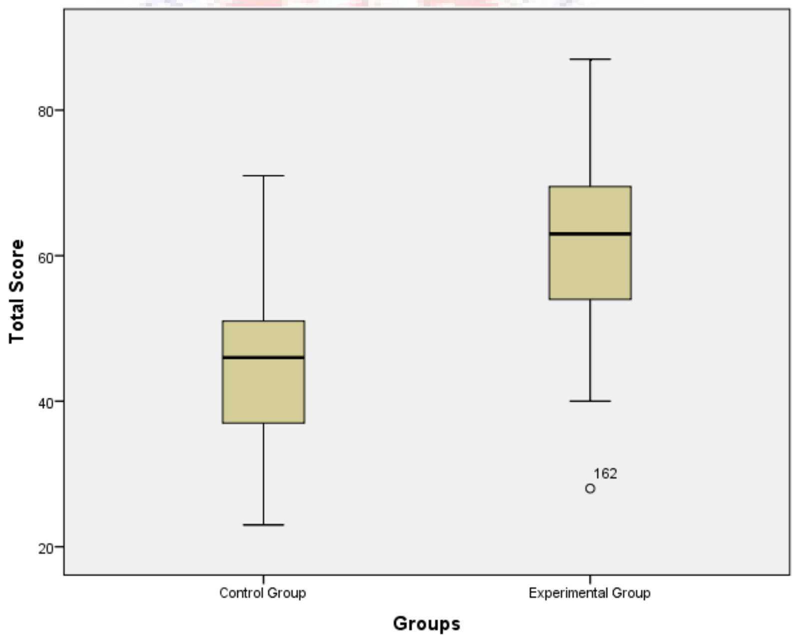

Figure 4.6: Box Plot for Total Scores for Phase 1 (2017/2018)

- **Normality of the Distribution**: The next assumption was the need for the dependent variables to be normally distributed. In this case, the test for normality using the Shapiro-Wilk test as indicated in Tables 4.4 and 4.5 were followed by the normal Q-Q plots as shown in Figures 4.7 and 4.8. From the results in Table 4.4 for 2016/2017 academic year, the Shapiro-Wilk test indicates that all the groups have significant values to be greater than 0.05 for the method scores, concept scores and total score. These indicate that the data from both the control and the experimental groups for all the scores are not significantly different from normality.

Table 4.4 Test for Normality of Phase 1 Test (2016/2017)

		Shapiro-Wilk Test		
Scores	**Groups**	**Statistics**	**df**	**Sig.**
Method Scores	Control Group	0.983	109	0.172
	Experimental Group	0.978	109	0.072
Concept Scores	Control Group	0.982	109	0.143
	Experimental Group	0.985	109	0.254
Total Scores	Control Group	0.982	109	0.144
	Experimental Group	0.990	109	0.612

Table 4.5 Test for Normality of Phase 1 Test Results (2017/2018)

		Shapiro-Wilk Test		
Scores	Groups	Statistics	df	Sig.
Method Scores	Control Group	0.980	91	0.177
	Experimental Group	0.991	91	0.801
Concept Scores	Control Group	0.977	91	0.105
	Experimental Group	0.979	91	0.153
Total Scores	Control Group	0.984	91	0.333
	Experimental Group	0.990	91	0.745

Similarly, the significant values for all the groups in method, concept and total scores in the 2017/2018 academic year group are greater than 0.05 (see Table 4.5). This also indicates that, all the scores for the control and experimental groups in the phase 1 are not significantly different from normality.

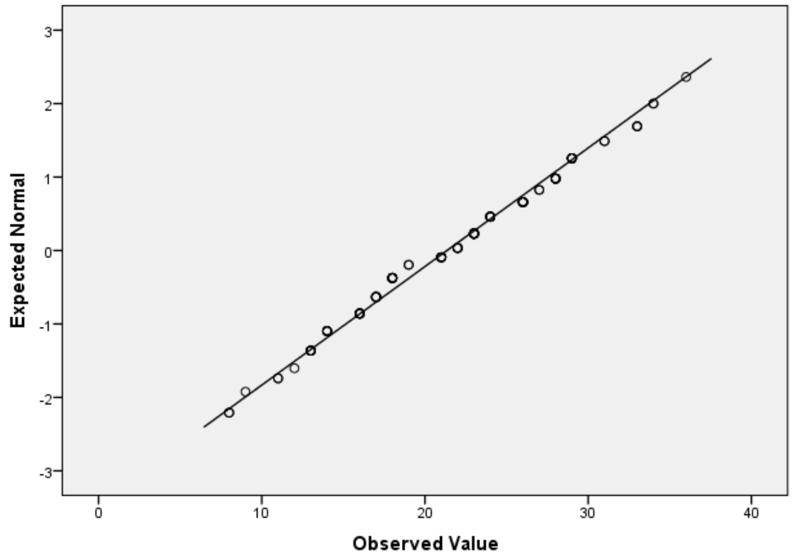

Figure 4.7: Normal Q-Q Plot of Method Scores for Control Group_Phase 1 (2016/2017)

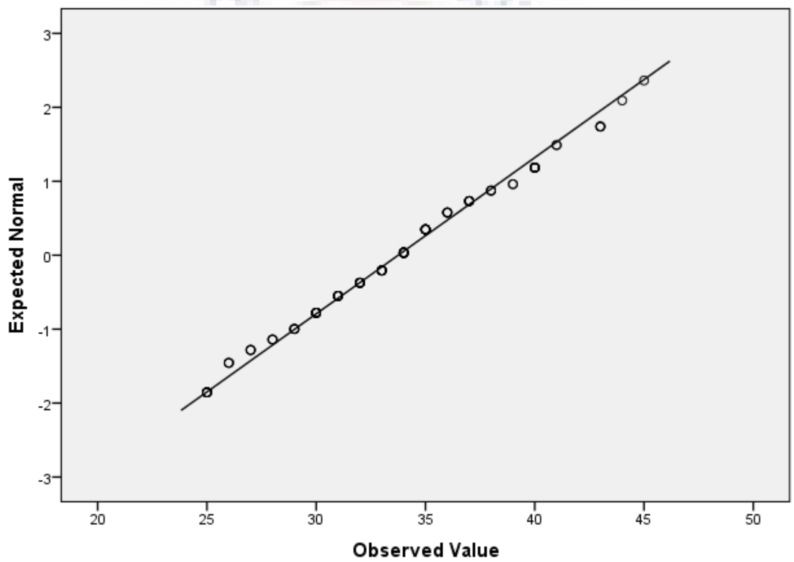

Figure 4.8: Normal Q-Q Plot of Method Scores for Experimental Group Phase 1 (2016/2017)

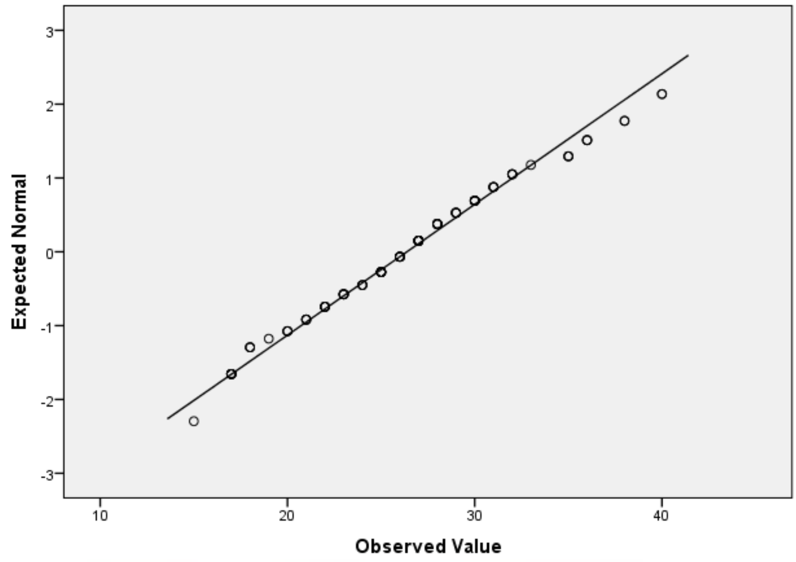

Figure 4.9: Normal Q-Q Plot of Method Scores for Control Group Phase 1 (2017/2018)

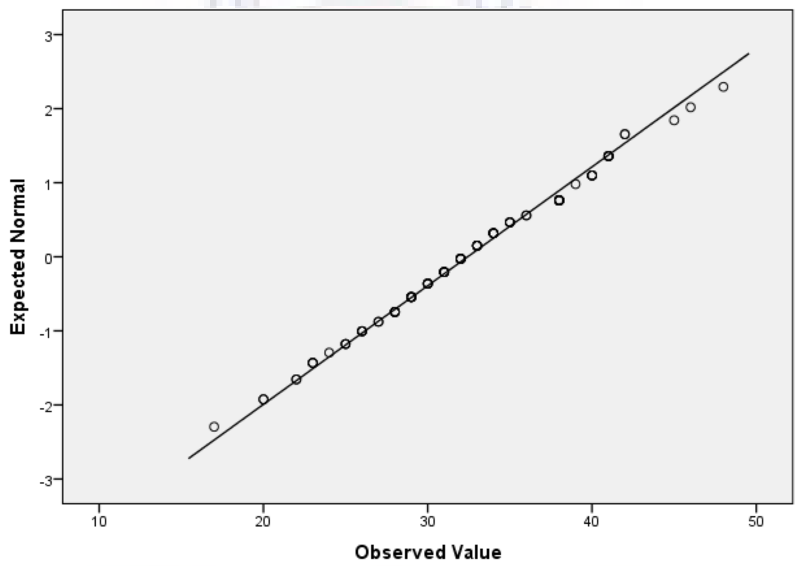

Figure 4.10: Normal Q-Q Plot of Method Scores for Experimental Group Phase 1 (2017/2018)

Figures 4.7 to 4.10 shows the normal Q-Q plots for the method scores for the control and experimental group for the two academic years. The plots indicate that the results obtained for method scores for both groups are close to normality. This confirms the results obtained from the Shapiro-Wilk test in Tables 4.4 and 4.5. From Table 4.4 the Shapiro-Wilk test gave a significant value of 0.172 and 0.072 for the control and experimental group respectively for the method scores in the 2016/2017 academic year. In the 2017/2018 academic year, the Shapiro-Wilk test gave a significant value of 0.419 and

0.089 for the control and experimental group respectively. It can therefore, be concluded that the scores obtained for the method scores in the 2016/2017 and 2017/2018 academic years are not significantly different from the normal distribution.

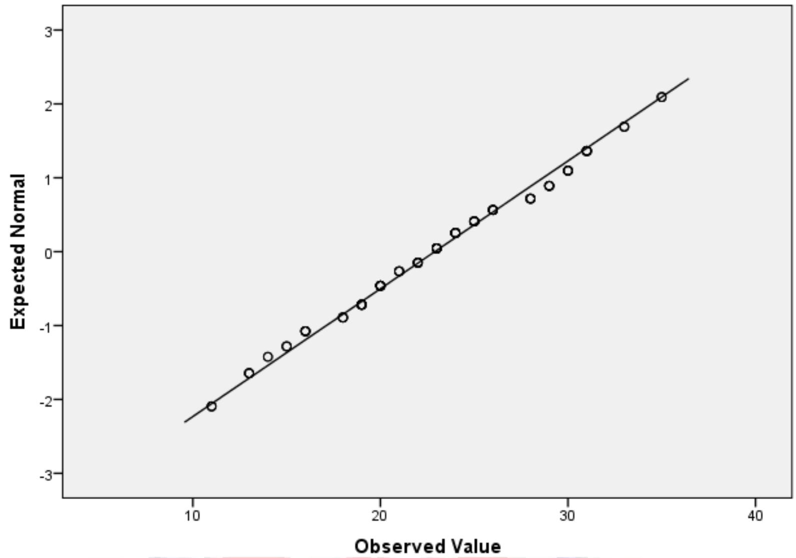

Figure 4.11: Normal Q-Q Plot of Concept Scores for Control Group_Phase 1 (2016/2017)

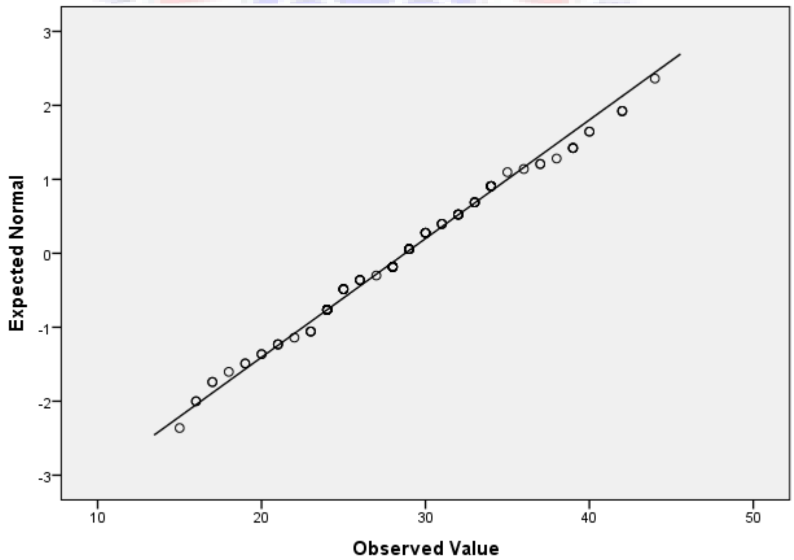

Figure 4.12: Normal Q-Q Plot of Concept Scores for Experimental Group_Phase 1 (2016/2017)

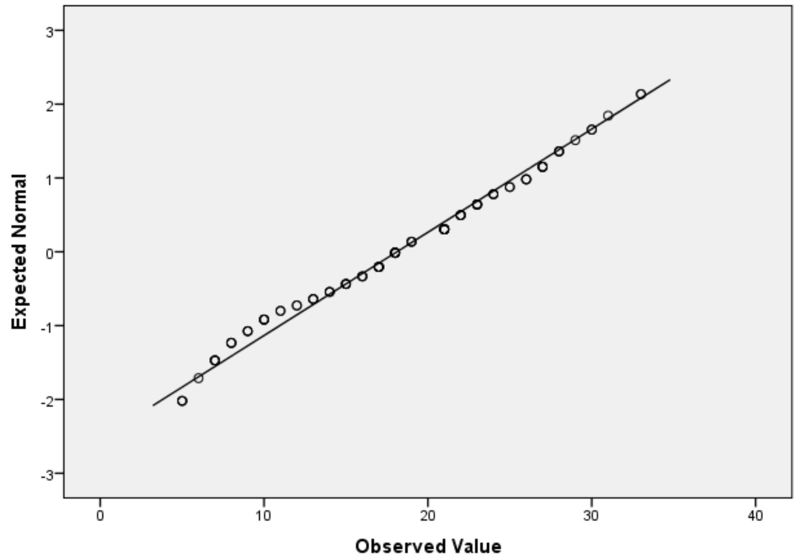

Figure 4.13: Normal Q-Q Plot of Concept Scores for Control Group Phase 1 (2017/2018)

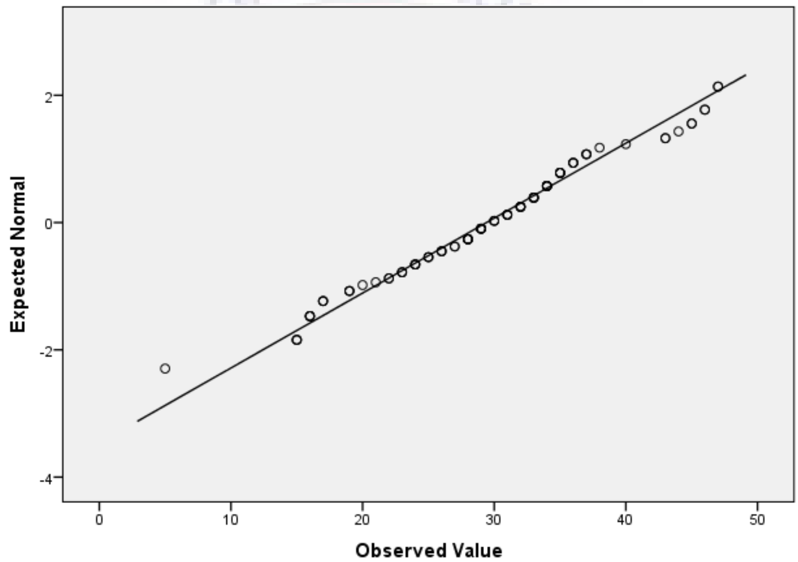

Figure 4.14: Normal Q-Q Plots of Concept Scores for the Experimental Group Phase 1 (2017/2018)

Figures 4.13 to 4.14 show the normal Q-Q plots for the scores obtained by the two groups on concept for the two academic years. From the plots, it could be seen that the scores are close to the straight line indicating how the data sets are close to normality. Again, the plots confirm the results obtained from the Shapiro-Wilk test in Tables 4.4 and 4.5. From Table 4.4, the significant value obtained for the control and experimental group on concept scores are 0.143 and 0.254 respectively. Since these values are all greater than 0.05 it can therefore, be concluded that the results obtained for the two groups on concept scores are not significantly different from normality.

The normal Q-Q plots for the sum of the method and concept scores (total scores) for 2016/2017 and 2017/2018 academic years are shown in Figures 4.15 to 4.18.

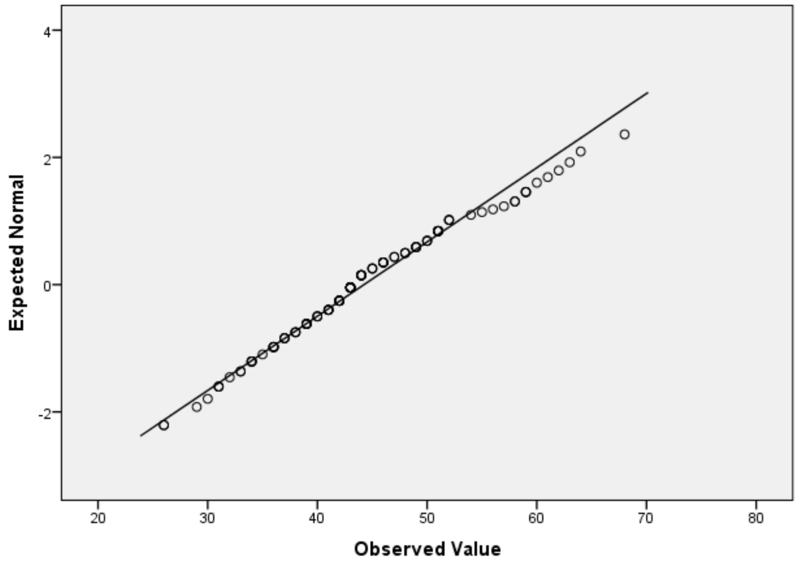

Figure 4.15: Normal Q-Q Plot of Total Scores for Control Group_Phase 1 (2016/2017)

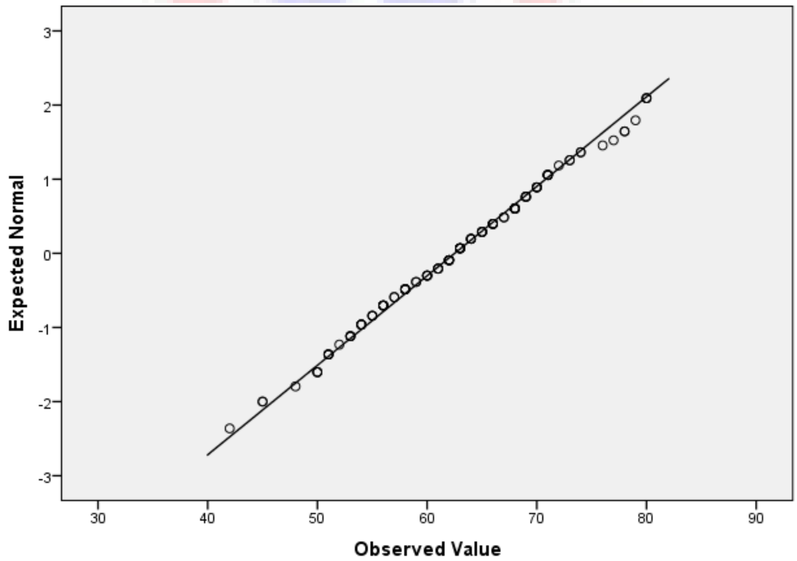

Figure 4.16: Normal Q-Q Plot of Total Scores for Experimental Group_Phase 1 (2016/2017)

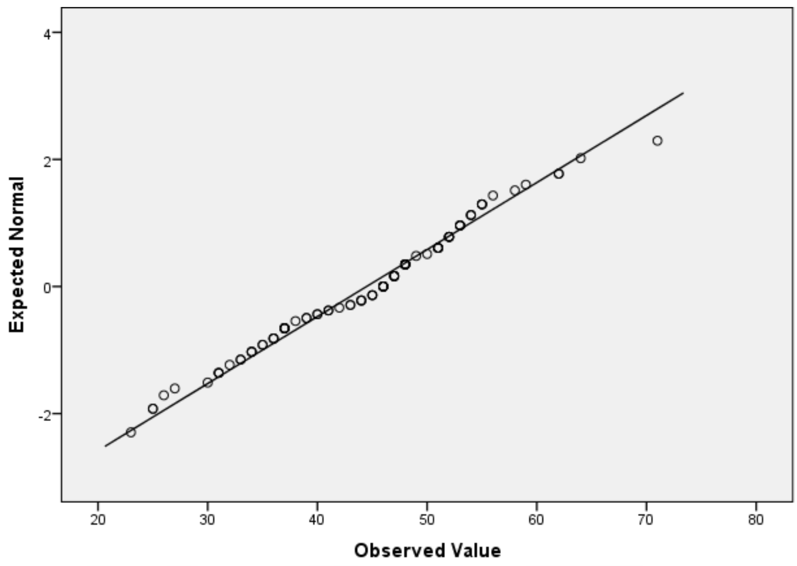

Figure 4.17: Normal Q-Q Plot of Total Scores for Control Group Phase 1 (2017/2018)

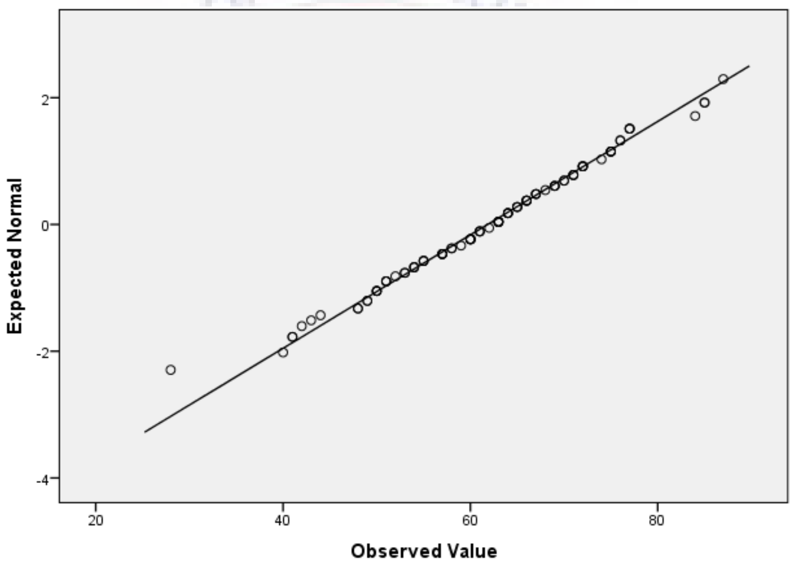

Figure 4.18: Normal Q-Q Plot for Total Scores for Experimental Group Phase 1 (2017/2018)

Again, the plots indicate that the total scores obtained for the control and experimental groups in the 2016/2017 and 2017/2018 academic years are significantly close to normality. This can be seen from how close the data points are to the straight line. This also confirms the results obtained from the Shapiro-Wilk test for the 2016/2017 and 2017/2018 academic years (see Tables 4.4 and 4.5). From Table 4.4, the significant values for the control and experimental groups for the total scores for 2016/2017 academic year are 0.144 and 0.612 respectively and that of the 2017/2018 academic year are 0.33 and 0.745

respectively. Again, it can be concluded that the sum of the scores obtained from the test in both academic years for both the control group and experimental group met the normality assumptions for the independent sample t-test.

- **Homogeneity of Variances**: The last but not the least is the need for homogeneity of variances. To test for the homogeneity of variances, the Levene's test for differences in variances was conducted and the results is displayed in Table 4.6 and (4.7).

Table 4.6 Levene's Test for Equality of Variances (Phase 1, 2016/2017)

Scores	**F**	**Sig.**
Method Scores	10.762	0.001
Concept Scores	0.272	0.602
Total Scores	0.000	0.998

From Table 4.6, the method scores have a significant value of 0.001 which is less than the α-value of 0.05. This means that the variances for the method scores for phase 1 in the 2016/2017 academic year are significantly different and so did not meet the homogeneity of variance assumption for independent sample t-test. However, the significant value for the concept scores is 0.602 which is greater than the α-value of 0.05. This shows that the variances for concept scores for the control and experimental groups are not significantly different. So, the concept scores met the homogeneity variances assumption for independent sample t-test. When the method scores and the concept scores were put together, the significant value was found to be 0.998 which is greater than the α-value of 0.05. Hence the variances for the control and experimental groups are not significantly different and so the total scores met the homogeneity of variance assumption.

Table 4.7 Levene's Test for Equality of Variance (Phase 1, 2017/2018)

Scores	F	Sig.
Method Scores	1.090	0.298
Concept Scores	1.289	0.258
Total Scores	1.672	0.198

For the phase 1 of the 2017/2018 academic year, the results in Table 4.7 shows that the significant value of the method, concept and total scores are 0.298, 0.258 and 0.198 respectively. It could be seen that all these values are greater than 0.05, the α-value. Hence the variances for the method, concept and total scores are not significantly different. The scores therefore met the assumption for the homogeneity of variances for independent sample t-test.

Test for Assumptions on Phase 2 Test Results

- **Outliers**: The box plots in Figures 4.19 and 4.20 respectively are the method scores for phase 2 of the treatment for the two academic years (206/2017 and 2017/2018). It could be seen from Figures 4.19 and (4.20) that there are no outliers in the method scores for both groups.

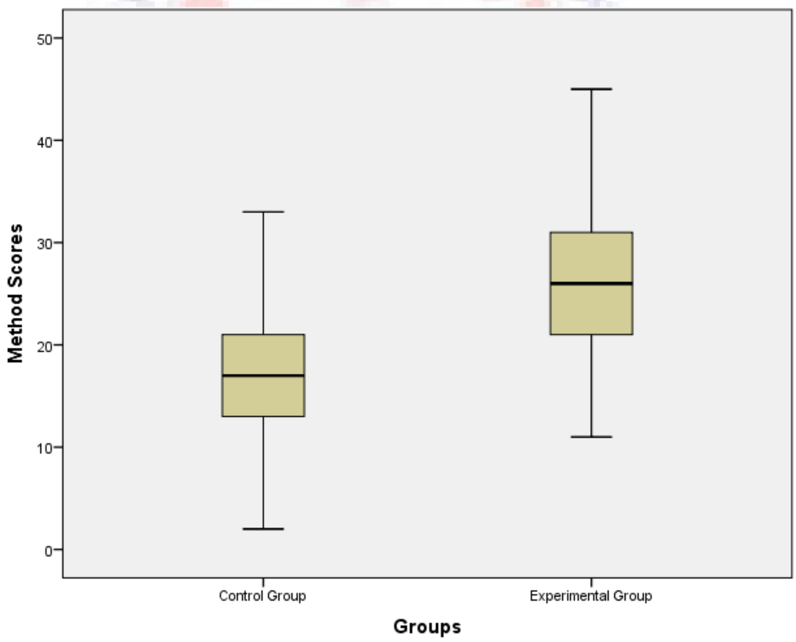

Figure 4.19: Box Plot for Method Scores for Phase 2 (2016/2017)

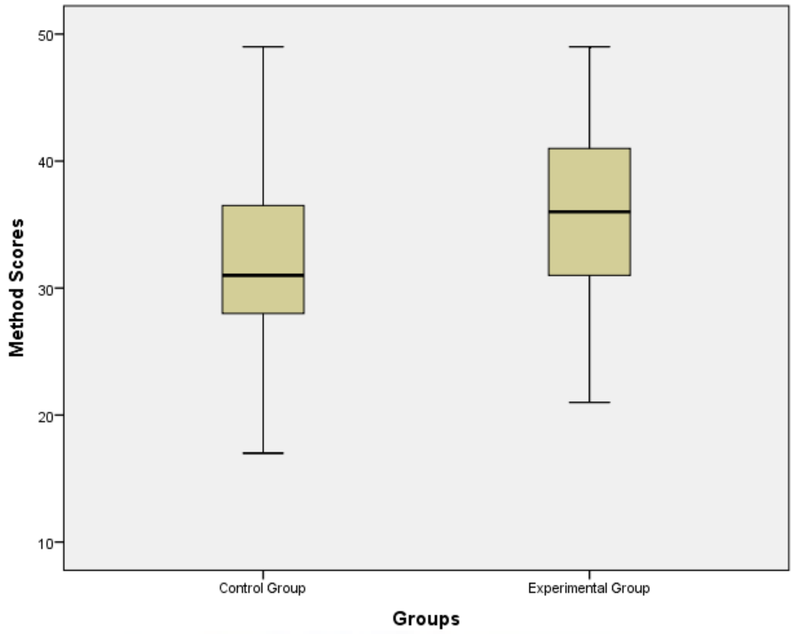

Figure 4.20: Box Plot for Method Scores for Phase 2 (2017/2018)

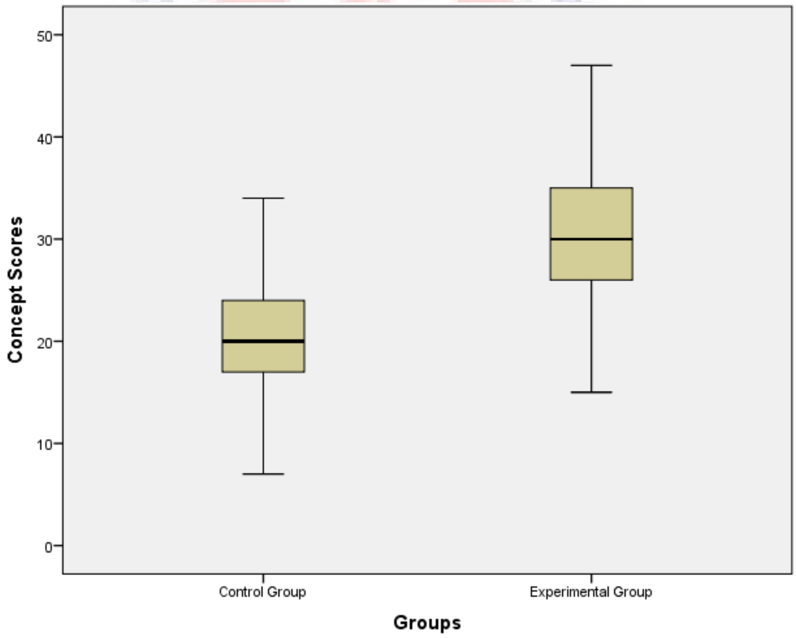

Figure 4.21: Box Plot for Concept Scores for Phase 2 (2016/2017)

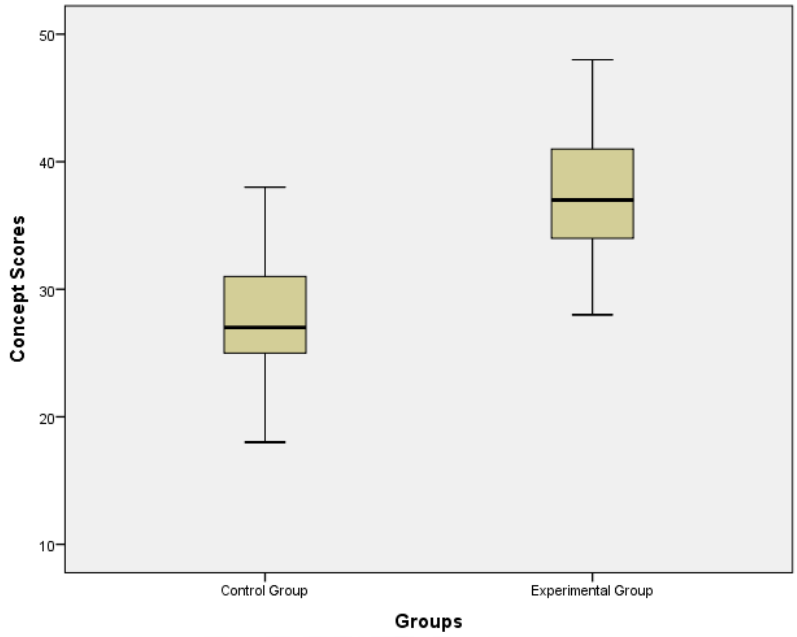

Figure 4.22: Box Plot for Concept Scores for Phase 2 (2017/2018)

The box plots in Figures 4.21 and 4.22 respectively are the concepts scores for phase 2 of 2016/2017 and 2017/2018 academic years respectively. Again, the plots show no significant outliers for the concept scores for the two groups.

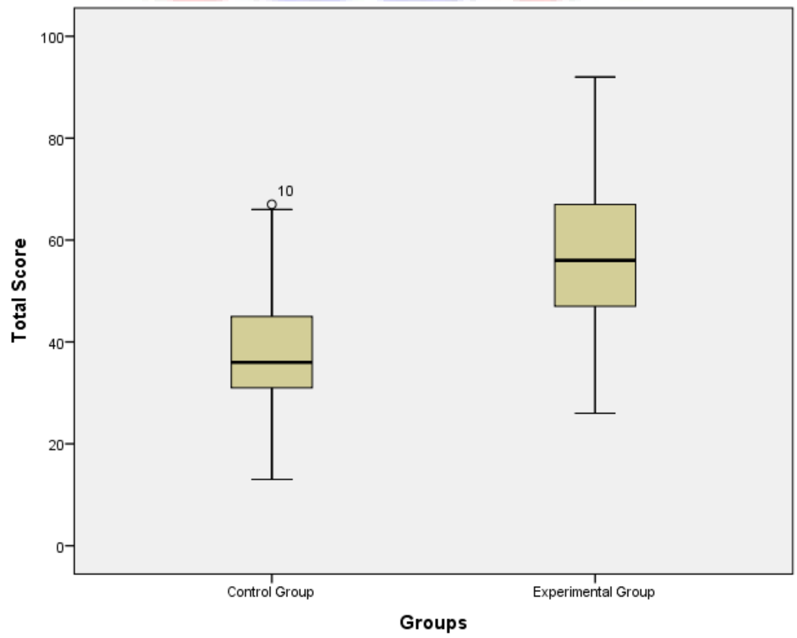

Figure 4.23: Box Plot for Total Scores for Phase 2 (2016/2017)

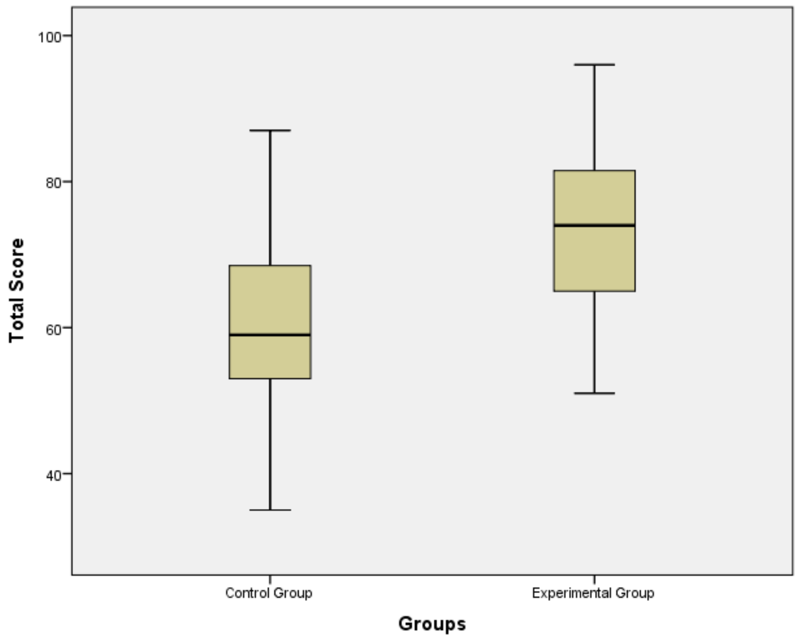

Figure 4.24: Box Plot for Total Scores for Phase 2 (2017/2018)

The box plot for the total scores for the control and experimental groups for the two academic years are displayed in Figures 4.23 and 4.24 respectively. The control group for the 2016/2017 academic year has one outlier but the rest of the groups have no outlier. The z-score for the outlier was 2.53 standard deviations above the mean. Since the outlier was within 3 standard deviations from the mean out of a sample of 218, it may not have significant impact on the results and so we conclude that there is no significant outliers for the two groups in terms of total scores. Hence the assumption for no significant outliers was met.

- **Normality of the Distribution**: The Shapiro-Wilk in Table 4.8 for 2016/2017 show that the significant values for method scores for the control and experimental groups are 0.482 and 0.043 respectively. In this case the control group was found to be close to normality while the experimental group was not found to be close to normality. That is, the significant value for the control group is greater than 0.05 while the significant value of the experimental group is less than 0.05. Figure 4.25 confirms that the distribution of the control group is close to normality while the histogram in Figure 4.26 is positively skewed indicating the scores for the method scores for the experimental group is not close to normality.

Table 4.8 Test for Normality of Phase 2 Test Results (2016/2017)

		Shapiro-Wilk Test		
Scores	Groups	Statistics	df	Sig.
Method Scores	Control Group	0.988	109	0.482
	Experimental Group	0.976	109	0.043
Concept Scores	Control Group	0.985	109	0.282
	Experimental Group	0.985	109	0.245
Total Scores	Control Group	0.985	109	0.283
	Experimental Group	0.988	109	0.426

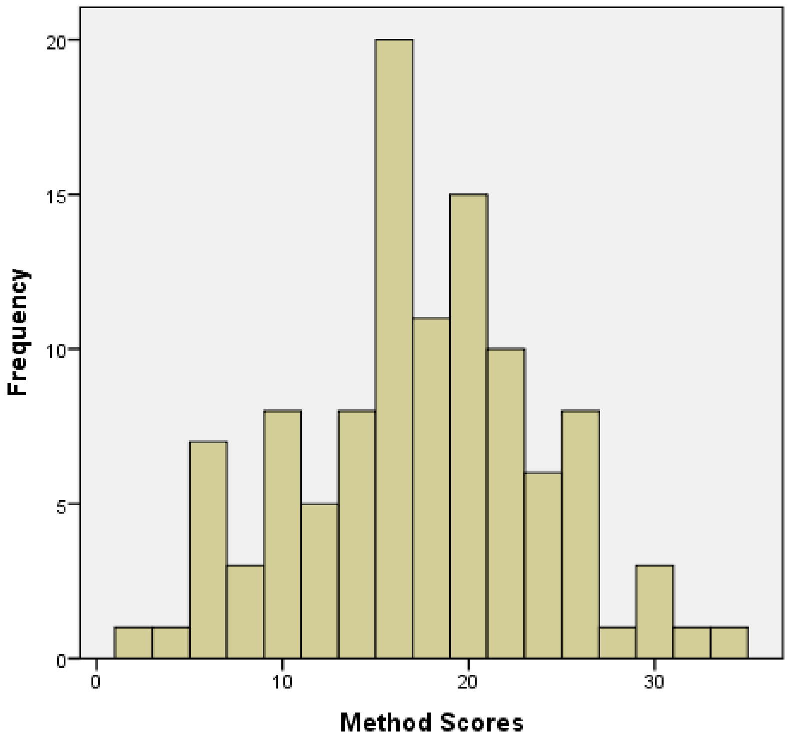

Figure 4.25: Histogram for Method Scores (Control Gp.), Phase 2 (2016/2017)

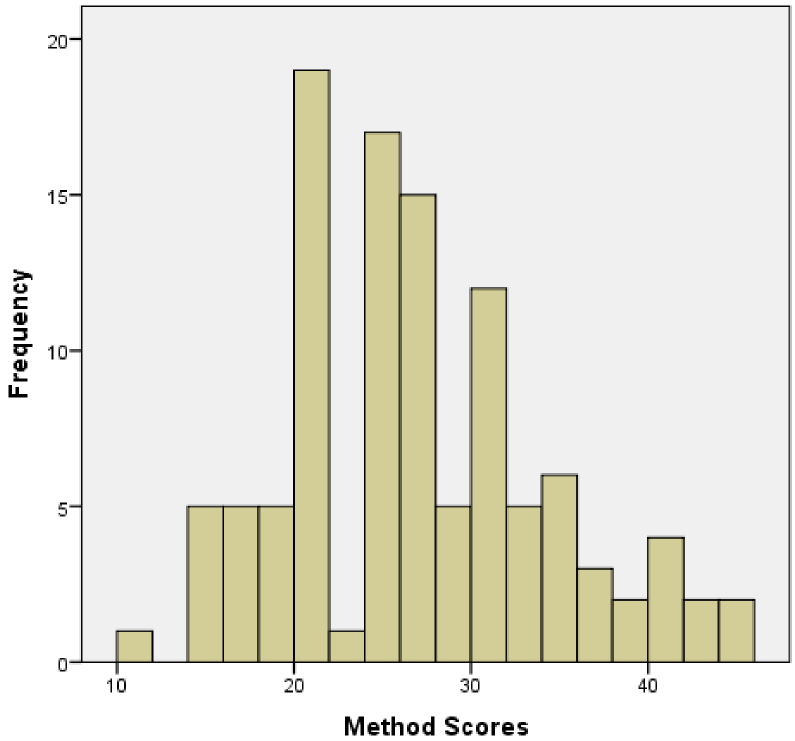

Figure 4.26: Histogram for Method Scores (Experimental Gp), Phase 2 (2016/2017)

For the concept scores, the significant values for Shapiro-Wilk test in terms of the concept scores for the control and experimental group are 0.282 and 0.245 respectively (see Table 4.8). Since these values are greater than 0.05, the α-value, the scores for both groups are not significantly different from the normal curve. In the same manner, the significant values of the total scores for the control and experimental groups according to the Shapiro-Wilk test are 0.283 and 0.426 respectively. This again shows that the total scores are not significantly different than the normal distribution.

For the 2017/2018 academic year, Table 4.9 indicates the normality test results obtained for the various scores. For the method scores the significant values obtained for the control and experimental groups respectively are 0.419 and 0.089. This indicates that the scores are not significantly different form the normal distribution.

Table 4.9 Test for Normality of Phase 2 Test Results (2017/2018)

		Shapiro-Wilk Test		
Scores	Groups	Statistics	df	Sig.
Method Scores	Control Group	0.986	91	0.419
	Experimental Group	0.976	91	0.089
Concept Scores	Control Group	0.973	91	0.059
	Experimental Group	0.976	91	0.084
Total Scores	Control Group	0.989	91	0.619
	Experimental Group	0.981	91	0.196

In the case of the concept scores, the significant values for the control and experimental groups are 0.059 and 0.084 respectively. And for the total scores, the significant values obtained for the control and experimental groups are 0.619 and 0.196 respectively which are all greater than 0.05. This confirms that, the concept scores and total scores are not significantly different from the normal distribution.

- **Homogeneity of Variances**: To test for the homogeneity of variances, the Levene's test for differences in variances was conducted and the results is displayed in Tables 4.10 and 4.11.

Table 4.10 Levene's Test for Equality of Variance (Phase 2, 2016/2017)

Scores	F	Sig.
Method Scores	1.228	0.269
Concept Scores	2.856	0.092
Total Scores	2.406	0.122

The results from Table 4.10 show that the significant values for the method, concept and total scores in the 2016/2017 academic year are 0.269, 0.092 and 0.122 respectively. Since the significant values are greater than the α-value (0.05), then we can conclude that there are no significant differences in the variances of the two groups in each case.

Table 4.11 Levene's Test for Equality of Variance (Phase 2, 2017/2018)

Scores	F	Sig.
Method Scores	0.506	0.478
Concept Scores	0.274	0.601
Total Scores	0.004	0.948

Again, the results from Table 4.11 indicated that for the 2017/2018 academic year, the significant values for the method, concept and total scores respectively are 0.478, 0.601 and 0.948. This indicates that the variability in the variances of the various scores for the two groups for the phase 2 confirmation test are not significant. It can therefore, be concluded that equal variances can be assumed for the results obtained from the phase 2 confirmation test in both academic years.

Test for Assumptions on Phase 3 Test Results

- **Outliers**: For the method scores in the 2016/2017 academic year, the box plot in Figure 4.27 shows one outlier for the experimental group but there is no outlier for the control group. Since the outlier for the experimental group falls within 3 standard deviations from the mean (2.9 standard deviations above the mean), it can therefore be concluded that the outlier was not significant in the analysis of the results. For the 2017/2018 academic year, there was no significant outlier for both the control and the experimental group (see Figure 4.28). This indicate no significant outlier for the two groups in the case of their method scores, hence the method scores meet the assumption that there should be no significant outliers.

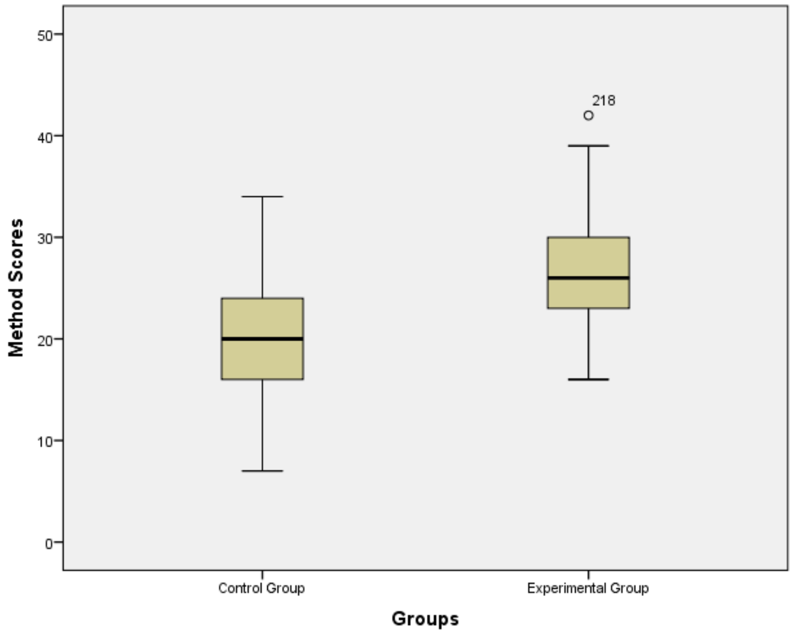

Figure 4.27: Box Plot for Method Scores for Phase 3 (2016/2017)

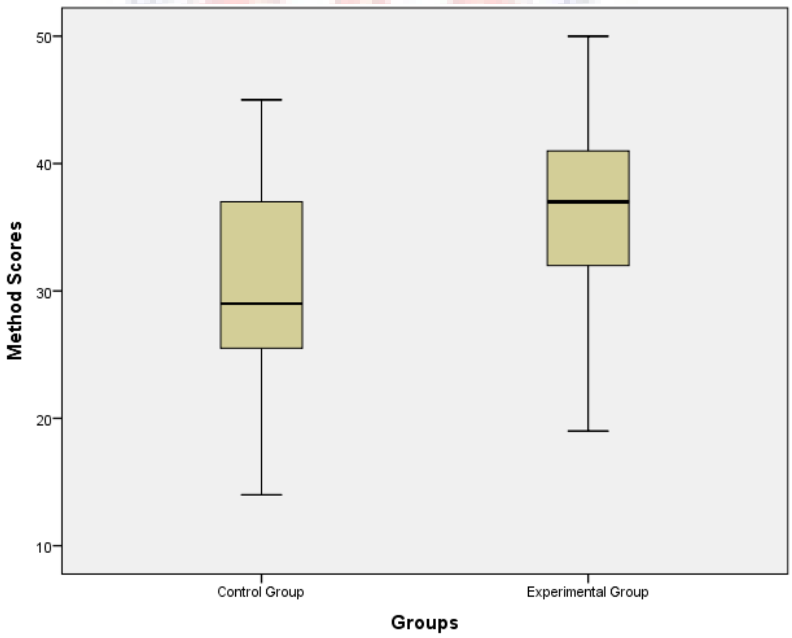

Figure 4.28: Box Plot for Method Scores for Phase 3 (2017/2018)

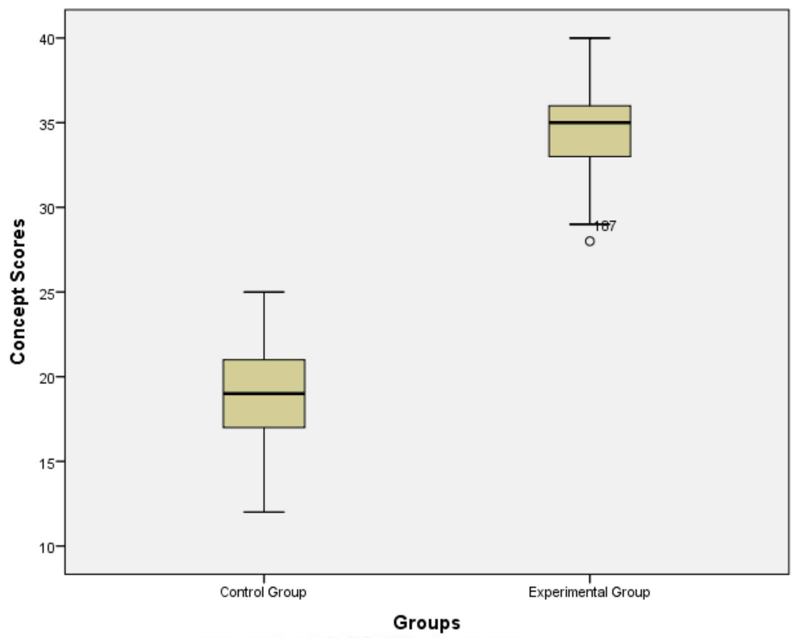

Figure 4.29: Box Plot for Concept Scores for Phase 3 (2016/2017)

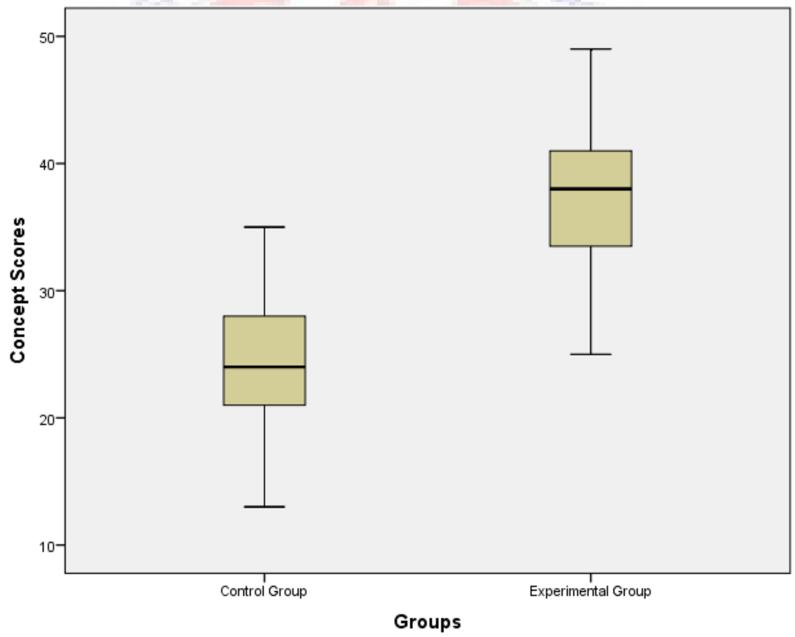

Figure 4.30: Box Plot for Concept Scores for Phase 3 (2017/2018)

For the concept scores, the box plot for the 2016/2017 as indicated in Figure 4.29 show that the control group has no outlier while the experimental has only one outlier. But the outlier for the experimental group is within 3 standard deviations from the mean (2.42 standard deviations below the mean), the outlier was not significant. The box plot in Figure 4.30 also show no outliers for the control and experimental groups for the concept scores for 2017/2018 academic year. In all, the concept scores for the 2016/2017 and 2017/2018 show no significant outliers for the concept scores.

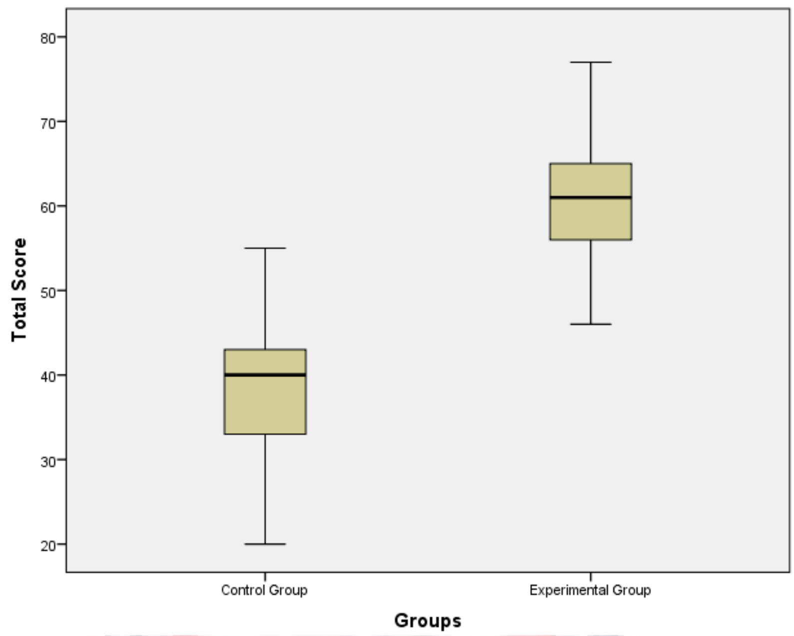

Figure 4.31: Box Plot for Total Scores for Phase 3 (2016/2017)

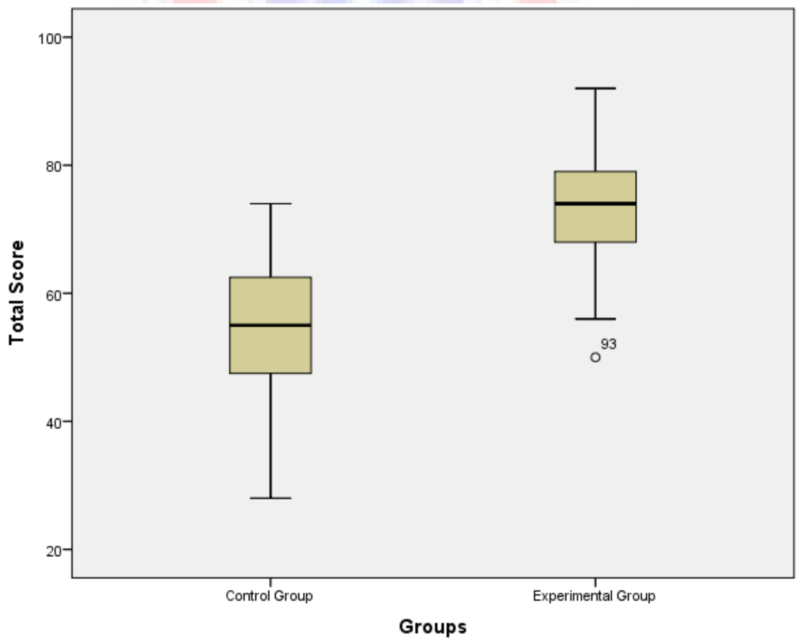

Figure 4.32: Box Plot for Total Scores for Phase 3 (2017/2018)

The box plots in Figures 4.31 and 4.32 are for the total scores for the 2016/2017 and 2017/2018 academic years respectively. Figure 4.31 shows no outlier for the total scores of the control and experimental groups of the 2016/2017 academic year. As in the case of the 2017/2018 academic year, Figure 4.32 shows no outlier for the control group and one outlier for the experimental group. However, the one outlier for the experimental group is not significant enough to change the outcome of the results since it was within 3 standard deviations from the mean. It can therefore be concluded that, there are no significant outliers for the total scores of both the control and experimental groups of the 2016/2017 and 2017/2018 academic years.

- **Normality of the Distribution**: The Shapiro-Wilk test in Table 4.12 for 2016/2017 show that the significant values of the method scores for the control and experimental groups are 0.412 and 0.301 respectively. Since both values are greater than 0.05, it can therefore be concluded that the method scores for the control and experimental groups are not significantly different from the normal distribution.

Table 4.12 Test for Normality of Phase 3 Test Results (2016/2017)

		Shapiro-Wilk Test		
Scores	Groups	Statistics	df	Sig.
Method Scores	Control Group	0.988	109	0.412
	Experimental Group	0.986	109	0.301
Concept Scores	Control Group	0.978	109	0.063
	Experimental Group	0.980	109	0.105
Total Scores	Control Group	0.977	109	0.060
	Experimental Group	0.978	109	0.067

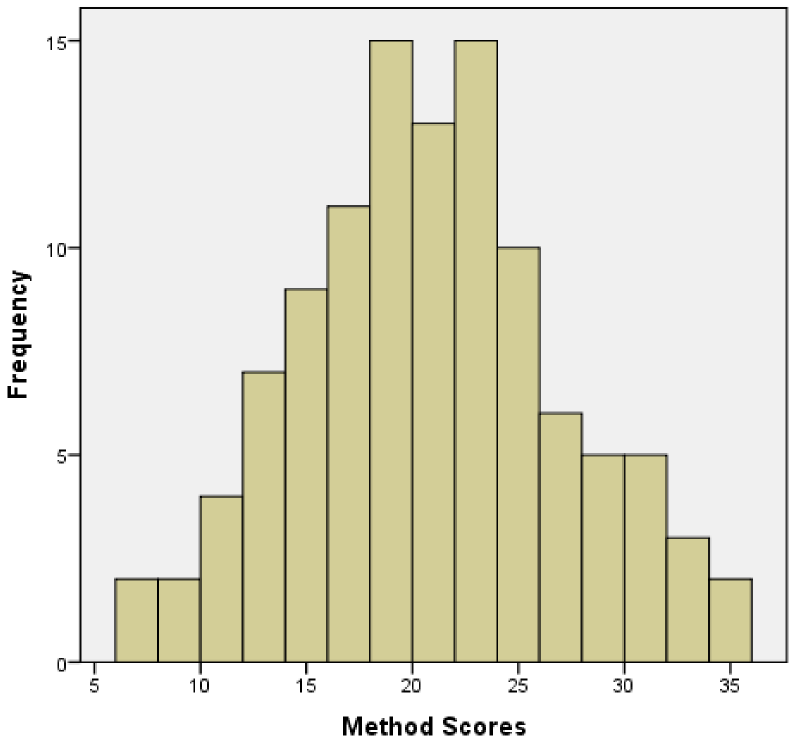

Figure 4.33: Histogram for Method Scores (Control Group), Phase 3 (2016/2017)

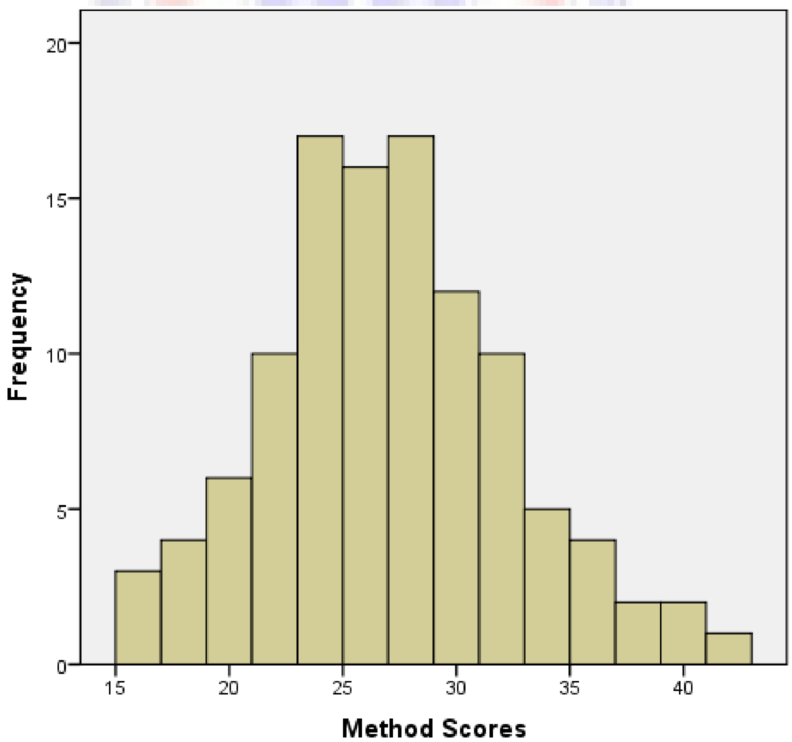

Figure 4.34: Histogram for Method Scores (Experimental Group), Phase 3 (2016/2017)

A look at the histograms in Figures 4.33 and 4.34 show that the distribution for the method scores are getting closer to the normal distribution. This confirms the results of the Shapiro-Wilk test for normality.

Again, the results in Table 4.12 shows that for the concept scores the significant values for the control and experimental groups respectively were 0.063 and 0.105 indicating that the distributions were not significantly different from the normal distribution. This can also be confirmed from the histogram in Figures 4.35 and 4.36. Hence, the concept scores also meet the assumption for normality.

As in the case of the total scores, the significant values for the Shapiro-Wilk test of the control and experimental groups were found to be 0.060 and 0.067 respectively. This again shows that the distribution of the total scores were not significantly different from the normal distribution for both the control and experimental groups.

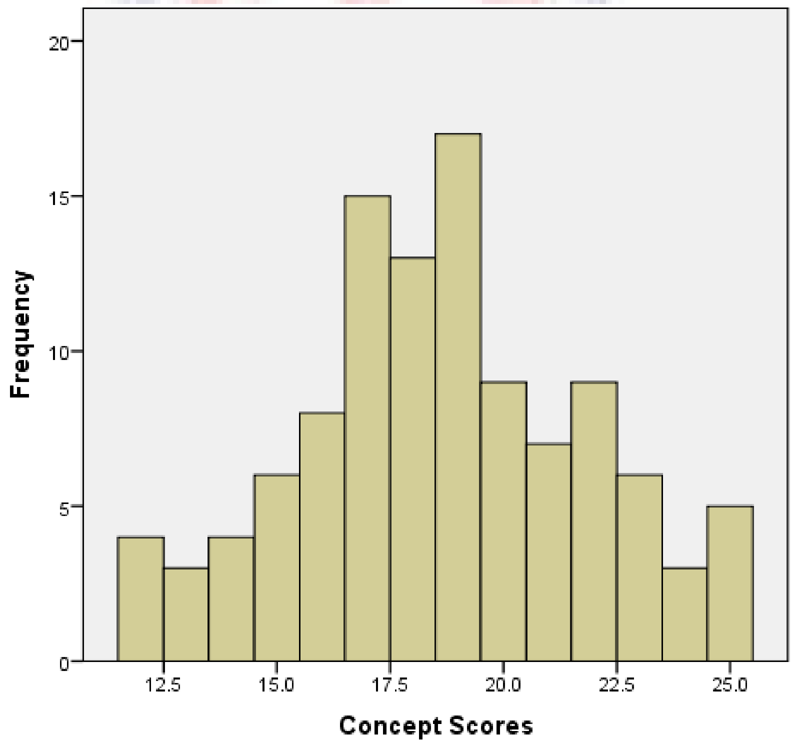

Figure 4.35: Histogram for Concept Scores (Control Group), Phase 3 (2016/2017)

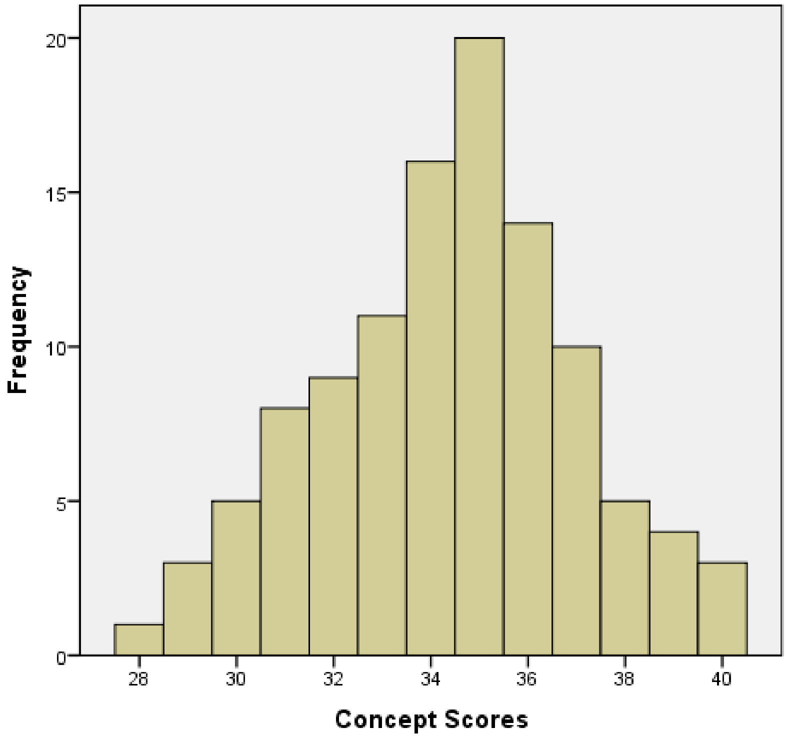

Figure 4.36: Histogram for Concept Scores (Experimental Group), Phase 3 (2016/2017)

The histograms in Figures (4.37) and (4.38) confirm that the distribution of the total scores for the control and experimental groups were not significantly different from the normal distribution.

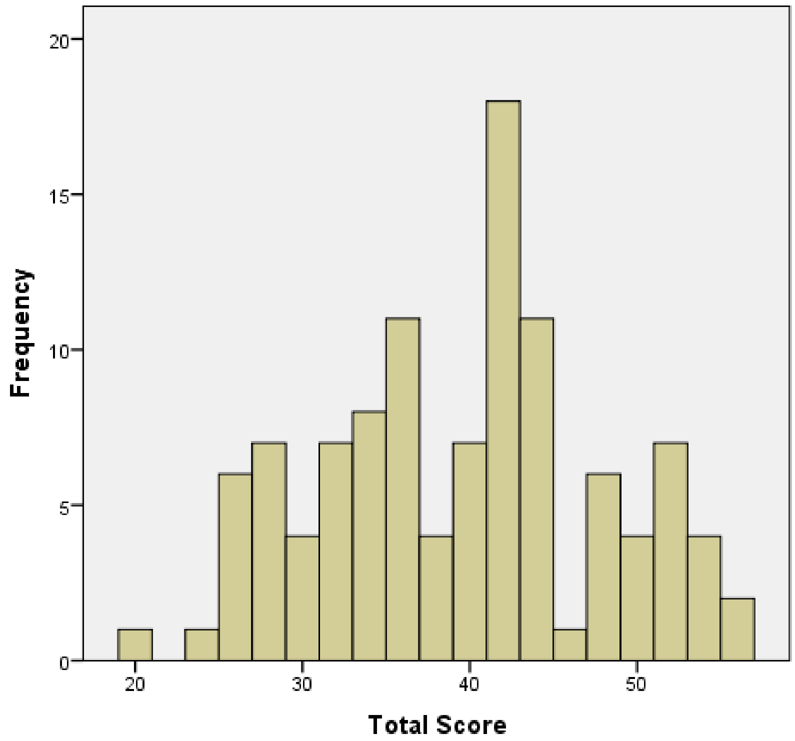

Figure 4.37: Histogram for Total Scores (Control Group), Phase 3 (2016/2017)

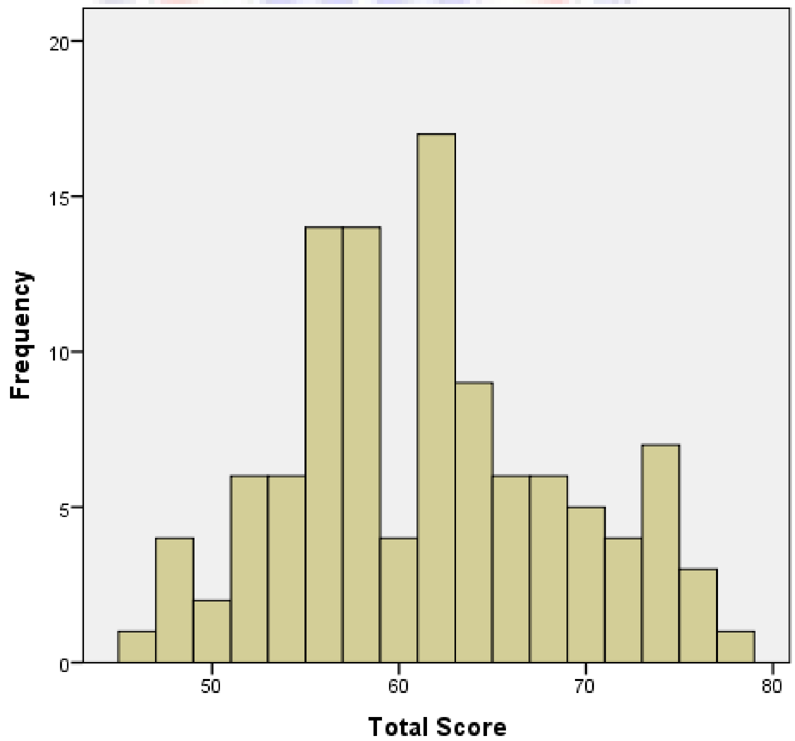

Figure 4.38: Histogram for Total Scores (Experimental Group), Phase 3 (2016/2017)

Table 4.13 shows the Shapiro-Wilk test of the significant values of the various scores of the control and experimental group for the 2017/2018 academic year.

From Table 4.13, the significant values of the method scores for the control and experimental groups were respectively 0.067 and 0.185 which were greater than the α-value of 0.05. This indicates the distribution of the method scores were not significantly different from the normal distribution and this is confirmed by the histogram in Figures 4.39 and 4.40.

Table 4.13 Test for Normality of Phase 3 Test Results (2017/2018)

		Shapiro-Wilk Test		
Scores	Groups	Statistics	df	Sig.
Method Scores	Control Group	0.974	91	0.067
	Experimental Group	0.980	91	0.185
Concept Scores	Control Group	0.980	91	0.187
	Experimental Group	0.981	91	0.222
Total Scores	Control Group	0.984	91	0.350
	Experimental Group	0.988	91	0.552

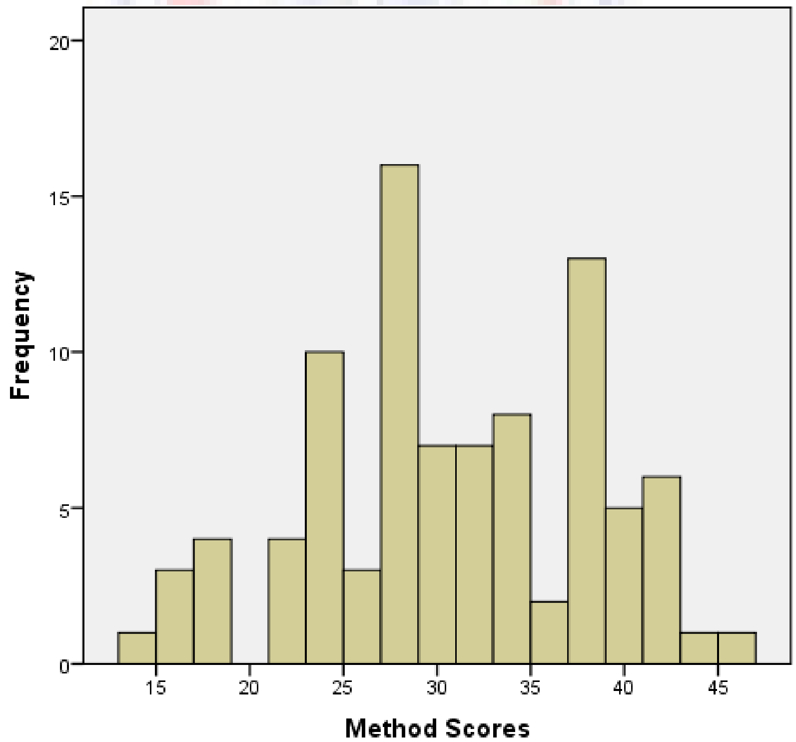

Figure 4.39: Histogram for Method Scores (Control Group), Phase 3 (2017/2018)

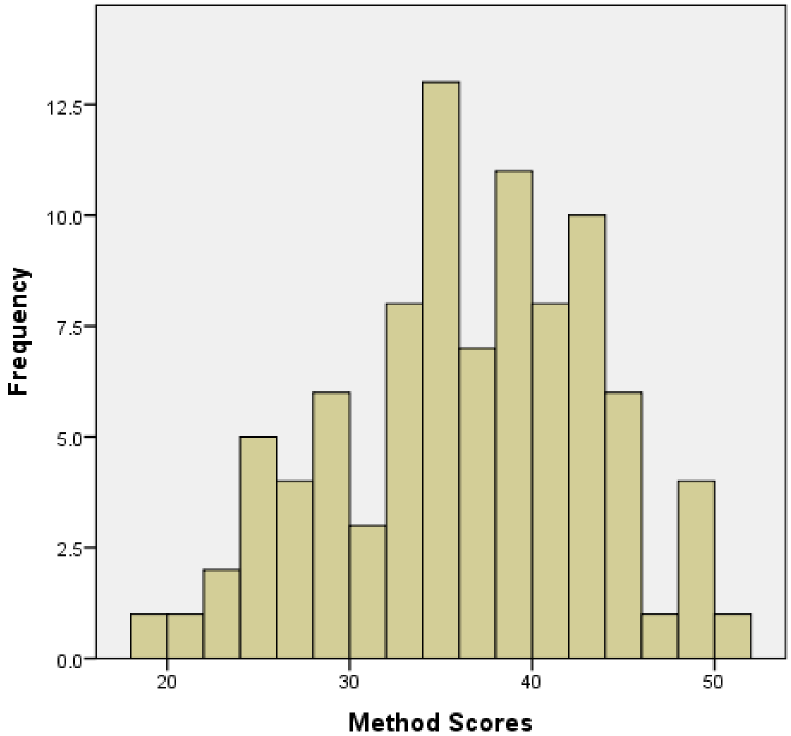

Figure 4.40: Histogram for Method Scores (Experimental Group), Phase 3 (2017/2018)

Again, the results from Table 4.13 shows that the significant values of the concept scores for the control and experimental groups are 0.187 and 0.222 respectively. Since the significant values once again are greater than 0.05 it can therefore, be concluded that the distribution of the concept scores for the two groups were not significantly different from the normal distribution. The results again can be confirmed from the histograms in Figures 4.41 and 4.42.

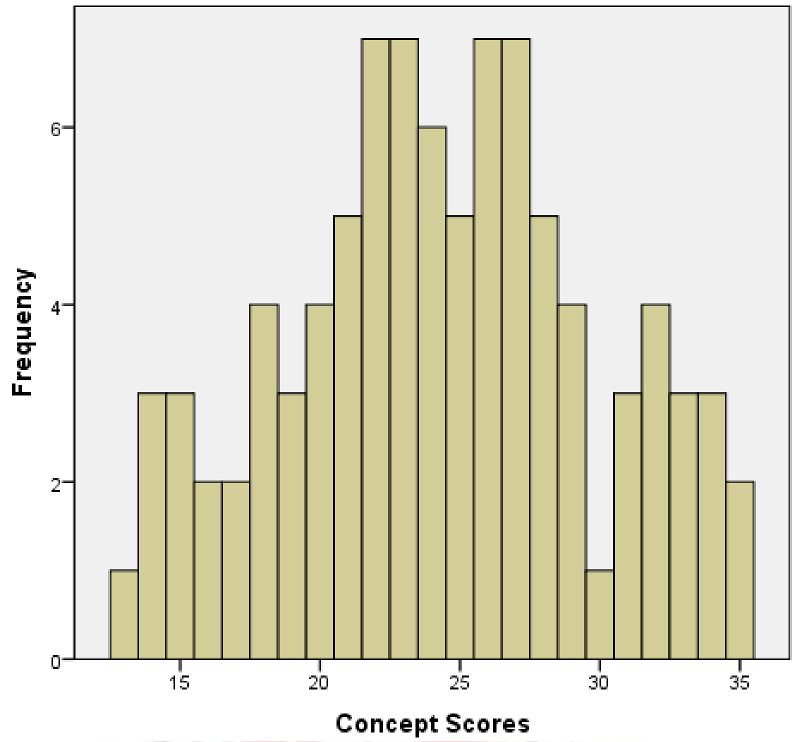

Figure 4.41: Histogram for Concept Scores (Control Group), Phase 3 (2017/2018)

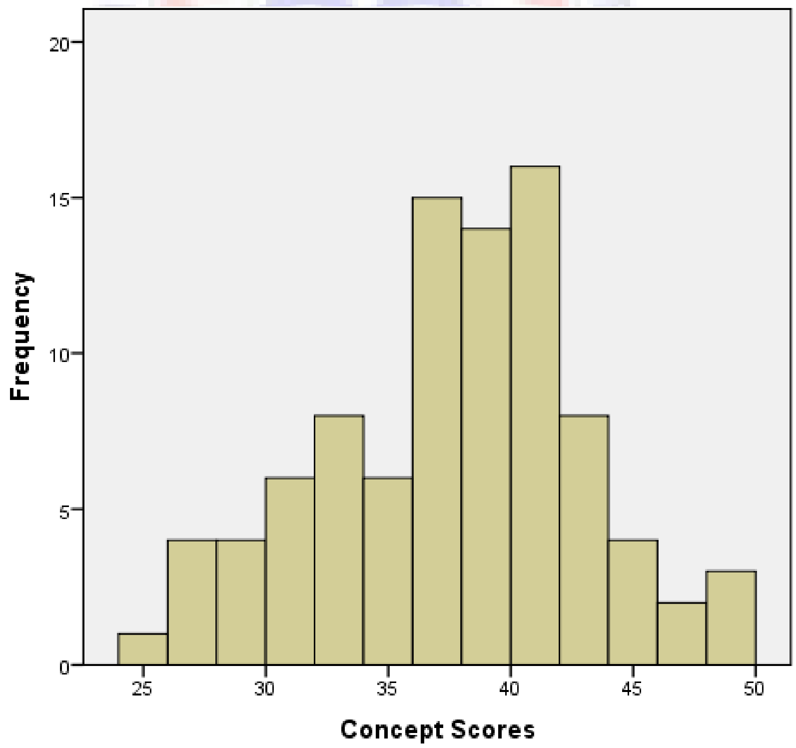

Figure 4.42: Histogram for Concept Scores (Experimental Group), Phase 3 (2017/2018)

From Table 4.13, the significant values of the total scores for the control and experimental groups are respectively 0.350 and 0.552. The distribution of the total scores were therefore not significantly different from the normal distribution.

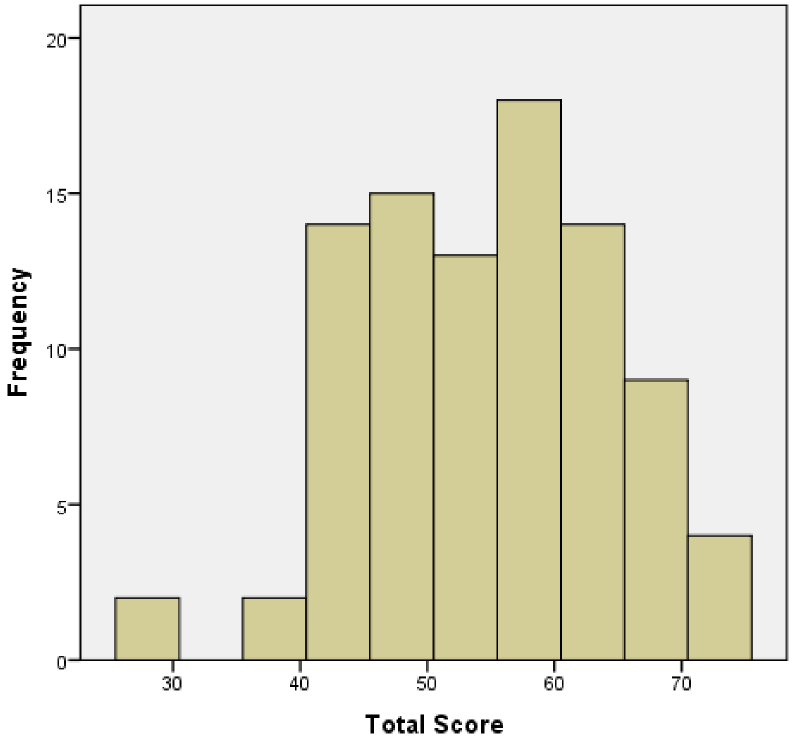

Figure 4.43: Histogram for Total Scores (Control Group), Phase 3 (2017/2018)

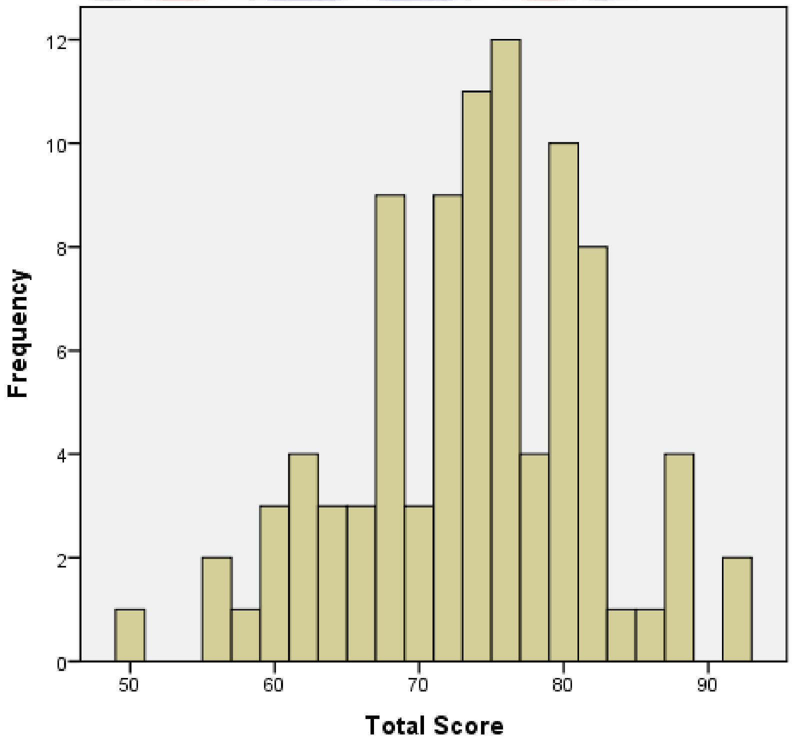

Figure 4.44: Histogram for Total Scores (Experimental Group), Phase 3 (2017/2018)

Also, the histogram in Figures 4.43 and 4.44 confirms that the distribution of the total scores are close to the normal distribution. It can therefore, be concluded that the distribution of the scores for both 2016/2017 and 2017/2018 academic years of the control and experimental groups were not significantly different from the normal distribution.

- **Homogeneity of Variances**: To test for the homogeneity of variances for the phase 3, the Levene's test for differences in variances was conducted and the results is displayed in Tables 4.14 and 4.15.

Table 4.14 Levene's Test for Equality of Variance (Phase 3, 2016/2017)

Scores	F	Sig.
Method Scores	2.372	0.125
Concept Scores	3.467	0.064
Total Scores	2.521	0.114

The results in Table 4.14 shows that for the 2016/2017 academic year, the significant values of the method, concept and total scores for the control and experimental groups were 0.125, 0.064 and 0.114 respectively. The values show that there was no significant difference between the variances of the scores in the control and the experimental groups.

Table 4.15 Levene's Test for Equality of Variance (Phase 3, 2017/2018)

Scores	F	Sig.
Method Scores	0.378	0.540
Concept Scores	0.131	0.718
Total Scores	0.526	0.114

From Table 4.15 the significant values for the 2017/2018 academic year for the method, concept and total scores of the control and experimental groups were 0.540, 0.718 and 0.114 respectively. This indicate that the variances of the control and experimental groups were not significantly different. It can therefore, be concluded that the variability in the variances for the scores of the control and experimental group were not significant.

4.4.2 Effect Size - Cohen's d

Effect size provide a definition of the scale of the observed results, regardless of the sample size factors that may be misleading. Research with different sample sizes but the same basic features (e.g. proportions, means, standard deviation, confidence intervals) may vary in their statistical significance values but not in their estimates of the effect size (Cohen, 1990; Fritz et al., 2012). According to Cohen (1988), effect size of 0.20 is small, 0.50 is medium and 0.80 is large. Effect size can also be thought of as the average percentile standing of the average treated (or experimental) participants relative to the average untreated (or control) participants (Cohen, 1988). Table 4.16 gives the interpretation of Cohen's d.

Table 4.16 The Interpretation of Cohen's d

Cohen's d Standard	Effect Size	Percentile Standing	Percent of Non-overlap (%)
	2.0	97.7	81.1
	1.9	97.1	79.4
	1.8	96.4	77.4
	1.7	95.5	75.4
	1.6	94.5	73.1
	1.5	93.3	70.7
	1.4	91.9	68.1
	1.3	90.0	65.3
	1.2	88.0	62.2
	1.1	86.0	58.9
	1.0	84.0	55.4
	0.9	82.0	51.6
Large	0.8	79.0	47.4
	0.7	76.0	43.0
	0.6	73.0	38.2
Medium	0.5	69.0	33.0
	0.4	66.0	27.4
	0.3	62.0	21.3
Small	0.2	58.0	14.7
	0.1	54.0	7.7
	0.0	50.0	0

Source: Becker (2000)

In this study, because the standard deviations differ but the sample size for each group are equal, then, from Cohen (1988) and Keppel and Wickens (2004), the Cohen's d will be computed using

$$d = \frac{\mu_e - \mu_c}{\sigma_{ec}}$$

with

$$\sigma_{ec} = \sqrt{\frac{\sigma_e^2 + \sigma_c}{2}}$$

where μ_e is the population mean of the experimental group, μ_c is the population mean of the control group, σ_e is the population standard deviation of the experimental group, σ_c is the population standard deviation of the control group and σ_{ec} is the population standard deviation of the two groups.

4.4.3 Independent Sample T-Test and Effect Size of the Test Scores

Preamble

A total of 218 students were selected for the study in the 2016/2017 academic year and they were put into two groups, group 1 and group 2. Each group consist of 109 students. For the 2017/2018 academic year, a total of 182 students were selected for the study and were also put into two groups, group 1 and group 2. Each group consist of 91 students. In each phase of the study, the confirmation test conducted after the treatments measured the performance of the students in three aspects:

1. the methods the students used in answering the questions and the scores were termed ”method scores”,
2. the conceptual understanding of the students and the scores obtained was termed ”concept scores” and
3. the total performance of the students which was obtained by summing up the method score and the concept score of each student. The sum of the scores was termed ”total score”.

For each of these aspects a descriptive analysis and independent sample t-test was conducted and the results were compared. The analysis was conducted phase-wise since the study was conducted in phases.

Phase 1 Test Scores, 2016/2017

Table 4.17 is the descriptive statistics of the scores obtained from phase 1 after the treatment for the 2016/2017 academic year group.

Table 4.17 Descriptive Statistics of 2016/2017 Phase 1 Test Scores

Scores	Groups	N	Mean	Var.	Min.	Max.
Method	Control	109	21.35	38.377	8	36
	Experimental	109	33.75	22.410	25	45
Concept	Control	109	22.90	33.406	11	35
	Experimental	109	28.76	38.869	15	44
Total	Control	109	44.25	73.447	26	68
	Experimental	109	62.51	68.641	42	80

From Table 4.17, the average method score of the control group is 21.35 with variance 38.377 (standard deviation of 6.1949). The average method score for the experimental group is 33.75 with variance, 22.41 (standard deviation of 4.7339). The control group had a minimum score of 8 and a maximum score of 36 while the minimum and maximum method scores of the experimental group are 25 and 45 respectively. So, the method score for the experimental group was found to be higher than the control group. The mean difference between the control group and the experimental group for the method scores is 12.404 (see Table 4.18).

In the case of the concept scores, the average score for the control group from Table 4.17 is 22.90 with variance 33.406 (standard deviation of 5.7798). The experimental group on the other hand had an average concept score of 28.76 with variance 38.869 (standard deviation of 6.2343). The control group had a minimum concept score of 11 and a maximum of 35. The minimum and maximum concept scores of the experimental group are 15 and 44 respectively. The mean difference between the control group and the experimental group for the concept scores is 5.862 (see Table 4.18).

Again, from Table 4.17, the average total score of the control group is 44.25 with variance 73.447 (standard deviation of 8.57) while that of the experimental group is 62.51 with variance 68.641 (standard deviation of 8.285). The minimum and maximum total score of the control group are respectively 26 and 68 while that of the experimental group are 42 and 80 respectively. The mean difference between the control group and the experimental group for the total scores is 18.266 (see Table 4.18). Again, the score of the experimental group was found to be higher than the control group indicating a higher performance in the experimental group. To check the significance of the difference in means of all the aspects, an independent sample t-test for equality of means was conducted and the results is displayed in Table 4.18.

Table 4.18 Independent Sample T-Test of 2016/2017 Phase 1 Test Scores

		Levene's Test for Equality of Vars.		**T-Test for Equality of Means**				
Scores		**F**	**Sig.**	**t**	**df**	**Sig.(2-T)**	**MD**	**ES**
Method	EVA	10.762	0.001	16.609	216	0.000	12.404	2.2
	EVNA			16.609	202.059	0.000	12.404	
Concept	EVA	0.272	0.662	7.199	216	0.000	5.862	1.0
	EVNA			7.199	214.773	0.000	5.862	
Total	EVA	0.000	0.998	15.998	216	0.000	18.266	2.2
	EVNA			15.998	215.753	0.000	18.266	

Keys:	2-T	:	2-tailed
	Sig.	:	Significant value or p-value
	df	:	degree of freedom
	MD	:	Mean Difference
	EVA	:	Equal variances assumed
	EVNA	:	Equal variances not assumed
	ES	:	Effect Size

From Table 4.18, the Levene's test for equality of variances for the method, concept and total scores have significant values 0.001, 0.662 and 0.998 respectively. Since the significant value of the concept and total scores are greater than the α-value of 0.05, the variability in variances of the control and experimental group are not significant. For this reason, equal variances were assumed for the concept and the total scores so the first row for each of these variables (concept and total scores) was read. But since the significance value for the method scores is less than 0.05, it is assumed that the variances are not equal and so the second row was used for the analysis.

From Tables 4.17 and 4.18, there was a significant difference between the means of the method scores obtained by the control group ($M = 21.35, SD = 6.1949$) and the experimental group ($M = 33.75, SD = 4.7339$); $t(202.059) = 16.609, p = 0.000 < 0.05$. These results suggest that the experimental group performed better in terms of the methods of solving PDEs than their colleagues in the control group. Again, the effect size of 2.2 (see Table 4.18) indicates that the effect size is large using Cohen's d interpretation in Table 4.16. The results show that, the mean for the experimental group is 2.2 standard deviations higher than the mean for the control group. Also, the effect size of 2.2 indicates that the mean of the experimental group is at more than 97.7th percentile of the control group with more than 81.1% non-overlaps of the experimental group scores with those of the control group. This suggest that, the IO-IA improved the students' performance on methods of solving first-order PDEs.

Again from Tables 4.17 and 4.18, there was a significant difference between the means of the concept scores between the control group ($M = 22.90, SD = 5.7798$) and the experimental group ($M = 28.76, SD = 6.2343$); $t(216) = 7.199, p = 0.000 < 0.05$. From Table 4.16, the effect size of 1.0 is large and it means the mean of the experimental group is 1.0 standard deviation above the mean of the control group. This means that the mean of the experimental group is about 84th percentile of the control group with about 55.4% non-overlaps of the experimental group scores with those of the control group. The result suggests that the mean difference of 5.862 between the control and the experimental group was significant and that the experimental group had a better understanding of the concept presented than their colleagues in the control group.

From Tables 4.17 and 4.18, there was a significant difference between the means of the total scores of the control group ($M = 44.25, SD = 8.57$) and the experimental group ($M = 62.51, SD = 8.285$); $t(216) = 15.998, p = 0.000 < 0.05$. Also, the effect size of 2.2 is very large according to Cohen's d interpretation in Table 4.16. This indicates that, the mean of the experimental group is 2.2 standard deviations above the mean of the control group and is at more than 97.7th percentile of the control group. The effect size of 2.2 also indicates that the scores of the experimental group is with more than 81.1% non-overlaps with those of the control group. These results suggest that the mean difference of 18.266 between the control and the experimental groups is significant. Since the total score is the sum of the method scores and the concept scores, it suggests that the performance of the experimental group was better than that of the control group after the treatment in phase 1. So the inquiry-oriented instructional approach (IO-IA) yielded better results in terms of students' performance than that of the traditional method of teaching.

Phase 1 Test Scores, 2017/2018

Table 4.19 displays the descriptive statistics for the test scores after phase 1 treatment for 2017/2018 academic year group. A total of 182 students were selected for the study with 91 students in each group.

Table 4.19 Descriptive Statistics of 2017/2018 Phase 1 Test Scores

Scores	**Groups**	**N**	**Mean**	**Var.**	**Min.**	**Max.**
Method	Control	91	26.37	31.903	15	40
	Experimental	91	32.43	38.914	17	48
Concept	Control	91	18.11	51.143	5	33
	Experimental	91	29.41	72.133	5	47
Total	Control	91	44.48	89.919	23	71
	Experimental	91	61.84	124.873	28	87

From Table 4.19, the average method scores for the control and the experimental group are 26.37 and 32.43 respectively. The variances of the control and experimental groups are 31.903 (standard deviation of 5.65) and 38.914 (standard deviation of 6.23) respectively. The control group obtained a minimum score of 15 and a maximum score of 40 while the minimum and maximum scores of the experimental group is 17 and 48 respectively. The mean difference between the control and experimental groups is 6.055 (see Table 4.20).

From table 4.19, the average concept score for the control group is 18.11 with a variance of 51.143 (standard deviation of 7.15) while the mean score of the experimental group is 29.41 with variance, 72.133 (standard deviation of 8.49). The minimum and maximum scores for the control group are 5 and 33 respectively while that of the experimental group are 5 and 47 respectively. The mean difference between the control and experimental group is 11.297 (see table 4.20).

From Table 4.19, the mean total scores of the control and experimental group are 44.48 and 61.84 respectively. The variances of the control and experimental group are 89.919 (standard deviation of 9.48) and 124.873 (standard deviation of 11.74) respectively. The minimum and maximum scores for the control group are 23 and 71 respectively while that of the experimental group are 28 and 87 respectively. The mean difference between the control and experimental group is 17.352 (see Table 4.20). To verify the significance of the mean differences between the control and the experimental group in the various scores, an independent sample t-test was conducted and the results is displayed in Table 4.20.

Table 4.20 Independent Sample T-Test of 2017/2018 Phase 1 Test Scores

		Levene's Test for Equality of Vars.		T-Test for Equality of Means				
Scores		**F**	**Sig.**	**t**	**df**	**Sig.(2-T)**	**MD**	**ES**
Method	EVA	1.090	0.298	6.864	180	0.000	6.055	1.0
	EVNA			6.864	178.253	0.000	6.055	
Concept	EVA	1.289	0.258	9.706	180	0.000	11.297	1.4
	EVNA			9.706	174.929	0.000	11.297	
Total	EVA	1.672	0.198	11.294	180	0.000	17.352	1.8
	EVNA			11.294	175.356	0.000	17.352	

Keys:			
	2-T	:	2-tailed
	Sig.	:	Significant value of p-value
	df	:	degree of freedom
	MD	:	Mean Difference
	EVA	:	Equal variances assumed
	EVNA	:	Equal variances not assumed
	ES	:	Effect Size

From Table 4.20, the significant value of the Levene's test for equality of variances of the method, concept and total scores are 0.298, 0.258 and 0.198 respectively for the control and the experimental group. Since all these values are greater than 0.05, the α-value, it is assumed that the variances are not significantly different and so the results for the t-test in the first row of each aspect was used.

From Tables 4.19 and 4.20 there was a significant difference between the means of the method scores of the control group ($M = 26.37, SD = 5.65$) and the experimental group ($M = 32.43, SD = 6.23$); $t(180) = 6.864, p = 0.000 < 0.05$. The effect size of 1.0 indicates that the mean difference of 6.055 between the experimental and the control groups is large. It means the mean of the experimental group is 1.0 standard deviation above the mean of the control group. Again, the effect size of 1.0 indicates that the mean experimental group is at the 84th percentile of the control group. The effect size of 1.0 again indicates a non-overlap of 55.4% in the two distributions. The results suggest that the experimental group performed better on the methods of solving first-order PDEs than the control group.

From Tables 4.19 and 4.20 there was a significant difference between the means of the concept scores of the control group ($M = 18.11, SD = 7.15$) and the experimental group ($M = 29.41, SD = 8.49$); $t(180) = 9.706, p = 0.000 < 0.05$. Also, the effect size of 1.4 indicates a large effect according to Cohen's d standard. It means that the mean of the experimental group is 1.4 standard deviations above the mean of the control group. The effect size of 1.4 also indicates that the mean of the experimental group is 91.9th percentile of the control group. It again indicates a non-overlap of 68.1% of the experimental group scores with those of the control group. The results suggest that the student in the experimental group had a better understanding of the concept of first-order PDEs presented to them through the use of the IO-IA than their colleagues in the control group.

Again, from Tables 4.19 and 4.20 there was a significant difference between the means of the total scores of the control group ($M = 44.48, SD = 9.48$) and the experimental group ($M = 61.84, SD = 11.74$); $t(180) = 11.294, p = 0.000 < 0.05$. Also, the effect size of 1.8 shows a large effect in accordance with Cohen's d standard. It means that the mean of the experimental group is 1.8 standard deviations above that of the control group. The effect size of 1.8 again indicates that the mean of the experimental group is 96.4th percentile of the control group. It also indicates a non-overlap of 77.4% in the two distributions. The results suggest that in total, the performance of the students in the experimental group was better than their colleagues in the control group. This means that the performance of the students in the experimental group is better than those in the control group.

Phase 2 Test Scores, 2016/2017

Table 4.21 displays the descriptive statistics for confirmation test scores of the phase 2 after the treatment for the 2016/2017 academic year.

Table 4.21 Descriptive Statistics of 2016/2017 Phase 2 Test Scores

Scores	**Groups**	**N**	**Mean**	**Var.**	**Min.**	**Max.**
Method	Control	109	16.87	42.594	2	33
	Experimental	109	26.54	53.621	11	45
Concept	Control	109	20.24	30.294	7	34
	Experimental	109	31.07	39.458	15	47
Total	Control	109	37.11	138.747	13	67
	Experimental	109	57.61	180.572	26	92

From Table 4.21, the average method scores obtained by the control and experimental are 16.87 and 26.54 respectively. The variances for the control and experimental groups are 42.594 (standard deviation of 6.53) and 53.621(standard deviation of 7.32) respectively. The minimum and maximum scores obtained by the control group are respectively 2 and 33 and the minimum and maximum scores obtained by the experimental group are 11 and 45 respectively. The mean difference between the control and experimental group is 9.670 (see Table 4.22).

From Table 4.21, the average concept scores obtained by the control group is 20.24 with variance 30.294 (standard deviation of 5.50). The average concept score obtained by the experimental group is 31.07 and the variance is 39.458 (standard deviation of 6.28). The minimum and maximum score of the control group are 7 and 34 respectively while that of the experimental group are 15 and 47 respectively. The difference in means between the control and experimental group is 10.835 (see Table 4.22).

From Table 4.21, the average total scores for the control and experimental group are 37.11 and 57.61 respectively. The variances for the control and experimental groups are 138.747 (standard deviation of 11.78) and 180.572 (standard deviation of 13.44) respectively. The minimum and maximum total scores for the control group are 13 and 67 respectively while that of the experimental group are 26 and 92 respectively. The mean difference of the total scores between the control and experimental group is 20.505 (see Table 4.22).

To verify the significance of the mean difference in all the aspects, an independent sample t-test for equality of means was conducted and the results is displayed in Table 4.22.

Table 4.22 displays the results of the independent sample t-test conducted on the confirmation test of phase 2 for the 2016/2017 academic year. From Table 4.22, the significant value obtained from the Levene's test for equality of variances for the method, concept and total scores are 0.269, 0.092 and 0.122 respectively. Since all the significant values are greater than 0.05, equal variances assumed for the three scores and hence the first row of each test was used for the analysis.

Table 4.22 Independent Sample T-Test of 2016/2017 Phase 2 Test Scores

		Levene's Test for Equality of Vars.		**T-Test for Equality of Means**				
Scores		**F**	**Sig.**	**t**	**df**	**Sig.(2-T)**	**MD**	**ES**
Method	EVA	1.228	0.269	10.292	216	0.000	9.670	1.4
	EVNA			10.292	213.2	0.000	9.670	
Concept	EVA	2.856	0.092	13.544	216	0.000	10.835	1.8
	EVNA			13.544	212.336	0.000	10.835	
Total	EVA	2.406	0.122	11.980	216	0.000	20.505	1.6
	EVNA			11.980	212.357	0.000	20.505	

Keys:
- 2-T : 2-tailed
- Sig. : Significant value or p-value
- df : Degree of Freedom
- MD : Mean Difference
- EVA : Equal variances assumed
- EVNA : Equal variances not assumed
- ES : Effect Size

From Tables 4.21 and 4.22 there was a significant difference between the means of the method scores of the control group ($M = 16.87, SD = 6.53$) and the experimental group ($M = 26.54, SD = 7.32$); $t(216) = 10.292, p = 0.000 < 0.05$. Also, the effect size of 1.4 is large and is means that the mean of the experimental group is 1.4 standard deviations above that of the control group. The effect size of 1.4 also shows that the mean of the experimental group is 91.9th percentile of the control group which also indicates a non-overlap of 68.1% in the scores of the two distributions. The results suggest that the experimental group performed better on the methods of solving first-order PDEs than the control group.

Again from Tables 4.21 and 4.22, there was a significant difference between the means of the concept scores of the control group ($M = 20.24, SD = 5.5$) and the experimental group ($M = 31.07, SD = 6.28$); $t(216) = 13.544, p = 0.000 < 0.05$. The effect size of 1.8 is large and it means that the mean of the experimental group is 1.8 standard deviations above that of the control group. This also means that the mean of the experimental group is 96.4th percentile of the control group. It also indicates a non-overlap of 77.4% in the scores of the two distributions. The results suggest that the mean difference of 10.835 between the control and the experimental group was significant and that the experimental group had a better understanding of the concept presented than their colleagues in the control group.

Again, from Tables 4.21 and 4.22 there was a significant difference between the means of the total scores of the control group ($M = 37.11, SD = 11.78$) and the experimental group ($M = 57.61, SD = 13.44$); $t(180) = 11.294, p = 0.000 < 0.05$. Also, the effect size of 1.6 is large and it indicates that the mean of the experimental group is 1.6 standard deviations above that of the control group. It also indicates that the mean of the experimental group is 94.5th percentile of the control group with a non-overlap of 73.1% in the scores of the two distributions. The results suggest that in total, the performance of the students in the experimental group was better than their colleagues in the control group.

Phase 2 Test Scores, 2017/2018

Table 4.23 displays the descriptive statistics for the test scores of phase 2 for 2017/2018 academic year group while Table 4.24 displays the independent sample t-test for the equality of means of the 2017/2018 academic year phase 2 test results. A total of 182 students with 91 students in each group were selected for the study.

Table 4.23 Descriptive Statistics of 2017/2018 Phase 2 Test Scores

Scores	Groups	N	Mean	Var.	Min.	Max.
Method	Control	91	31.97	46.143	17	49
	Experimental	91	35.75	50.835	21	49
Concept	Control	91	28.07	20.551	18	38
	Experimental	91	37.84	22.606	28	48
Total	Control	91	60.03	125.877	35	87
	Experimental	91	73.58	126.313	51	96

From Table 4.24, the significant values of the Levene's test for equality of variances for the method, concept and total scores are 0.478, 0.601 and 0.948 respectively. Since the significant values of these scores are greater than 0.05, the results in the first row for their t-test was used.

Table 4.24 Independent Sample T-Test of 2017/2018 Phase 2 Test Scores

		Levene's Test for Equality of Vars.		T-Test for Equality of Means				
Scores		**F**	**Sig.**	**t**	**df**	**Sig.(2-T)**	**MD**	**ES**
Method	EVA	0.506	0.478	3.662	180	0.000	3.780	0.5
	EVNA			3.662	179.58	0.000	3.780	
Concept	EVA	0.274	0.601	14.186	180	0.000	9.769	2.1
	EVNA			14.186	179.593	0.000	9.769	
Total	EVA	0.004	0.948	8.139	180	0.000	13.549	1.2
	EVNA			8.139	179.999	0.000	13.549	

Keys:
- 2-T : 2-tailed
- Sig. : Significant value or p-value
- df : Degree of Freedom
- MD : Mean Difference
- EVA : Equal variances assumed
- EVNA : Equal variances not assumed
- ES : Effect Size

From Table 4.23, the average method score for the control group is 31.97 while that of the experimental group is 35.75. The variances for the control and the experimental group respectively are 46.143 (standard deviation of 6.79) and 50.835 (standard deviation of 7.13). The minimum and maximum method scores of the control group are 17 and

49 respectively while that of the experimental group are 21 and 49 respectively. From Table 4.24, the mean difference of the method scores between the control group and the experimental group is 3.78. From Tables 4.23 and 4.24 there was a significant difference between the means of the method scores of the control group ($M = 31.97, SD = 6.7929$) and the experimental group ($M = 35.75, SD = 7.1299$); $t(180) = 3.662, p = 0.000 < 0.05$. Also, an effect size of 0.5 is medium according to Cohen (1988) and it indicates that the mean of the experimental group is 0.5 standard deviation above that of the control group. It also shows that the mean of the experimental group is 69th percentile of the control group and it indicates a non-overlap of 33.0% in the scores of the two distributions. The effect size of 0.5 is still significant and so, it indicates that the IO-IA have a positive impact in methods students used in solving PDEs problems. The results suggest that the experimental group performed better on the methods of solving first-order PDEs than the control group. This implies that the mean difference of 3.78 is significant and so there was an improvement in the method of solving PDEs of the students in the experimental group than those in the control group.

From Table 4.23 the average concept score of the control group is 28.07 while that of the experimental group is 37.84. The variances for the control and experimental group respectively are 20.551 (standard deviation of 4.53) and 22.606 (standard deviation of 4.75). The minimum and maximum concept scores for the control group are 18 and 38 respectively while that of the experimental group are 28 and 48 respectively. From Table 4.24 the mean difference between the control and the experimental group is 9.769. Again from Tables 4.23 and 4.24, there was a significant difference between the means of the concept scores of the control group ($M = 28.07, SD = 4.5333$) and the experimental group ($M = 37.84, SD = 4.7546$); $t(180) = 14.184, p = 0.000 < 0.05$. Also, the effect size of 2.1 is very large and it indicates that the mean of the experimental group is 2.1 standard deviations above that of the control group. It also indicates that the mean of the experimental group is above 97.7th percentile of the control group and shows a non-overlap of over 81.1% of the scores of the experimental and the control group. The result suggests that the mean difference of 9.769 between the control and the experimental group was significant and that the experimental group had a better understanding of the concept of first-order PDEs than their colleagues in the control group.

From Table 4.23 the average total scores of the control and experimental group respectively are 60.03 and 73.58. The variances for the control and experimental group are 125.877 (standard deviation of 11.22) and 126.313 (standard deviation of 11.24) respectively. The minimum and maximum total score of the control group are 35 and 87 respectively while that of the experimental group are 51 and 96 respectively. From table 4.24, the mean difference between the control and experimental group is 13.549. Again, from Tables 4.23 and 4.24 there was a significant difference between the means of the total scores of the control group ($M = 60.03, SD = 11.2195$) and the experimental group ($M = 73.58, SD = 11.2389$); $t(180) = 8.139, p = 0.000 < 0.05$. In addition, an effect size of 1.2 is large and it indicates that the mean of the experimental group is 1.2 standard deviations above that of the control group. It also shows that the mean of the experimental group is 88th percentile of the controls group with a non-overlap of 62.2% of the scores of the two distributions. The results suggest that in total, the performance of the students in the experimental group was better than their colleagues in the control group. Hence the performance of the students in the experimental group is better than those in the control group.

Phase 3 Test Scores, 2016/2017

A total of 218 students with 109 in each group (see Table 4.25) were selected for the study for phase 3 of the 2016/2017 academic year. Table 4.25 displays the descriptive statistics of the results from the confirmation test for the 2016/2017 academic year while Table 4.26 displays the independent sample t-test for equality of means for the two groups.

Table 4.25 Descriptive Statistics of 2016/2017 Phase 3 Test Scores

Scores	Groups	N	Mean	Var.	Min.	Max.
Method	Control	109	20.31	37.735	7	34
	Experimental	109	26.62	28.403	16	42
Concept	Control	109	18.65	10.192	12	25
	Experimental	109	34.39	6.924	28	40
Total	Control	109	38.96	70.425	20	55
	Experimental	109	61.01	54.083	46	77

From Table 4.26, the significant values for the Levene's test for equality of variances for the method, control and total scores are 0.125, 0.064 and 0.114 respectively. Since these values are greater than 0.05, it means the variability in the variances of the control and experimental group in each case is not significant. So, the result in the first row for the t-test in each score was used.

Table 4.26 Independent Sample T-Test of 2016/2017 Phase 3 Test Scores

		Levene's Test for Equality of Vars.		T-Test for Equality of Means				
Scores		**F**	**Sig.**	**t**	**df**	**Sig.(2-T)**	**MD**	**ES**
Method	EVA	2.372	0.125	8.103	216	0.000	6.312	1.1
	EVNA			8.103	211.784	0.000	6.312	
Concept	EVA	3.467	0.064	39.705	216	0.000	15.734	5.4
	EVNA			39.705	208.403	0.000	15.734	
Total	EVA	2.521	0.114	20.624	216	0.000	22.046	2.8
	EVNA			20.627	212.342	0.000	22.046	

Keys:
- 2-T : 2-tailed
- Sig. : Significant value or p-value
- df : Degree of Freedom
- MD : Mean Difference
- EVA : Equal variances assumed
- EVNA : Equal variances not assumed
- ES : Effect size

From Table 4.25, the minimum and maximum method scores for the control group respectively are 7 and 34 while that of the experimental group are 16 and 42 respectively. The mean method scores of the control and experimental group are 20.31 and 26.62 respectively.

The variances for the control and the experimental groups are 37.735 (standard deviation of 6.11) and 28.403 (standard deviation of 5.16) respectively. From Table 4.26, the mean difference between the control and the experimental group is 6.312. From Tables 4.25 and 4.26 there was a significant difference between the means of the method scores of the control group ($M = 20.31, SD = 6.1429$) and the experimental group ($M = 26.62, SD = 5.3294$); $t(216) = 8.103, p = 0.000 < 0.05$. Also, the effect size of 1.1 is large and it shows that the mean of the experimental group is 1.1 standard deviations above that of the control group. It also indicates that the mean of the experimental group is 86th percentile of the control group and sows a non-overlap of 58.9% of the scores of the two distributions. The results suggest that the experimental group performed better on the methods of solving first-order PDEs than the control group.

The minimum and maximum concept scores obtained by the control group are 12 and 25 respectively while that obtained by the experimental group are 28 and 40 respectively. The average concept scores for the control and the experimental group are 18.65 and 34.39 respectively. The variances of the control and experimental group are 10.192 (standard deviation of 3.19) and 6.924 (standard deviation of 2.63) respectively (see Table 4.25). From Table 4.26, the mean difference between the control and experimental group is 15.734. From Tables 4.25 and 4.26, there was a significant difference between the means of the concept scores of the control group ($M = 18.65, SD = 3.1925$) and the experimental group ($M = 34.39, SD = 2.6313$); $t(216) = 39.705, p = 0.000 < 0.05$. The effect size of 5.4 is very large and it indicates that the mean of the experimental group is 5.4 standard deviations above that of the control group. It also indicates that the mean of the experimental group is above 97.7th percentile of the control group with a non-overlap of over 81.1% of the scores of the two distributions. The results suggest that the mean difference of 15.734 between the control and the experimental group was significant. Hence the students in the experimental group had a better understanding of the concept of second-order PDEs and its applications than their colleagues in the control group.

From Table 4.25, the minimum and maximum total scores of the control group are 20 and 55 respectively while that of the experimental group are respectively 46 and 77. The average total scores of the control and experimental group respectively are 38.96 and 61.01. The variances for the control and experimental group respectively are 70.425 (standard deviation of 8.39) and 54.083 (standard deviation of 7.35). The mean difference between the control and the experimental is 22.046. Again, from Tables 4.25 and 4.26 there was a significant difference between the means of the total scores of the control group ($M = 38.96, SD = 8.3920$) and the experimental group ($M = 61.01, SD = 7.3541$); $t(216) = 20.624, p = 0.000 < 0.05$. The effect size of 2.8 is very large and it means that the mean of the experimental group is 2.8 standard deviations above that of the control group. It also means that the mean of the experimental group is above 97.7th percentile of the control group with a non-overlap of over 81.1% of the scores of the two distributions. The results suggest that the performance of the students in the experimental group on second-order PDEs for 2016/2017 academic year was better than their colleagues in the control group.

Phase 3 Test Scores, 2017/2018

A total of 182 students consisting of 91 in each group were selected for the study for the third phase in the 2017/2018 academic year. Table 4.27 displays the descriptive statistics of the confirmation test for phase 3 of the 2017/2018 academic year while Table 4.28 displays the independent sample t-test for equality of means.

Table 4.27 Descriptive Statistics of 2017/2018 Phase 3 Test Scores

Scores	Groups	N	Mean	Var.	Min.	Max.
Method	Control	91	30.29	54.829	14	45
	Experimental	91	35.95	49.941	19	50
Concept	Control	91	24.32	30.486	13	35
	Experimental	91	37.23	29.179	25	49
Total	Control	91	54.66	91.508	28	74
	Experimental	91	73.18	69.102	50	92

From Table 4.28, the significant values for the Levene's test for equality of variances of the method, concept and total scores are 0.540, 0.718 and 0.114 respectively. Since these values are greater than 0.05, the first row of the t-test for equality of means for the various scores was used.

From Table 4.27, the minimum and maximum method score of the control group are 14 and 45 respectively while that of the experimental group are 19 and 50 respectively. The average method scores of the control and experimental group respectively are 30.29 and 35.95 with a mean difference of 5.659 (see Table 4.28). From Table 4.27, the variances of the control and experimental group are 54.829 (standard deviation of 7.40) and 49.941 (standard deviation of 7.07) respectively.

Table 4.28 Independent Sample T-Test of 2017/2018 Phase 3 Test Scores

		Levene's Test for Equality of Vars.		**T-Test for Equality of Means**				
Scores		**F**	**Sig.**	**t**	**df**	**Sig.(2-T)**	**MD**	**ES**
Method	EVA	0.378	0.540	5.274	180	0.000	5.659	0.8
	EVNA			5.274	179.609	0.000	5.659	
Concept	EVA	0.131	0.718	15.946	180	0.000	12.912	2.4
	EVNA			15.946	179.914	0.000	12.912	
Total	EVA	2.526	0.114	13.979	180	0.000	18.571	2.1
	EVNA			13.979	176.564	0.000	18.571	

Keys:

	2-T	:	2-tailed
	Sig.	:	Significant value or p-value
	df	:	Degree of Freedom
	MD	:	Mean Difference
	EVA	:	Equal variances assumed
	EVNA	:	Equal variances not assumed
	ES	:	Effect Size

From Tables 4.27 and 4.28 there was a significant difference between the means of the method scores of the control group ($M = 30.29, SD = 7.4047$) and the experimental group ($M = 35.95, SD = 7.0669$); $t(180) = 5.274, p = 0.000 < 0.05$. The effect size of 0.8 is large and it means that the mean of the experimental group is 0.8 standard deviation above that of the control group. It also means that the mean of the experimental group is 79th percentile of the control group with a non-overlap of 47.4% of the scores of the two distributions. The results suggest that the students in the experimental group had a better understanding of the method of solving second-order PDEs than those in the control group.

In the case of the concept scores, the minimum and maximum scores of the control group are 13 and 35 respectively while that of the experimental group are 25 and 49 respectively (see Table 4.27). From Table 4.27, the average concept scores of the control and experimental group respectively are 24.32 and 37.23. The variances of the control and experimental group are 30.486 (standard deviation of 5.52) and 29.179 (standard deviation of 5.40) respectively. From Tables 4.27 and (4.28), there was a significant difference between the means of the concept scores of the control group ($M = 24.32, SD = 5.5214$) and the experimental group ($M = 37.23, SD = 5.4018$); $t(180) = 15.946, p = 0.000 < 0.05$. Also, the effect size of 2.4 is very large and it indicates that the mean of the experimental group is 2.4 standard deviations above that of the control group. It again means that the mean of the experimental group is above 97.7th percentile of the control group with a non-overlap of over 81.1% of the scores of the two distributions. The results suggest that the mean difference of 12.912 between the control and the experimental group was significant and that the students in the experimental group had a better understanding of the concept of second-order PDEs than those in the control group.

The minimum and maximum total score of the control group from Table 4.27 are 28 and 74 respectively while that of the experimental group are 50 and 92 respectively. The average total score of the control and the experimental group are 54.66 and 73.18 respectively. The variances of the control and experimental group are 91.508 (standard deviation of 9.57) and 69.102 (standard deviation of 8.31) as indicated in Table 4.27. Again, from Tables 4.27 and (4.28) there was a significant difference between the means of the total scores of the control group ($M = 54.66, SD = 9.5660$) and the experimental group ($M = 73.18, SD = 8.3128$); $t(180) = 13.979, p = 0.000 < 0.05$. Also, the effect size of 2.1 is large and it indicates that the mean of the experimental group is 2.1 standard deviations above that of the control group. Again, it indicates that the mean of the experimental group is 97.7th percentile of the control group with a non-overlap of 81.1% of the scores of the two distributions. The results suggest that the performance of the students in the experimental group on second-order PDEs for 2017/2018 academic year was better than their colleagues in the control group.

4.5 Challenges Associated with the use of the IO-IA

Research Question 3: What are the challenges associated with the use of the Inquiry-Oriented Instructional Approach (IO-IA) in the teaching and learning of PDEs in the undergraduate level?

Research question three sought to find out the challenges associated with the use of the Inquiry-Oriented Instructional Approach (IO-IA) in teaching and learning of PDEs in the undergraduate level. In order to answer this question, data collected from the observations were analysed. It was realized from the study that, even though the IO-IA provided opportunities for students to build the conceptual understanding of the PDEs, the implementation presented a number of challenges. The major challenges associated with the implementation of the IO-IA are presented below and the detailed discussions are presented in chapter five.

1. Inability of the instructor to present real-life applications in most of the aspects of the lesson.

2. **Time Constraints**: The IO-IA is time consuming and so it requires a lot of time during its implementation.

3. **Inadequate Motivation**: Motivation which was a factor is also a challenge to the instructor. Sufficient motivation is needed for the students to be able to conduct investigation or engage in the inquiry process.

4. **Inadequate ICT tools and Simulation Software**: Another major challenge to the

implementation of the IO-IA is the inadequate ICT tools and simulation software.

5. **Lack of Investigation Techniques**: For the students to involve in meaningful investigation during the inquiry process, they must possess the ability to perform the task required.

6. **Students' inability to apply pre-requisite Knowledge**: One of the major factors of teaching and learning of PDEs was students ability to recall and apply pre-requisite knowledge. This led to the students' inability to apply it into the learning of PDEs.

7. **Class Management**: In the inquiry-based teaching, students are allowed to come out with their own ideas and processes.

8. **Assessment**: The progressive assessment of individual students' group from investigation through formulation of inquiry questions, conduction of the investigation and formulation of models to articulate answer in detailed was a major challenge.

Chapter 5

DISCUSSION

5.1 Overview

This chapter discussions on the results obtained from the study. The results and findings on each research question were intensively discussed and theorems or conclusions from other researchers were compared to the findings.

5.1.1 Factors Affecting Teaching and Learning of PDEs

Research Question 1: What factors affect teaching and learning partial differential equations (PDEs) and the challenges that associated with these factors in the undergraduate level?

Research question one sought to find the factors that affect teaching and learning of PDEs in the undergraduate level and the challenges associated with these factors. Table 4.2 presents the factors that affect teaching and learning of partial differential equations. The factors affecting teaching and learning of PDEs was put into four categories: pedagogical, conceptual, technological and modelling factors. Instructional strategies and methods, motivation and socio-economic factors were classified under pedagogical factors. Student's ability to apply pre-requisite knowledge and computational ability were classified as conceptual factors. Availability of ICT tools and computer applications to mathematics were classified as technological factors. Comprehension was categorized as a modelling factor. Computational ability and computer applications to mathematics were also considered as modelling factors as well since they are also required for modelling processes.

Pedagogical Factors

1. **Instructional strategies and methods**: According to the students, a very good instructional strategy and method of teaching together with relevant teaching/learning materials aid conceptual understanding and so it is the most essential aspect of the teaching-learning process. It was the most influential factor (see Table 4.2).

   The instructional strategies and methods include instructional approaches and methods used during lesson delivery as well as teaching/learning materials (TLMs) used. An effective instructional strategy provides meaningful and authentic learning activities to enable students construct or come out of their own methods as well as understanding of the subject matter under consideration. And this goes a long way to shape students' learning progress and accomplishment. Students were of the opinion that instructional strategies and methods can positively or negatively affect their learning process. They were of the view that, students' performance does not only rely on the student's hard work but also as a result of good instructional approach during lesson delivery by the teacher. The type of instructional strategy employed by the teacher may help the students to construct their own ideas which will result in the improvement in their conceptual understanding of the partial differential equations, a situation, the IO-IA was able to achieve in this study. This is in line with Belhu (2017) and Gunaseelan and Pahhanivelu (2016) view that instructional strategies and methods provides students with learning situations where they can develop and apply higher-order operations which are critical for mathematics achievements. It also confirms Bloom (1976) assertion that "instructional strategies where students actively participate in their own learning is critical for success". According to Rohrer and Pashler (2007), because students forget much of what they learn, they should be made to benefit from learning strategies that yields long-lasting knowledge. The findings from the study suggest that effective instructional strategies improve students learning process and helps students to reason critically that leads to very good acquisition of knowledge.

The challenges associated with appropriate instructional strategies and methods as indicated in Table 4.3 are lack of conceptual understanding of PDEs on the part of the students, the inability of students to apply concept learnt to other areas, short-term memory on the side of students, poor knowledge acquisition, poor students' performance on PDEs and also de-motivates students to learn. Lack of conceptual understanding leads to short-term memory on the part of students due to poor knowledge acquisition. The results of which are poor performance which also demotivates students in the learning process.

2. **Motivation**: In the current educational system of Ghana, the challenges associated with the study of mathematics requires disciplined study, concentration and motivation. The ability to relate what is learnt to real-life applications could be enhance if students are well motivated during the teaching and learning process. The challenges lack of motivation brings to the learning process of students could be minimized if students are motivated. It is therefore imperative that teachers do not underestimate motivation. It is no doubt that students should be motivated enough to learn mathematics. Edelson et al (1999) was of the view that students must be sufficiently motivated in order for them to be engaged in inquiry-based learning. The engagement phase of the 5E instructional model requires that teachers motivate to arouse their interest of the new topic (Duran & Duran, 2004). The data gathered from the interview suggests that motivation is one of the key factors affecting teaching and learning of PDEs (see Table 4.2).

The process of motivation stems from stimulation which is turns followed by an emotional reaction that lead to a specific behaviour response (Bed, 2017). The task of the teacher as an instructor is to create an environment for students to actively engage in mathematical thinking and reasoning activities. These according to Carr (1996) help students see mathematics as an area which requires exploration, conjecture, representation, generalization, verification and reflection. The students during the interview confirmed that the learning environment together with their engagement in the lesson through modelling which leads to the formation of differential equations served as a motivational ground for them to reason logically and analytically. This confirms Duran and Duran (2004) assertion that the engagement phase of the 5E instructional model which is a student - centred phase should be a motivational period that can create a desire to learn more about the upcoming topic. In so doing, there is the need to include discrepant events, demonstrations, questioning, importance of the topic, application to the real-life problems and/or graphic organizers to create interest and generate curiosity (Duran & Duran, 2004). According to Gunaseelan and Pahnivelu (2016), "mathematics requires highly motivated students because students' need to reason logically, formulate mathematical ideas and interpret them, deal with issues on mathematics, understand the concepts and solve the problems" and relate the content learnt to real-life situation. It was obvious from the responses of the students that motivation is one of the important factors in high failure in mathematics and there is the need to attach importance to it.

Low interest in mathematics, discouragement, poor performance, and low self-efficacy in mathematics were some of the challenges associated of lack of motivation in students. Most students perceived mathematics as a very difficult course. For this reason, students needs to be highly motivated to arouse their interest in mathematics. Inadequate motivation leads to low interest and discouragement in mathematics. When students are discouraged in their course to pursue mathematics, it leads low self-efficacy which in turn leads to poor academic performance in the subject.

3. **Socio-Economic Factors**: The socio-economic factors as stated earlier in this study refers to the financial constraints on students and their parents alike. The data gathered from the interview show that socio-economic factors is a contributing

factor in the teaching and learning of PDEs (see Table 4.2). The socio-economic factors were not included as one of the questions for the interview but was considered as a factor by 40% of the respondents. Students were of the view that socio-economic factor (financial constraints) affects their learning process. According to the students, these financial constraints makes them loose concentration during lessons. Those who considered this as a factor stated that coming to school with all these financial challenges does not only de-motivate you but makes you lose concentration during teaching and learning processes.

This factor affected the engagement and the exploration stage of the 5E instructional model since it de-motivates the students as well. According to Howie (2006) and Gunaseelan and Pahanivelu (2016), ”socio-economic status is determined to be a predictor of mathematics achievement” and affects students' performance in mathematics. According to Ma and Klinger (2000) cited in Gunaseelan and Pahanivelu (2016), socio-economic status was found significant in primary mathematics and science achievement scores. Bed (2017) was of the view that economics status of parents is one an important aspect of parents' factor in studying mathematics and it determine the children's education. Belhu (2017) also considered income as a major factor affecting the conceptual understanding of mathematics. The challenges associated with this factor are lack of concentration on the part of the students, de-motivation and poor mathematics performance. It is obvious that, if students loose concentration in the classroom, it will be difficult to get the conceptual understanding of the subject matter and will definitely lead to poor academic performance in the subject.

Conceptual Factors

1. **Students' ability to recall and apply pre-requisite knowledge**: The study revealed that, most of the students were unable to recall and apply previous relevant knowledge which are pre-requisite to partial differential equations (PDEs). That is, most of the

students were unable to apply their knowledge on ordinary differential equations, differential and integral calculus for example, in finding solutions to partial differential equations (see Table 4.2). This actually affected both the exploration and the elaboration as well as the evaluation stage of the 5E model of instruction and served as a setback in the entire inquiry process. Forgetfulness is the major key in this scenario. For example, most of the students could barely recall and apply most of the concept they learnt on Ordinary Differential Equations (ODEs). Most of the students for example could not link boundary-value problems in ordinary differential equations to the partial differential equations when its application was needed in solving heat and wave problems. Again, some of the students could not realize they could use separation of variables in solving the resulting characteristic

$$\frac{dx}{x} = \frac{dy}{y} = \frac{du}{xe^{-u}}$$

of the partial differential equation

$$xu_x + yu_y = xe^{-}u$$

Students were of the view that, they could have retained most of the concept learnt on ODEs if PDEs follows ODEs in the next semester. Other areas students have forgotten include differential and integral calculus, trigonometric functions and their identities as well as some concept of geometry. For example, almost all the students could not realize that, in series which appeared during the process of solving heat and wave equations,

$$e^{in\pi} = e^{-in\pi} = (-1)^n$$

because they have forgotten their lessons on Euler formula for polar coordinates and how they are related to trigonometric functions. So, resulting integrations like

$$\int_{\pi}^{\pi} (e^{-in\pi x} - e^{in\pi x})dx$$

is zero (0) but some obtained results like

$$\int_{\pi}^{\pi} (e^{-in\pi x} - e^{in\pi x})dx = \frac{2}{in}\left(e^{in\pi} + e^{-in\pi}\right)$$

and many more different answers.

According to the students, their inability to recall previous knowledge on ordinary differential equations (ODEs) for example is due to the fact that, PDEs is learnt one and half years after ODEs. The situation could have been improved if PDEs follows immediately after ODEs. Rohrer and Pashler (2007) attributed students' inability to retain what they have learnt to duration of the study and the temporal distribution of the study time and this confirms the students' claim. The main challenge that this factor (see Table 4.3) imposes on the student is that, they are unable to apply this requisite knowledge in solving either a given problem or the problems that comes out from the models. It also made it difficult for them to deduce some of the models during the inquiry process. Most of the students have challenges in solving Ordinary Differential equations which is an immediate pre-requisite for partial differential equations. Also, some of the students have challenges in differentiation and integration of certain key functions like trigonometric and inverse trigonometric functions, hyperbolic and inverse hyperbolic functions, exponential, logarithmic functions and combinations of these functions.

2. **Computational Ability**: Computational ability which was referred to as arithmetic ability by Gunaseelen and pazhanivelu (2016), consist of skills and techniques such as manipulation of mathematical knowledge and concepts in the manner that

transforms their meaning and implications. It is the third most influential factor according to the study (see Table 4.2). In the study, it was realized that, most of the students were unable to solve some of the partial differential equations because they could not integrate the resulting characteristic equation due to its complex nature. For example, some of the trigonometric functions have to be written in different forms (identities) or needs to be written in exponential form in order for their integration to be easy or possible. And students' inability to re-write these trigonometric functions in their corresponding identities makes it difficult to integrate them.

In finding solutions to first-order quasilinear PDEs for example, it is very important to recognize how to re-write the resulting characteristic equations in different form ("algebraic manipulation") so that it can easily be solved. For example, students in solving a quasilinear PDEs after coming with the characteristic equation

$$\frac{dx}{x(y^2+u)} = \frac{dy}{-y(x^2+u)} = \frac{du}{(x^2-y^2)u}$$

could not recognized that for any equivalent fractions

$$\frac{a}{b} = \frac{c}{d}$$

it is always true that

$$\frac{a+c}{b+d} = \frac{a}{b} = \frac{c}{d} \text{ and } \frac{a-c}{b-d} = \frac{c-a}{d-b}$$

The students must be able to re-write or transform some trigonometric functions to exponential functions or an equivalent trigonometric function in order to integrate or differentiate. They could hardly see that some functions need to be re-written or rationalize in order to make integration simple. This confirms Gunaseelan and Pazhanivelu (2016) view that "computational ability (arithmetic ability) allow students to interpret, analyse, synthesize, generalize or hypothesize the facts and ideas of mathematics". The lack of computational ability affected the progress of the elaboration stage of the 5E instructional model. To help in minimizing the challenges that associated with this factor, some of the concepts required to do these computations were reviewed as they unfold in the process. The computational ability is also classified as a modelling factor because after the model has been developed through the inquiry process, there is the need to solve the model as well. You also need the computational ability to formulate the model. For example, if the model is about the rate of change of one variable with respect to another variable, then one's ability to compute the derivative of integral of functions is needed.

The inquiry-learning process requires that, students have problem-solving skills and should be able to some extent, self-instruct themselves. But the challenges students face due to their computational ability is lack of problem-solving skills and ineffective self-instructions. Students during the inquiry-process are expected to do some basic computations which leads to the formulation of the models, but these challenges associated with their inability to perform some computations hinders their progress during the learning process.

Technological Factors

1. **Availability of I. C. T. Tools**: An ICT driven teaching and learning is very important in our instructional delivery systems. Simulation and graphical representation of concepts are very important during teaching and learning in the classroom. But the inadequate ICT tools have made demonstration of ideas and its subsequent practice by the students very difficult. Every demonstration in a classroom should be followed by hands-on activity but the inadequate ICT tools serves as a barrier to these hands-on activities. Students' view on the availability of the ICT tools (see Table 4.2) was that instructors' demonstration should be followed by hands-on activity but unfortunately that has not been the case. They were of the view that,

the instructor's demonstration of the simulations on the projector should have been followed by hands on but unfortunately only a few students have access to laptops which gave just of few of them to do immediate hands on.

Majority of the students were unable to do hands-on activity during and after the instructional period because the ICT tools (computers and programmable calculators) available were in a poor state and so most of the students have challenges in using these tools to investigate solutions of the PDEs which could have enhance their conceptual understanding and reduce heavy computational load. The inadequate ICT tools affected the exploration stage of the inquiry process. Because there were not enough computers for the students to simulate most of the processes, two or three students have to use one computer and this also affect the output of the study.

Inadequate ICT tools, poor state of available ICT tools, lack of hands-on activity and difficulties in using the ICT tools were some of the challenges associated with availability of ICT tools. Because the ICT tools available were inadequate and those which were available were not in good state, most of the students were not able to do hands-on activity during instructional delivery.

2. **Computer Application for Mathematics**: The use of computer applications in simulation and graphing of solutions help students to understand the concept underlying PDEs. It is the seventh most influential factor in the teaching and learning of PDEs (see Table 4.2). However, the difficulties in getting these relevant applications for teaching and learning contributes to the setbacks in the teaching and learning process. Students were of the view that they lack these applications that could have been used to simulate or demonstrate most of the concept in the absence of real-life demonstrations. This also has contributed to the difficulties they faced in understanding of concepts being taught. They stated that the use of computer applications for mathematics helps in simulation of solutions and nature of PDEs and so have influence positively in the teaching and learning of PDEs.

It was obvious from the interaction (interview) with the students that, even though students took a course on computer tools for mathematics (Octave, GeoGebra and Microsoft Excel), most of them have forgotten how to apply or use these software to simulate or draw graphs of the solutions to the PDEs. In this study, students were limited to GeoGebra and Microsoft Excel which cannot at all times simulate the nature of the partial differential equations and their solutions for better understanding. This in the study, also affected the exploration stage of the study as students were finding it difficult to visualize most of the processes. By using educational software (computer application), teachers' roles are significantly changed since the very organization of different classes changes (didactics-methodology components include methods, forms, principles and organizing of the teaching process with all phases of the teaching process: the processing of new materials, repetition, exercises, and testing of taught and acquired materials) (Metodički, 2011). The challenges associated with lack of computer applications to mathematics are inadequate classroom practices and students' inability to present most especially three-dimensional graphical representation of solutions.

Modelling Factor

Comprehension: Comprehension in this study refers to one's ability to read and understand the problem presented. It is the 5th major factor affecting teaching and learning of PDEs (see Table 4.2). It was realized in the study that, some of the students have difficulties in the interpretation and analysis of the questions when it comes to application questions. According to the students, they sometimes have difficulties in interpreting some of the problems most especially when it comes to problems involving application of the concept learnt. They have challenges in comprehending the problems which leads to their inability to form the mathematical model out of them.

This is obviously the reason why students most of the time avoid application questions.

Students' inability to comprehend what they read really affect their understanding of PDEs. These challenges manifested during the exploration and elaboration phases of the 5E instructional model. At the exploration stage, some of the students were finding it difficult to interpret and analyse the activities and how to translate them to mathematical statements. At the elaboration phase, some of them have challenges in comprehending the given problem which requires the application of concept learnt. Research have shown that, how students experience situations affects their problem-solving processes and so the comprehension of the situation is a relevant factor when solving word problems or real-life application problems (Belhu, 2017; DeCorte et al., 1985). According to Österholm (2006), "there is a complex relationship between comprehension and problem-solving". Polya (1990) cited in Mustafa (2017), found out that reading comprehension process is followed by planning for solution and planning implementation steps. So, students' inability to comprehend the problem given results in difficulties in finding solution to that problem. This factor affected both exploration and the elaboration stage of the teaching and learning process. Several challenges are associated with the students inability to comprehend the problem assigned to them. A few among them is lack of understanding of the problem which hinders the students ability to interpret and analyse the problem and lack of self-efficacy. Self-efficacy reflects confidence in the ability to exert control over one's own motivation, behaviour, and social environment and if the students lack it, then he loses his or confidence.

Other modelling factor includes computer application to Mathematics. Some models require several iterations in order to get the rule and variables for the model and in such cases, one's ability to use computer applications is a necessity or relevant. Apart from that, some of the models cannot be solve analytically, there is the need to apply numerical approach which may require several iterations or simulations. The need for computer applications is relevant in such situations as well.

5.1.2 Impact of the use of IO-IA on teaching and learning of PDEs

Research Question 2: How does the use of inquiry-oriented instructional approach (IO-IA) impact on teaching and learning PDEs in the undergraduate level?

Research question two sought to find out how the use of the inquiry-oriented instructional approach (IO-IA) impact on the teaching and learning of PDEs. In all the three phases of the treatment, the experimental group performed better than the control group in the method they employed in solving partial differential equations (PDEs) (see tables 4.18, 4.20, 4.22, 4.24, 4.26 and 4.28). The p-values were all less than 0.05 indicating a significant difference in the performance of the two groups. Most of the students in the experimental group were able to identify the appropriate method of solving the given PDEs as compared to those in the control group. For the concepts of PDEs, the p-value of 0.000 in all the phases (see tables 4.18, 4.20, 4.22, 4.24, 4.26 and 4.28) indicate that the experimental group had a better conceptual understanding than their colleagues in the control group. The results showed that most of the students in the experimental group were able to interpret the solutions to the PDEs as compared to their colleagues in the control group. This show that the IO-IA had a positive impact on the conceptual understanding of PDEs as compared to the traditional method of teaching.

The results from the study indicate that in all the phases, there were significant differences in the performance of the two groups in favour of the experimental groups. Since in all the phases, the experimental group performed better, it can therefore be confirmed that the inquiry - oriented instructional approach of teaching and learning (IO-IA) PDEs improved the performance of the students. So, IO-IA is more effective in teaching PDEs as compared to the traditional method. It can therefore be concluded that there was an improvement in the performance of the experimental groups on the methods of solving partial differential equations as well as the concept of partial differential equations. This indicates an improvement in the total performance of the experimental group over the control group.

The results confirm the findings made by several researchers in the inquiry-learning process. Friesen and Scott (2013) confirmed that inquiry-based approach to learning positively impact students' ability to understand core concepts and procedures. Also, Kwon et al (2005) and Rasmussen et al (2006) stated that the students who were taken through the inquiry-oriented approach (inquiry-oriented differential equations) regardless of their academic background and gender differences, outperformed traditionally taught comparison students on the post-test.

5.1.3 Challenges Associated with the use of IO-IA

Research Question 3: What are the challenges associated with the use of the inquiry-oriented instructional approach (IO-IA) in the teaching and learning of PDEs in the undergraduate level?

Even though there were several challenges affecting the implementation of the IO-IA, the most pertinent ones are time constraints, motivation and lack of access to ICT tools and simulation software. These challenges negatively impacted on the implementation of the IO-IA. Other challenges include lack of investigative techniques on the side of the students, students' inability to recall pre-requisite knowledge of some topics previously treated, management, assessment and response to emerging questions.

1. **Inability to present real-life applications**: The difficulties in presenting a real-life situation for student to verify the concept of PDEs was a major challenge. This challenge made presentation of systematic investigations a major challenge to the instructor and the students as well. Also, the scarcity of computer tools and simulation software that would have further helped students to visualize the outcome of their investigations also negatively affected the outcome of the inquiry process.

2. **Time Constraints**: The implementation of the IO-IA is time demanding not only in terms of instructional delivery but most especially, during the inquiry process by the students. The students needed ample time during their investigation to be able to get the right results. But unfortunately, the lecture is once a week with 3-hour duration and the instructor had 12 weeks in the semester. This means that the instructor has only 12 meetings with the learner. This is not enough for full implementation of the IO-IA For this reason, some of these investigations were carried out by the students in their halls and hostels. This did not support effective group work among students. Students did not have enough time to meet to continue the investigation due to the numerous activities students faced outside the classroom. It was realized from the study that, sometimes it takes more than a lesson for the students to come out with the findings and generalization of the task assigned them. For the instructor to be able to minimize these challenges, extra tuition was organized.

   Keiser and Lambdin (1996) research on time constraints revealed that one of the biggest frustrations associated with teaching is the difficulty in moving at a reasonable pace because of uncertainty of which lesson requires more time as well as the students' ability to quickly understand the concept being presented. To minimize the effect of time constraint on the implementation of the IO-IA, extra tuition was sometimes organized for students to catch up with the lost time. Teaching in the spirit of the current mathematics education reform movement may be highly dependent upon flexibility in class scheduling. Innovations in teaching mathematics (e.g., increased group work, writing, extended projects, and alternative forms of assessment) seem to require additional time, and new ways of thinking about using class time (Jane & Diana, 1996).

3. **Inadequate access to ICT tools and Simulation Software**: The computer laboratory where most of the simulations took place during the instructional delivery, had only 18 working computers which was used by 109/91 students. This was a setback to the progress and effectiveness of the students' investigations. According

to Farrel et al. (2007), among the numerous challenges of integrating ICT into teaching and learning in tertiary institutions are the inadequate ICT materials and limited access to computers and internet to large number of students. Sarfo and Elen (2014), on their study on the topic, “Towards an instructional design model for learning environments with limited ICT resources in higher education”, considered limited ICT resources as a major barrier to successful integration of ICT in teaching and learning. Apart from the inadequate ICT tools, it was difficult to get appropriate simulation software to help students to visualize what they were being taught. For example, it was difficult for students to generate waves and measure the wave length and amplitude with the resources available. Even though guitars and/or drums are useful real application of waves for example, the institution lack the equipment that could be used to measure the amplitude and wavelength of the waves generated. For this reason, the instructor relied on videos simulations from YouTube and other online resources to help students to visualize heat transfers and wave generation among others. Also, mathematical model requires a lot of experimentation and simulations and the inadequate ICT tools was a major setback in that direction.

ICT tools include computers, tablets and graphic calculators and other machinery that help in our computations and graphing. The simulation software on the other hand are the mathematics applications like GeoGebra, MATLAB and so on that are installed on the computers and tablets. Because of the inadequate computers in the laboratory coupling with few simulation software, it made it difficult for the researcher to simulate or demonstrate most of the concept learnt. For example, it was difficult for students to generate waves and measure the wave length and amplitude with the resources available. According to Sarfo and Ansong-Gyimah (2010), despite the passing into law, “Ghana’s ICT for Accelerated Development (ICT4AD) Policy” in 2014 by parliament, the implementation of ICT policies and infrastructure for ICT to facilitate research, teaching and learning have been without problems. These problems include poor maintenance system of ICT tools, inadequate

ICT resources (computer hardware and software and internet). For this reason, not all students have access to computers due to the high cost of ICT tools (Sarfo & Ellen, 2014). Even though guitars and/or drums are useful real application of waves for example, the institution lack the equipment that could be used to measure the amplitude of the waves generated. For this reason, the instructor had to rely on videos from YouTube and other online resources to help students to visualize heat transfers and wave generation as well.

4. **Lack of Investigation Techniques**: Scientific investigation techniques are mostly complicated and requires a high level of precision and carefulness. For this reason, the students require investigative skills in order to conduct these scientific investigations. According to Althauser (2018), constructivist approach to teaching encourages deep learning approach on the side of the learner and this can only be achieved if students are allowed to investigate on their own and comes out with solutions and models. Unfortunately, these are the qualities the students did not possess. This had a negative impact on their results since they do not have the requisite skills to formulate the right questions and organize their data. In some cases, the instructor has to intervene in these areas. For example, the instructor has to guide the students on the variables they are supposed to measure and the questions they should ask themselves which would lead to these variables. Some of the task were also framed as step-by-step questions which the students answered as they move along with the procedures (see Appendix II(a-d)). The IO-IA is student-centred especially during the exploration phase and for that matter students were to do most of the investigation by themselves to come out their own idea as described by the constructivist theory of learning. Students' lack of investigative techniques affected the exploration phase of the treatment.

5. **Students' inability to apply pre-requisite knowledge**: The students' inability to apply their pre-requisite knowledge on subject areas like Calculus and ODEs, was a major challenge. During the formulation of mathematical models, pre-requisite knowledge on some areas like ordinary differential equations (ODEs), Calculus,

geometry, linear algebra and the likes which are supposed to serve as the basis for the formulation of the mathematical models have been forgotten. For this reason, students were not able to properly formulate the mathematical models and were also finding it difficult to solve some of the questions that arouse from the investigations. Integration of some key functions in trigonometry, natural logarithms and so on have been forgotten by students. Also, it was realized from the study that, students were unable to apply their pre-requisite knowledge on data organization and results interpretation. This was a barrier to the inquiry process as students found it difficult to organize their data in order to give a meaningful interpretation to the results. This challenge affected the exploration and the elaboration phase of the 5E instructional model. According to Bed (2017), "mathematical pre-knowledge is the infrastructure as well all-round development of students in the mathematics sectors. Those students who have lack of sufficient prior knowledge did not want to learn and could not get success in the further level". Students' inability to apply pre-requisite knowledge means the instructor would have to feed the students with all that and this marred the innovativeness of the students' learning process.

6. **Inadequate Motivation**: The data gathered from the observations suggested that, inadequate motivation is a major challenge in the teaching and learning of PDEs. The nature of the IO-IA requires that students are highly motivated as compared to the traditional method of teaching. To highly motivate students, one of ways is to arouse the interest of the learner in the investigation. This mostly is done through presentation of real-life application and importance of the PDEs. But because of the difficulties in finding the real-life application of most of the concepts in our environment, arousing the interest of the students in some cases was a major challenge. Most of these real-life applications was demonstrated through short-videos from "YouTube tutorials" which fails to address the issue of hands-on experience by the students. According to Duran and Duran (2014), students must be highly motivated during the engagement phase of the 5E instructional model cycle and one of the ways is the introduction of the students to the application of

the upcoming subject matter. It was observed from the treatment that, the design could not address some of the motivational challenges adequately because of the difficulties in getting a real-life situation. For example, in the derivation of the wave equation, even though the researcher could easily get access to drums and guitar, the instrument to measure the amplitude of the vibrating string at any point on the string was not available. For this reason, the researcher resulted to online videos for the demonstration which is a hindrance when it comes to hands-on activities. So even though some sorts of motivation were provided during the study, it was not done to the expectation of the researcher due to these challenges. This challenge negatively affected both the engagement and exploration stage of the treatment.

7. **Assessment**: The various sub-groups must be assessed from time to time throughout the inquiry process in each lesson and this was a big challenge for one instructor. The role of a teacher in the assessment of students is to watch, listen, ask questions and give feedback. A role which is difficult when it comes to inquiry process with so many students. A teacher can observe only one activity at a time and the inquiry process with multiple activities makes it difficult to go along with such a role. Also, you can listen and ask questions one at a time. A task which is also difficult for the instructor in classroom in which so many students are going through an inquiry process. This challenge also reduced the intensity of the feedback to the students. It also made it difficult for the instructor to lay more emphasis on formative assessment and thus leaves the instructor to look more into summative assessment. Also, the progressive assessment of individual students' group from investigation through formulation of inquiry questions, conduction of the investigation and formulation of models to articulate answer in detailed was a major challenge. Summative assessment in inquiry process may not be the best way to assess students. This is because inquiry-based learning is not content bound only but cognitive as well. According to Gatt and Zammit (2017), summative assessment does not tend to be conducive to inquiry-based learning as inquiry skills tend to be cognitive process and not content bound. This means that the paper and pencil

made test was not the best form of assessment for the study.

8. **Class Management**: The inquiry process leads to multiplicity of investigations from different students. The multiple investigations being carried out at the same time by different sub-groups makes it difficult for the instructor to response to all emerging queries in order to provide an inquiry-based support. This problem emerges from the large number of students coupling with inadequate ICT tools. There is the need to be able to organize and manage complex and extended activities. According to Edelson, et al. (1999), students are not tasked to manage extended complex process in traditional educational activities. But scientific investigation requires planning and coordination of activities and management of resources and work products. These new tasks requires good class management because if students are unable to organize their work and manage extended process, they cannot engage in open-ended inquiry or achieve the potential of inquiry-based learning (Edelson et al., 1999). This situation requires that more than one instructor is needed during the inquiry process. To minimize these challenges, the instructor sorted the service of other personnel to assist in these processes.

5.2 Summary

Inquiry-oriented instructional approach (IO-IA) requires planning and skills in

- the provision of students with experiences in learning that stimulates curiosity which is a motivating factor when it comes to inquiry processes.
- the development of relevant questions in both the individual bases and group basis which are in a sequence that guides students to perform their own inquiry-oriented learning.
- the management of multiple students' investigations and questions so as to be able to provide an inquiry-oriented feedback to the participating students at the same time.

- the continuous assessment (formative and summative) of the progress of the students right from the exploration of the task through the inquiry process to articulating of solutions to the problem or task.
- provision of inquiry-based support to students.
- responding to students' emerging queries and giving feedback to students during the inquiry process.

The results of the study showed that the IO-IA helped students to develop conceptual understanding of PDEs. It inculcated in them the ability to reason logically and helps them to use the results obtained to make informed decisions on the real-life application of PDEs.

Chapter 6

SUMMARY OF FINDINGS, CONCLUSIONS AND RECOMMENDATIONS

6.1 Overview

This chapter consist of summary of major findings of the study, the implication of these findings, the conclusion drawn on the study, the recommendation by the researcher and suggestions for further studies.

6.2 Summary of the Study

The study investigated the impact of the inquiry-oriented instructional approach (IO-IA) on teaching and learning of partial differential equations (PDEs) in undergraduate student. The study focused on the use of IO-IA to help students develop problem-solving skills and mathematical thinking/reasoning ability so as to inculcate in them positive attitude towards PDEs in particular and mathematics in general. This study adapted the Inquiry-Oriented Differential Equations (IO-DE) project conducted by Rasmussen and Kwon (2007) on the teaching and learning of ordinary differential equations (ODEs). The study extended the inquiry-based teaching approach to PDEs as well. This study was relevant because in the field of applied sciences and economics, PDEs play a major part and hence the need for effective way of teaching it.

The selected population for the study was the entire undergraduate mathematics education students in the University of Education, Winneba (UEW). A purposive sampling technique was used to select 400 students from two consecutive academic years: 2016/2017 and 2017/2018 academic years. The selected students in both academic years were the level

400 students and were selected because PDEs is taught at that level. There were 218 level 400 students in the 2016/2017 academic year and 182 level 400 students in the 2017/2018 academic year. The selected students in each year group were put into two groups: control and experimental group. The study employed sequential explanatory design. This according to Shavelson et al (2003) will help teachers to discover, test and build theories of teaching and learning, and produce instructional tools that survive challenges found in everyday practice. It also improves students' retention. The concept underlying the IO-IA was adapted from the 5E instructional model proposed by Bybee and Landes (1990). The 5E instructional model consist of five stages: engagement, exploration, explanation, elaboration and evaluation. The study adapted the inquiry-oriented differential equations (IO-DE) project by Rasmussen and Kwon (2007). The theory behind the study was based on cognitive psychology and constructivist- learning theory. Cognitive psychology is the area of psychology that focuses on internal mental processes like thinking, decision-making, problem-solving, language, attention, and memory while the constructivist-learning theorem is based on the view that, students are active thinkers and so, they construct their own understanding from interactions with phenomena, the environment and other individuals.

Three data collection methods were used during the study: observation, test and interview (semi-structured interview). The test was conducted after each phase of the study in both academic years. After each test, a random sampling procedure was used to select five students from the experimental group for the interview. In all, a total of 30 students were interviewed in the two academic years. Observation were also made during the inquiry-process and during the marking of the confirmation test. The observations and the interview helped the researcher to identify the factors affecting teaching and learning of PDEs. The confirmation test helped the researcher to access the impact of the IO-IA on teaching and learning of PDEs and challenges in implementing IO-IA. The average score for each group were compared using the independent sample t-test for equality of means.

6.3 Summary of Major Findings of the Study

1. On the issue on the factors affecting teaching and learning of PDEs, the study identified 7 major factors.

   (a) Instructional strategies and methods: This refers to the instructional approach and the various methods used by the teacher during the lesson delivery.

   (b) Students' ability to recall and apply pre-requisite knowledge in solving PDEs: This refers to students ability to retain knowledge gain on pre-requisite courses like ordinary differential equations, differential and integral calculus, trigonometry and so on and apply them in solving PDEs.

   (c) Computational ability: This refers to the skills and techniques such as the manipulation of mathematical ideas and concepts for the transformation their studies on PDEs.

   (d) Availability of I.C.T. tools: This refers to the ICT tools like computers, graphic calculators and organizers that help the students to graph solutions of equations and simulations of equations.

   (e) Comprehension: This refers to the students ability to read and understand the problem presented to them.

   (f) Motivation: This refers to anything or information that arouse the interest and curiosity of the students to make them eager to want to know more about the subject matter.

   (g) Computer Applications for Mathematics: This refers to computer applications that helped the students to simulate solutions or behaviour of solutions of given partial differential equations so that they can visualize what is being taught in the classroom.

   (h) Other factor that was identified but was not considered as a major factor was socio-economic factors which in this case is the financial constraints on students during the entire semester which reduces their attention in the classroom during instructional hours.

2. In the investigation of the how the use of the IO-IA impacts on the teaching and learning of PDEs, it was realized that the IO-IA impacted positively on the teaching and learning of PDEs. The significant-value for t-test for equality of means in all the phases for the two academic years were found to be less than 0.05 (see Table **??**). This indicated that there were significant differences in the means of the control and the experimental groups in each of the phases. The IO-IA therefore, have a positive impact on the teaching and learning of PDEs because it improves their performance in PDEs.

3. On the issue of the challenges associated with the implementation of the IO-IA, the study identified the following challenges:

   (a) Time constraints: This refers to the instructional time allocated per week as well as the number of weeks within the semester that is required by the instructor to deliver the lesson. This by the nature of the IO-IA, will not be enough if the approach is to be used in the entire course.

   (b) Inadequate ICT tools and simulation software: This refers to inadequate computers and graphic calculators as well as computer software that is used for graphing and simulations of the equations and their solutions.

   (c) Lack of investigation techniques on the side of the students. This refers to students lack of in-depth knowledge on formulation of right inquiry questions and organization of data.

   (d) Students' inability to apply pre-requisite knowledge: This refers to students' inability to apply pre-requisite knowledge like data organization and analysis which is an area in statistics during the investigation and calculus and ordinary differential equations which are pre-requisite partial differential equations, in finding solution to models which came out of the inquiry process.

(e) Inadequate motivation: This refers to the need to arouse the interest of the students as well as to make them more curious so that they will be eager to learn more on the PDEs.

(f) Assessment: This refers to the type of assessment that must be used to assess the level of understanding of the students. The study revealed that, summative assessment is not the best way but the progressive assessment to articulate answers in detail was a great challenge.

(g) Management: This refers to the instructor's ability to assist all the students during the inquiry process and the provision of responses to students' emerging questions.

6.4 Conclusion

In our day-to-day teaching of PDEs and mathematics in general, there is the need to present the subject matter as practicable as possible. There is also the need for students to develop problem-solving skills and also think critically and analytically in order to apply what they have learnt to solve problems in our environment. According to Barrow (2006), since 1910, there have be call to move teaching and learning of science and mathematics from the traditional method of learning which lacks critical thinking and problem-solving skills to an inquiry-based teaching and learning which inculcate in students critical thinking and analysis and problem-solving skills. This study adds on to the various inquiry-based teaching and learning approaches which have conducted by several researchers (Barrow, 2006; Kwon, 2008; Rasmussen & Kwon, 2007; Rasmussen et al., 2017; National Research Council, 2000).

The study identified four categories of factors affecting teaching and learning of partial differential equations (PDEs) in particular and mathematics in general. These factors include pedagogical factors, conceptual factors, technological factors and modelling factors. The pedagogical factors include instructional strategies and methods, motivation and socio-economic factors. The conceptual factors consist of students' ability to apply pre-requisite knowledge and computational ability. The technological factors include the availability of ICT tools and computer applications to mathematics while the modelling factors consist of comprehension, computational ability and computer applications to mathematics.

The study also shown that, the inquiry-oriented instructional approach (IO-IA) provides conceptual understanding and inculcate in them critical thinking, analysis and problem-solving skills of partial differential equations. The IO-IA serves as a guided instructional sequence for the students and the reinvention of the mathematical procedures which helps students emerge from conceptual knowledge and led to a longer retention of knowledge of partial differential equations. This confirms similar studies made by Rasmussen and Kwon (2007), Kwon (2008) and National Research Council (2000) just to mention a few.

Despite these advantages of the IO-IA, the study revealed that inability of teachers to present real-life application in most of the topics in PDEs, time constraint and inadequate access to ICT tools and simulation software are some of the major challenges facing the implementation of IO-IA in teaching and learning of PDEs. Others challenges include lack of investigative techniques on the part of the students, students' inability to recall pre-requisite knowledge like ordinary differential equations, calculus, linear algebra, geometry and so on, inadequate motivation, assessment and class management.

In all, the study revealed that, the IO-IA developed the problem-solving skills among students. It inculcated in the students, positive attitude towards PDEs and help them to think critically and analytically.

6.5 Recommendations

The study recommends that:

1. Teachers should integrate inquiry-oriented instructional approach (IO-IA) in the teaching of PDEs in particular and mathematics in general.

2. Teachers should adapt measures to help students improve on their computational ability and investigation techniques so that they will be able to apply what they have learnt in subsequent topics or subject which requires these skills.

3. Teachers should find a better way of motivating students (internally and externally) so that it will serve as a springboard for them to begin investigations on the application of PDEs in particular and mathematics in real-life.

4. Teachers should teach students investigation techniques so that with little or no guidance they will be able to use the inquiry-process to come out with their own ideas.

5. Students should realize the need for constant revision of courses learnt in order to reduce forgetfulness of concepts learnt.

6. Students should work on their comprehension so that they can interpret their assign task and do same during application of concepts on real-life.

7. Students should learn to investigate real-life problems on their own and take measures to find solutions to them. This will help them to develop problem-solving skills

8. Students should also practice application questions so that they will be able to develop problem-solving skills and investigative techniques.

9. Mathematics departments of higher learning should also provide the needed ICT tools and software that will help teachers to explain the concepts and also help students during their investigations.

10. The mathematics departments in our institutions of higher learning should adopt small class size so that teachers will be able to effectively integrate IO-IA in their teaching.

11. The mathematics departments in our educational institutions should provide mathematics laboratories equipped with all relevant ICT tools and stimulation software and internet connectivity to make teaching and learning of mathematics as practicable as possible.

6.6 Suggestions for Further Studies

The following suggestions were made for further studies:

- The study should be replicated in other subject areas like vectors and mechanics, differential and integral calculus as well.
- A further study should be done on the causes of students' inability to recall previous mathematical concepts learnt.

References

Abraham, M. R. (1997). *The learning cycle approach to science instruction (research matters - to the science teacher, no. 9701).* Columbia, MO: The National Association for Reserch in Science Teaching.

Alfred, K., Linety, M., & Symon, C. K. (2012). Factors inhibiting acquisition of mathematical knowledge in kenyan institutions of higher learning. *The International Institute for Science, Technology and Education (IISTE)*, *2*(9), 94-104.

Alibert, D., & Thomas, M. (1991). Research on mathematical proof. In D. Tall (Ed.), *Advanced mathematical thinking.* Dordrecht, The Netherlands: Kluwer Academic Publishers.

Alsina, C. (2002). Why the professor must be a stimulating teacher. In H. Derek (Ed.), *The teaching and learning of mathematics at university level: An icmi study* (p. 3-12). Dordrecht, The Netherlands; Kluwer Academic Publishers.

Althauser, K. L. (2018, August). The emphasis of inquiry instructional strategies: Impact on preservice tteacher' mathematics efficacy. *Journal of Education and Learning*, *7*(1), 53-70.

Angrosino, M. (2005). Recontextualizing observation: Ethnography, pedagogy, and the prospects for a progressive political agenda. In N. Denzin & Y. Lincoln (Eds.), *The sage handbook of qualitative research* (3rd ed., p. 729-745). The Sage: Thousand Oaks: Sage.

Antwi, S. K., & Hamza, K. (2015). Qualitative and quantitative research paradigms in business research. *European Journal of Business and Management*, *7*(3), 217-225.

Arrowsmith, D. K., & Place, C. M. (1990). *An introduction to dynamical systems*. Cambridge University Press.

Asiedu-Addo, S. K. (2016). *Differential equations: A first course for tertiary institutions* (Revised ed.). Pedaddo Ventures.

Asiedu-Addo, S. K., & Ali, M. (2019). *Partial differential equations: A first course for tertiary institutions*. Pedaddo Ventures.

Assem, H. D., Nartey, L. T., Amoah, C. A., & Mensah, S. A. (2018). The impact of inquiry method of learning science on the academic achievement of bs8 students in the ejisu munincipality of ashanti region of ghana. *Ra Journal of Applied Research*, *4*(8), 1903-1909.

Bada, S. O. (2015). Constructivisim learning theory: A paradign for teaching and learning. *ISOR Journal of Researcyh and Method in Education*, *5*, 66-70. Retrieved 2019-11-27, from https://www.academia.edu/34174228/Constructivism_Learning_Theory_A_Paradigm_fo

Bandura, A. (1986). *Social foundation of thought and action: A social cognitive thoery*. Englewood Cliffs, NJ: Prentice Hall.

Barron, B., & Darling-Hammond, L. (2008). *Teaching for meaningful learning: A review of research on inquiry-based and cooperative learning. powerful learning: What we know about teaching for understanding.* San Francisco, CA: Jossey-Bass.

Barrow, L. (2006). A brief histor of inquiry: From dewey to standards. *Journal of Science Teacher Education*, *17*(3), 265-278.

Becker, L. A. (2000). *Effect size (es).* Retrieved from web.uccs.edu/lbecker/Psy590/es.htm

Bed, R. A. (2017). Factors affecting difficulties in learning mathematics by mathematics learners. *International Journal of Elementary Education*, *6*(2), 8-15. Retrieved from www.sciencepublishinggroup.com/j/ijeedu doi: 10.11648/j.ijeedu.20170602.11

Belhu, H. S. (2017). Factors affecting learning mathematics in the case assosa university college of natural science. *International Journal of Education, Culture and Society*, *2*(1), 6-12. doi: 10.11648/j.ijecs.20170201.12

Bibi, A., Sharifa, N. S. Z., Nabeel, A. M. A., Mushtaq, A., & Umbreen, S. (2017). Factors affecting differential equation problem solving ability of students at pre-university level: A conceptual model. *Malaysian Online Journal of Educational Sciences*, *5*(4), 13-24.

Blanchard, P., Devaney, R., & Hall, R. (1998). *Differential equations*. Pacific Grove, CA: Brooks/Cole.

Bloom, B. (1976). *Human characteristics and school learning*. McGraw-Hill, Inc., New York.

Bloom, B., Englehart, M., Furst, E., Hill, W., & Krathwohl, D. (1956). *Taxonomy of educational objectives: The classification of educational goals. handbook i: Cognitive domain*. New York, Toronto: Longmans, Green.

Blum, W. (1993). Mathematical modelling in mathematics education and instruction. In Breteig (Ed.), (p. 3-14). Ellis Horword Limited, Chichester, S.

Blumberg, B., Cooper, D. R., & Schindler, P. S. (2005). *Business research methods*. Berkshire: McGrawHill Education.

Blumenfeld, P. C., Soloway, E., Marx, R., Krajcik, J. S., Guzdial, M., & Palincsar, A. (1991). Motivating project-based learning: Sustaining the doing, supporting the learning. *Educational Psychologist*, *26*(3 and 4), 369-398.

Bok, D. (2005). *Our underachieving colleges: A candid look at how much students learn and why they should be learning more.* Princeton: Princeton University Press.

Braun, M. (1983). *Differential equations and their applications* (3rd ed.; F. John, J. E. Marsden, & L. Sirovic, Eds.). Springer-Verleg.

Bruner, J. (1966). *Toward a theory of instruction*. Cambridge, MA: Harvard University Press.

Burns, N., & Grove, S. K. (2001). *The practice of nursing research, conduct, critique and utilization* (4th ed.). W.B. Saunders Company, Philadelphia.

Bybee, R., & Landes, N. M. (1990). Science for life and living: An elementary school science program from bilogical sscience imprimprove study (bscs). *The American Biology Teacher*, *52*(2), 92-98.

Camacho-Machin, M., Perdomo-Diaz, J., & Santos-Trigo, M. (2012). An exploration of students' conceptual knowledge built in a first ordinary differential equations cours (part ii). *The Teaching Mathematics*, *15*(2), 63-84.

Cannell, C. F., & Kahn, R. L. (1968). Interviewing. In G. Lindzey & A. Aronson (Eds.), *The handbook of social psychology: Research methods* (Vol. 2, p. 526-95). Addison-Wesley.

Carr, M. (1996). *Motivation in mathematics*. Hampton Press Inc. New York.

Chakrabartty, S. N. (2013). Best split-half and maximum reliability. *IOSR Journal of Research and Method in Education*, *3*(1), 1-8.

Christie, A. (2005). *Constructivism and its implications for educators.* Retrieved from http://alicechristie.com/edtech/learning/constructivism/index.htm

Cohen, J. (1988). *Statistical power analysis for the behavioral sciences* (2nd ed.). Hillsdale, NJ: Lawrence Earlbaum Associates.

Cohen, J. (1990). Things i have learned (so far). *American Psychology*, *45*(12), 1304-1312.

Cohen, L., Manion, L., & Morrison, K. (2007). *Research methods in education* (6th ed.). London: Routledge.

Cohen, L., Manion, L., & Morrison, K. (2011). *Research methods in education* (7th ed.). New York, NY: Routledge.

Coleman, M. P. (2013). *An introduction to partial differential equations with matlab* (2nd ed.). 6000 Broken Sound Parkway NW, Suite 300 Boca Raton, FL 33487-2742: Taylor and Francis Group, LLC.

Cooper, H., Robinson, J. C., & Patall, E. A. (2006). Does homework improve academic achievement? a synthesis of research, 1987-2003. *Review of Educational Research*, *76*(1), 1-62.

Creswell, J. W., Plano, C. V. L., Gutmann, M., & Hanson, W. (2003). Advanced mixed methods research designs. In A. Tashakkori & C. Teddlie (Eds.), *Handbook on mixed methods in the behavioral and social sciences* (p. 209-240). Thousand Oaks, CA: Sage.

Crotty, M. (1998). *The foundation of social research: Meaning and pespective in the research process*. Longdon: Sage.

Crotty, M. (2003). *The foundation of social research: Meaning and pespective in the research process*. Sage Publications.

Cuban, L. (1993). *How teachers thought: Constancy and change in american classroom 1890-1996*. New York: Teachers' College Press.

DeCorte, E., Verschaffel, L., & DeWin, L. (1985). Influence of rewording verbal pproblem on children's problem representations and solutions. *Jouranl of Educational Psychology*, *77*(4), 460-470.

Driscoll, M. P. (2000). *Psychology of learning for instruction* (2nd ed.). Neehan Heights, MA: Allyn and Bacon.

Dummont, H., Istance, D., & Benavides, F. (Eds.). (2010). *The nature of learning: Using research and through inspire preactice*. OECD Publications.

Duran, L. B., & Duran, E. (2004). The 5e instructional model: A learning cycle approach for inquiry-based science teaching. *The Science Education Review*, *3*(2), 49-57.

Ebert-May, D., Brewer, C., & Allred, S. (1997). Innovation in large lectures-teaching for active learning. *BioScience*, *47*(9), 601-607.

Edelson, D., Gordin, D., & Pea, R. (1999). Addressing the challenges of inquiry-based learning through technology and curriculum design. *Journal of the Learning Sciences*, *8*(3), 391-450. Retrieved from https://www.learntechlib.org/p/165279/

English, L. D. (1995). Cognitive psychology and mathematics education: Reflection on the past and the future. In L. D. English & G. S. Halford (Eds.), *Mathematics education: Models and processes.* Lawrence Erlbaum Associates: Mahwah: NJ.

Erbaş, A. K., Kertil, M., Çentinkaya, B., Çakiroğlu, E., Alacaci, C., & Baş, S. (2014). Mathematical modling in matheamtics education: Basic concepts and approaches. *Educational Consultancy and Research Center*, *14*(4), 1621-1627.

Etheredge, S., & Rudnitsky, A. (2003). *Instroducing students to scientific inquiry: How do we know what we know?* New York: Pearson Education.

Farr, R. M. (1982). Interviewing: The social psychology of the interview. In F. Fransella (Ed.), *Psychology of occupational therapist.* London: Macmillian.

Farrel, G., Isaacs, S., & Trucano, M. (2007). *Survey of ict and education in africa* (53 Country Reports No. 2).

Formia, E. D. (Ed.). (2012). *Innovation in design education: Theory, research and processes to and from a latin perspective.* Umberto Allemandi & C., Toronto.

Freudenthal, H. (1991). *Revisiting mathematics education*. The Netherlands: Kluwer Academic Publishers.

Friesen, S., & Scott, D. (2013, June). *Inquiry-based learning: A review of the research literature* (Tech. Rep.). Alberta Ministry of Education.

Fritz, C. O., Morris, P. E., & Richler, J. J. (2012). Effect size estimates: Current use, calculations and interpretation. *Journal of Experimental Psychology: General*, *141*(1), 2-18.

Furr, R. M. (2012). Split-half reliability. In N. J. Salkind (Ed.), *Encyclopedia of research design* (chap. 1411-1413). SAGE Publications Inc.

Gagne, R. M. (1977). *The conditions of learning and theory of instruction*. New York: Holt, Rinehart and Watson.

Gagne, R. M., & Dick, W. (1983). Instructional psychology. *Annual Review of Psychology*, *34*, 261.

Gagne, R. M., & Merrill, M. D. (1990). Integrative goals for instructional design. *Educational Technology Research and Development*, *38*(1), 23-30.

Gatt, S., & Zammit, C. (2017). The challanges of implementing inquiry science in primary school. *International Trends in Preparation*, 207-223.

George, E. H. (1991, 10). Constructivist learning theory. In *The museum and the needs of people.*

Ghana Education Service, G. E. S. (2010, September). *Teaching syllabus for senior high school elective mathematics.* Curriculum Research and Development Division.

Ghana Education Service, G. E. S. (2012, September). *National syllabus for mathematics (primary 1-6).* Curriculum Research and Development Division.

Glass, A. L., Holyoak, K. J., & Santa, J. L. (1980). *Cognition*. Addison-Wesly Reading, Massachusetts.

Greer, B. (1981). *Cognitive psychology and mathematical thinking*. FLM Publishing Association of Monreal, Quebec, Canada.

Guba, E., & Lincoln, Y. (1994). Competing paradigm in qualitative reseach. In N. Denzin & Y. Lincoln (Eds.), *Handbook on qualitative research* (p. 19-136). Sage Publications.

Gunaseelan, B., & Pazhanivelu, G. (2016, July). Identifying factors affecting the mathematics achievement of students for better instructional design. *International Journal of Development Research*, *6*(7), 8804-8809. Retrieved from https://www.journalijdr.com/identifying-factors-affecting-mathematics-achieve

Haines, C., & Crouch, R. (2007). Mathematial modeling and applications: Ability and conetence fframework. In W. Blum, P. L. Galbraith, H. Henn, & M. Niss (Eds.), *Modelling and applications in mathematics education: The 14th icmi study* (p. 417-424). New York: NY: Springer.

Halcomb, E. J., & Hickman, L. (2015). Mixed methods research. *Nursing Standard: promoting excellence in nursing care*, *29*(32), 41-47.

Harel, G. (1989). Learning and teaching linear algebra: difficulties and an alternative approach to visualizing concepts and process. *Focus on Learning Problems in Mathematics*, *11*, 139-148.

Harris, C. J., & Rooks, D. L. (2010). Managing inquiry-based science: Challenges in enacting complex science instruction in elementary and middle school classrooms. *J Sci Teacher Educ*, 227-240.

Holton, D. (Ed.). (2001). *The teaching and learning of mathematics at the university level.* Dordrecht: Kluwer.

Honebein, P. C. (1996). Seven goals for the design of constructivist learning environments. In G. W. Brent (Ed.), *Constructivist learning environments: Case studies in instructional design* (p. 11-24). Englewood Cliffs: Educational Technology Publications.

Howie, S. J. (2006). Multi-level factors affecting the performance of south african pupils in mathematics. In S. J. Howie & T. Plomp (Eds.), *Contexts of learning mathematics and science: Lessons learned from timss.* London: Routledge.

Hudson, J. P. (2009). *Pathway between eastern and western education.* Charlotte; NC, USA, Information Age Publishing.

Huitt, W., & Hummel, J. (1998). *Cognitive development.* Retrieved from `http://www.edpsychinteractive.org/topics/congnition/piaget.html`

Huitt, W., & Hummel, J. (2003). *Piaget's theory of cognitive development: Educational psychology interactive.* Voldosta GA: Voldosta State University. Retrieved from `http://www.edpsycinteractive/topics/cognitive/piaget.html`

James, S. (2012). Exploring the philosophical underpinnings of research: Relating ontology and epistemology to the methodology and methods of the scientific, interpretative, and critical reseach pradigms. *Canadian Center for Science and Education*, *5*(9), 9-16.

Jane, M. K., & Diana, V. L. (1996). The clock is ticking: Time constraint issues in mathematics teaching reform. *The Journal of Educational Research*, *90*(1), 23-31. Retrieved from https://www.jstor.org/stable/27542065

Jingoo, K., & Tuula, K. (2016). Examining factors affecting implementation of inquiry-based learning in finland and south korea. *PROBLEMS OF EDUCATION IN THE 21st CENTURY*, *74*, 31-48.

Jonassen, D. (1999). *Design constructivitst learning environments. in reigeluth, c. (ed.), instructional design thoeries and models: A new paradigm of instructional theory. vol. ii pp. 215-239.* Lawrence Eribaum Associates. Mahwah, NJ, USA.

Joseph, K. T. (2008). Explicity solutions for a system of first-order partial differential equations. *Electronic Journal of Differential Equations*, *2008*(157), 1-8. Retrieved from http://ejde.math.txtstate.edu

Kaiser, G. (2006). Introduction to the working group. ”application and modelling”. In M. Bosch (Ed.), *Proceeding of the fourth congress of the european society for research in mathematics education (cerme 4)* (p. 1613-1622). Sant Feliu de Guixols, Spain: FUNDEMI IQS, Universitat Ramon Llull.

Keiser, J. M., & Lambdin, D. V. (1996, September - October). The clock is ticking: Time constraint issues in mathematics teaching reform. *Journal of Educational Research*, *90*(1), 23-31.

Keppel, G., & Wickens, T. D. (2004). *Design and analysis: A researcher's handbook* (4th ed.). Upper Saddle River, NJ: Pearson.

Ketkar, M. A., & Reddy, G. B. (Eds.). (2003). *Microsoft excel-based numerical solutions of linear, homogeneous 1-dimensional transient partial differential equations.* American Society for Engineering Education Annual Conference and Exposition.

Khan, B. H. (2000, January). *A framework for web-based learning.* (Vol. 44) (No. 3). Retrieved from https://www.learntechlib.org/p/90327

Kimberlin, C. L., & Winterstein, A. G. (2008). Validity and reliability of measurement instruments used in research. *American Journal of Health-System Pharmacists*, *65*(1), 2276-2284.

Kiwanuka, H. N., Damme, J. V., Noortgate, W. V. D., Anumendem, D. N., & Namusisi, S. (2015). Factors affecting mathematics achievement of first-year secondary school students in central uganda. *South African Journal of Education*, *35*(3), 1-16.

Krajcik, J., Blumenfeld, P. C., Marx, R. W., Bass, K. M., Fredericks, J., & Soloway, E. (1998). Inquiry in project-based science classrooms: Initial attempts by middle school students. *Journal of the Learning Sciences*, *7*(3 and 4), 313-350.

Krauss, S. E. (2005). Research paradigm and meaning making: A primer. *The Qualitative Report*, *10*(4), 758-770. Retrieved from `nsuworks.nova.edu/tqr/vol10/iss4/7`

Kuklthau, C. C., Maniotes, I. K., & Caspari, A. K. (2007). *Guided inquiry: Learning in the 21st century.* Westport, CT & London: Libraries Unlimited.

Kwon, O. N. (2003). Guided reinvention of euler algorithm: An analysis of progressive mathematization in rme-based differential equations course. *J. Korea Soc. Math. Ed. Ser. A: The Mathematical Education*, *42*(3), 387-402.

Kwon, O. N. (Ed.). (2005, May 12-16). *Towards inquiry-oriented mathematics instruction in the university.* Daejeon, Korea: KAIST.

Kwon, O. N. (2008). Instructional design of inquiry-oriented differential equations. In *Asian technology conference in mathemtics (actm).* Suan Sunandha Rajabhat University.

Kwon, O. N., Rasmussen, C., & Allen, K. (2005). Students' retention of matheamtical knowledge and skills in differential equations. *School Science and Mathematics*, *105*(5), 1-13.

L'Amour, L. (n.d.). *Summary reading.* Retrieved 2019-07-16, from `https://www.goodreads.com/author/quotes/858.Louis_L_Amour`

Land, S. M., & Hannafin, M. J. (2000). Student-centered learning environments. In D. H. Oassen & S. M. Land (Eds.), *Theoretical foundations of learning environments.* (p. 1-24). Lawrence Eribaum Associates.

Laurillard, D. M. (2008). The teacher as action reresearch: Using technology to capture pedagogical form,. *Studies in Higher Education*, *33*(2), 139-154.

Lehrer, R., & Schauble, L. (2003). Origins and evaluation of model-based reasoning in mathematics and science. In R. Lesh & H. M. Doerr (Eds.), *Beyond constructivism: Models and modling perspectives on matheamtics problem solving, learning and teaching* (p. 59-70). Mahwah, NJ: Lawrence Erlbaum.

Lesh, R., & Doerr, H. M. (2003a). Foundations of a models and modeling perspective on mathematics teaching, learning, and problem solving. In R. Lesh & H. M. Doerr (Eds.), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching* (p. 3-33). Mahwah, NJ: Lawrence Erlbaum.

Lesh, R., & Lehrer, R. (2003). Models and modelling perspectives on the development of students an teachers. *Mathematical Thinking and Learning*, *5*(2 and 3), 109-129.

Lu, Z., & Dorsher, B. A. (2007). Cognitive pyschology. *Scholarpedia*, *2*(8), 2769. Retrieved from http://www.scholarpedia.org/article/Cognitive_psychology (revision #88969) doi: 10.4249/scholarpedia.2769

Ma, X., & Klinger, D. A. (2000). Hierarchical linear modeling of student and school effects on academic achievement. *Canadian Journal of Education*, *25*, 41-55.

McDermott, L. C., Rosenquist, M. L., & Van Zee, E. H. (1987). Student difficulties is connecting graphs and physics: Examples from kinetics. *American Journal of Physics*, *55*(6), 503-513.

McQueen, P., & McQueen, H. (2010). *Key concepts in philosophy.* Basingstoke: Palgrave Macmillan.

Mensah-Wonkyi, T., & Adu, E. (2016). Effect of the inquiry-based teaching approach on students' understanding of circle theorems in plane geometry. *African Journal of Educational Studies in Mathematics and Sciences*, *12*, 61-73.

Metodički, O. (2011). Application of computers in initial teaching of mathematics. *Primena Računra U Pačetnoj Nastavi Mathematike: Review Article*, *6*(1), 105-117.

Mohammed, Z. G. A., Mamat, M. B., Abdullah, L., & Maad, H. A. (2012). The factors influences students' achievement in mathematics: A case for libyan's students. *World Applied Sciences*, *17*(19), 1224-1230.

Mrozek, Z. (2014). *Teaching, modeling and visualization of ordinary differential equations* (Tech. Rep.). Faculty of Electrical and Computer Engineering, Cracow Universit of Technology.

Murray, J. (2013, August). The factors that influence mathematics achievement at the berbice campus. *International Journal of Business and Social Sciences*, *4*(10), 150-164.

Mustafa, U. (2017). The effect of reading comprehension and problem solving strategies on classifying elementary 4th grade students with high and low problem solving success. *Journal of Education and Training Studies*, *5*(6), 44-63.

Myers, J., Trubatch, D., & Winkel, B. (2008). Teaching modelling with partial differential equations: Several succesful approaches. *PRIMUS*, *XVIII*(2), 161-182.

Myint-U, T., & Debnath, L. (2007). *Linear partial differential equations for scientists and engineers* (4th ed.). Birkäuser Boston.

Na, L. (2012). Approaches to learning: Literature review. *International Baccalaureate Organization*, 1-40.

Nadine, M., & Laurie, B. (2013). How can teacher-directed professional development lead to the identification, utilization, reflection and revision of 5e learning prgressions? *Electronic Journal of Science Education*, *17*(1), 1-24.

Nataliya, V. I., Creswell, J., & Sheldon, L. S. (2006). Using mixed-methods sequential explanatory design: From theory to practice. *Field Methods*, *18*(1), 3-20. doi: 10.1177/1525822X05282260

National Research Council, N. R. C. (1996). National science education standards. Washinnton, DC: National Academy Press.

National Research Council, N. R. C. (2000). Inquiry and the national sicence education.

Oppenheim, A. N. (1992). *Questionnaire design, interviewing and attitude measurement*. London: Pinter.

Orton, A. (1980). *A cross-sectional study of the understanding of elementary calculus in adolescents and young adults.* (Unpublished doctoral dissertation). University of Leeds, England.

Österholm, M. (2006). Characterizing reading comprehension of mathematical texts. *Educational Studies in Mathematics.*, 136-145.

Paraskakis, I. (2003, July). Rethinking the teaching of differential equations through the constructivism paradigm. In ICALT (Ed.), *2003 ieee international conference on advanced learning technologies.* (p. 506-510). Athens, Greece.

Parkay, F. W., & Hass, G. (2000). *Curriculum planning* (7th ed.). Needham Heights, MA: Allyn & Balon.

Paul, C. P., Rajiv, S. J., & I-Chant, A. C. (2015). *Research methods in psychology* (2nd ed.). The Saylor Foundation, Canada.

Pinchover, Y., & Rubinstein, J. (2005). *An introduction to partial differential equations*. Cambridge University Press.

Polya, G. (1990). *How to solve it?* New York.

Radwan, F. A. A. (2001). Solutions of partial differential equations using excel. *Pakistan Journal of Applied Sciences*, *1*(4), 458-465.

Rasmussen, C., Keene, K. A., Dunmyre, J., & Fortune, N. (2017). *Inquiry oriented differential equations: Course materials.* Retrieved from `https://iode.wordpress.ncsu.edu`

Rasmussen, C., & Kwon, O. N. (2007). An inquiry-oriented approach to undergraduate mathematics. *Journal of Mathematical Behavior*, *26*, 189-194.

Rasmussen, C., Kwon, O. N., Allen, K., Marrongelle, K., & Burtch, M. (2006). Capitalizing on advances in mathematics and k-12 mathematics education in undergraduate mathematics: An inquiry-oriented approach to differential equations. *Asia Pacific Education Review*, *7*(1), 85-93.

Rasmussen, C., Zandieh, M., King, K., & Teppo, A. (2005). Advancing mathematical activity: A view of advanced mathematical thinking. *Mathematical Thinking and Learning*, *7*, 51-73.

Rasmussen, R. (2001). Quantification on the light cycler. In S. Meuer, C. Wittwer, & K. Nakagawara (Eds.), *Rapid cycle real-time pcr, methods and applications* (p. 21-34). Springer Press, Heidelberg.

Reigeluth, C. M. (1983). Instructional theory and technology for the new paradigm education. *RED (Revista de Educacion a Distancia)*, *32*, 2-18.

Reiser, R. A. (2001). A history of instructional design and technology. *Educational Technology Research and Development*, *49*(2), 53-64.

Reiser, R. A., & Dempsey, J. V. (2007). *Trends and issues in instructional desging and technology*. Pearson Educational Inc.

Reiser, R. A., & Dempsey, J. V. (2012). *Trends and issues in instructional desging and technology*. Pearson Educational Inc.

Renner, J., Stafford, D., Lawson, A., McKinnon, J., Friot, E., & Kellog, G. (1976). *Research, teaching and learning with piaget's model*. Norman, OK: Univerisity of Oklahoma Press.

Richard, J. (1991). Mathematical discussions. In E. von Glasersfeld (Ed.), *Radical constructivism in mathematics education* (p. 13-51). Dordrecht, The Netherlands: Kluwer Academic Publishers.

Richard, K. (2003). *Qualitative inquiry in tesol*. Basingstoke: Palgrave Macmillan.

Robson, B. (2011). *Real world research: A resource for users of social reseach methods in applied settings*. Sussex, A. John Wiley and Sons Ltd.

Rohrer, D., & Pashler, H. (2007). Increasing retention without increasing study time. *Association for Psychological Science*, *16*(4), 183-186.

Rowe, M. B. (1973). *Teaching science as continuous inquiry.* New York: McGraw-Hill.

Rowland, G. (1992). What do instructional design actually do? an initial investigation of expert practice. *Performance Improvement Quarterly*, *5*(2), 65-86.

Rowland, G. (1993). Design and instructional design. *Educational Technology Research and Development*, *41*(1), 79-91.

Rowland, G., & Wilson, G. (1994). Liminal states in designing. *Performance Improvement Quarterly*, *7*(3), 30-45.

Rutherford, F. J., & Algren, A. (1990). *Project 2061: Science for all americans*. New York: Oxford University Press.

Sarfo, F. K., & Elen, J. (2014). Towards an instructional design model for learning environments with limited ict resources in higher educaion. *African Educational Research Journal*, *2*(2), 85-95.

Savery, J. R. (2007, October). *Introduction to instructional design.*

Sayyed, R. S., & Abdullah, A. (2013). Research paradigms: Research's wordviews, theoretical fframework and study. *Arab World English Journal*, *4*(4), 252-264.

Scardamalia, M. (2002). Collective cognitive responsibility for the advancement of knowledge. In B. Smith (Ed.), *Liberal education in a knowledge society* (p. 67-98). Chicago, IL: Open Court.

Schauble, L., Glaser, R., Duschl, R. A., Schulze, S., & John, J. (1995). Students' understanding of the objectives and procedures of experimentation in the science classroom. *Journal of the Learning Sciences*, *4*(2), 131-166.

Schneider, D. K. (2006). Instructional design models and methods. In *Online learning in diplomacy workshop.*

Schoenfeld, H. (1985). *Mathematical problem solving*. Orlando: Academic Press.

Seel, N. M., Lehmann, T., Blumschein, P., & Podolskiy, O. A. (2017). *Instructional design for learning: Theoretical foundations*. Sense Publishers, Netherland. Retrieved from http://www.sensepublishers.com

Segers, M., Dochy, F., & DeCorte, E. (1999). Assessment practices and students knowledge profiles in a problem-based activity. *Learning Environment Research*, *2*, 191-213.

Selden, A., & Selden, J. (1995). Unpacking the logic of mathematical statements. *Educational Studies in Mathematics.*, *29*, 123-151.

Shaffer, D. (2000). *Social and personality development* (4th ed.). Belmont, CA: Wadsworth/Thompson.

Shavelson, R. J., Phillips, D. C., Towne, L., & Feuer, M. J. (2003). On the science of education design studies. *Educational Researcher*, *32*(1), 25-28.

Sinkovics, R. R., Pervez, E. P., & Ghauri, P. N. (2008). Enhancing the trusthworhiness of qualitative research in international business. *Management International Review (MIR)*, *48*(6), 689-714.

Skinner, B. F. (1972). *Utpia through the control of human behaviour. in john martin rich (ed.), readings in the philosophy of education*. Belmont, CA: Wadsworth/Thompson.

Smith, P. L., & Ragan, T. J. (1999). *Instructional design*. John Wiley & Sons Inc.

Snape, D., & Spencer, L. (2003). The foundations of qualitative research. In J. Richie & J. Lewis (Eds.), *Qualitative research practice* (p. 1-23). Los Angeles: Sage.

Sriraman, B., Kaiser, G., & Blomhϕj, M. (2006). A brief surey of the state of mathematical modeling around the world. *ZDM - The International Journal on Mathematics Education*, *38*, 212-213.

Strauss, W. A. (2008). *Partial differential equations: An introduction* (2nd ed.). Joh.

Svec, M. T. (1995). Effect of micro-computer based laboratory on graphing interpretation, skills and understanding of motion. paper presented at the 1995 annual meeting of the national association for research in science teaching..

Tall, D., & Vinner, S. (1981). Concept image and concept definition inmathematics with particular reference to limits and continuity. *Educational Studies in Mathematics.*, *12*, 151-169.

Tam, M. (2000). Constructivism, instructional design, and technology: Implications for transforming distance learning. *Educational Technology and Society*, *3*(2).

Tavakol, M., & Dennick, R. (2011). Making sense of cronbach's alpha. *International Journal of Medical Education*, *2*, 53-55.

Thatcher, R. (2010). Validity and reliability of quantitative electroencephalography (qeeg). *Journal of Neurotherapy*, *14*, 122-152.

Thomas, J. W., & Mergendoller, J. R. (2000). Managing project-based learning: Principles from the field. *American Educational Research Association, New Orleans.*.

Trochim, W. M. (2003). *The research method knowledge base.* Retrieved from www.socialresearchmethods.net/kb/

Twycross, A., & Shields, L. (2004). Validity and reliability: What's it all about? part 2 reliability in quantitative studies. *Paediatric Nursing*, *16*(10), 36.

Uno, G. (1999). *Handbook on teaching undergraduate science courses: A survival training manual.* Independence, KY: Thompson Custom Publishing.

Verschaffel, L., Greer, B., DeCorte, E., & Borghart, I. (2002). Everday knowlede and mathematical modeling of school world pproblem. In K. P. Gravemeijer, H. Lehrer, H. J. Van Oers, & L. Verschaffel (Eds.), *Symbolizing, modeling and tool use in mathematics educaiton* (p. 171-195). Dordrecht, The Netherlands: Kluwer Academic Publishers.

Wand, Y., & Weber, R. (1993). On the ontological expressiveness of information systems analysis and design grammars. *Information Systems Journal*, *3*(4), 217-237. Retrieved from doi.org/10.1111/j.1365-2575.1993.tb00127.x

Wood, K. C., Smith, H., & Corssniklaus, D. (2001). *Piaget's stages of codgnitive development. in orey, m. (ed.), emerging perspective on learning, teaching and technology.* Retrieved from http://projects.coe.uga.edu/epltt

Wubbles, T., Korthagen, F., & Broekman, H. (1997). Preparing teachers for realistic mathematics education. *Educational Studies in Mathematics.*, *32*, 1-28.

Yin, R. K. (1989). *Case study research: Design and methods* (Vol. 5). SAGE Publications.

Zhou, M., & Brown, D. (2014). *Education learning theory*. Galileo Openg Learning Material, Georgia.

APPENDICES

Appendix I: Test Items

Appendix I(a): Diagnostic Test for Pilot Study

1. Show that $y(x) = x^{-\frac{3}{2}}$ is a solution to $4x^2y'' + 12xy + 3y = 0$ for $x > 0$.
2. What is the actual solution to the IVP
   (a) $2ty' + 4y = 3, y(1) = -4$?
   (b) $2y' - y = 4\sin(3x), y(0) = y_0$?
3. Solve the differential equation $2xy - 9x^2 + (2y + x^2 + 1)\frac{dy}{dx} = 0$
4. Given that $y'' - 4y' - 12y = 3e^{5t}$, determine
   (a) the homogeneous solution.
   (b) the particular solution.
   (c) hence find the solution of the given equation.

Appendix I(b): Pilot Study Test

Appendix I(b)(i): Pilot Study Test 1

Instructions: Answer all questions.

1. Which of the following operators are linear?
   (a) $\mathcal{L}u = u_x + xu_y$
   (b) $\mathcal{L}u = u_x + uu_y$
   (c) $\mathcal{L}u = u_x + u_y + 1$
   (d) $\mathcal{L}u = u_x + u_y^2$
   (e) $\mathcal{L}u = \sqrt{1+x^2}(\cos y)u_x + u_{yxy} - \left[\tan^{-1}\left(\frac{x}{y}\right)\right]u$
2. For each of the following state whether it is homogeneous or non-homogeneous, linear, semi-linear, quasilinear or fully nonlinear.
   (a) $u_t - u_{xx} + 1 = 0$
   (b) $u_t - u_{xxt} + uu_x = 0$
   (c) $u_{tt} - u_{xx} + x^2 = 0$
   (d) $iu_t - u_{xx} + \frac{u}{x} = 0$
   (e) $u_t + u_{xxx} + \sqrt{1+u} = 0$
3. Show that $u = f(xy)$, where f is an arbitrary differentiable function that satisfies $xu_x - yu_y = 0$ and form this differential equation using the functions $\sin(xy), \ln(xy), e^{xy}$ and $(xy)^3$. What can you say about these functions in relation to the given PDE?

Appendix I(b)(ii): Pilot Study Test 2

Instruction: Answer all questions.

1. Solve the transport equation $u_t + 3u_x = 0$ given the initial condition $u(x,0) = xe^{-x^2}, x \in R$.

2. Solve $2yu_x + (3x^2 - 1)u_y = 0$ by the method of substitution. If the arbitrary function f of the general solution is given by $f(p) = \sin p$, draw the graph of the particular solution.

3. Solve the equation $(1 + x^2)u_x + u_y = 0$. Sketch some of the characteristic curves.

4. (a) Solve the equation $yu_x + xu_y = 0$ with $u(0,y) = e^{-y^2}$.
   (b) In which region is the solution uniquely determined?

Appendix I(b)(iii): Pilot Study Test 3

Instruction: Answer all

1. Solve the Cauchy problem $x^2u_x - y^2u_y = 0$, where $u \to e^x$ as $y \to \infty$. Draw the characteristic curve for the solution.

2. Find the general solution of $(y + u)u_x + (x + u)u_y = x + y$.

3. Consider the equation $u_x + u_y = 1$, with the initial condition $u(x,0) = f(x)$.
   (a) What are the projections of the characteristic curves on the (x,y) plane?
   (b) Solve the equation.

4. A river is defined by the domain

$$D = \{(x,y)||y| < 1, -\infty < x < \infty\}.$$

   A factory spills a contaminant into the river. The contaminant is further spread and convected by the flow in the river. The velocity field of the fluid in the river is only in the x direction. The concentration of the contaminant at a point (x,y) in the river and at time t is denoted by $u(x,y,t)$. Conservation of matter and momentum implies that u satisfies the first-order PDE

$$u_t - (y^2 - 1)u_x.$$

   The initial condition $u(x,y,0) = e^y e^{-x^2}$.

   (a) Find the concentration u for all (x,y,t).
   (b) A fish lives near the point $(x,y) = (2,0)$ at the river. The fish can tolerate contaminant concentration levels up to 0.5. If the concentration exceeds this level, the fish will die at once. Will the fish survive? If yes, explain why. If no, find the time in which the fish will die.
   Hint: Notice that y appears in the PDE just as a parameter.

Appendix I(c): Diagnostic Test for the Study

Appendix I(c)i.: Diagnostic Test 1 for the Study

Instructions: Attempt all questions

1. Suppose that a large mixing tank with a capacity of 900 gallons initially holds 300 gallons of water is which 50 pounds of salt have been dissolved. Another brine solution with concentration 2 lb/gal is pumped into the tank at a rate of 6 gal/min, and when the solution is well stirred, it is then pumped out at a slower rate of 2 gal/min.

2. (a) Solve the IVP

$$\begin{cases} (150+2t)y' + y = 1800+24t \\ y(0) = 50 \end{cases}$$

   (b) Compute $y(150)$

   (a) Determine a differential equation for the amount of salt $A(t)$ in the tank at time $t > 0$.

   (b) What is the amount of salt in the tank when it overflows?

3. Solve the following differential equation using the method of undetermined coefficients.

   (a) $y'' - 4y' - 12y = 2t^e - t + 3$

   (b) $y'' - 4y' - 12y = 3e^{5t}$

Appendix I (c)ii.: Diagnostic Test 2 for the Study

Instruction: Attempt all questions

1. A 1500 gallon tank initially contains 600 gallons of water with 5 lbs of salt dissolved in it. Water enters the tank at a rate of 9 gal/hr and the water entering the tank has a salt concentration of $(1+2t)$ lbs/gal. If a well mixed solution leaves the tank at a rate of 6 gal/hr, how much salt is in the tank when it overflows? Sketch the graph of the amount of salt against time for $t = 0$ to $t = 300$.

2. Solve the IVP

$$\begin{cases} y' + \dfrac{2y}{200+t} = 18t+9 \\ y(0) = 5 \end{cases}$$

3. Solve the differential equation $2x^2y'' - 5xy' - 15y = t + 5e^{5t}$

Appendix I(d): Test 1 for the Study

1. Classify the following partial differential equations by the following: order, linear, semi-linear, quasilinear, fully non-linear and homogeneity. For example, the equation $u_x + u_y = u$ is first-order, semi-linear, homogeneous PDE.

   i. $u_{xx} + u_{yy} = 0$
   ii. $u_t + tu_x = u^2$
   iii. $uu_t + u_{xx} = \sin u + e^t$
   iv. $u_{xx}^2 + yu_{yy} = e^{xy}$
   v. $u_t + u_{xxx} - 6uu_x = 0$
   vi. $u_t - (x^2 + u)u_{xx} = x - t$
   vii. $u^2 u_{tt} - \frac{1}{2}u_x^2 + (uu_x)_x = e^u$
   viii. $(u_{xy})^2 - u_{xx} = 0$
   ix. $u_{tt} + xu_{xx} + u_t = f(x,t)$
   x. $uu_{xx} + u_y^2 = \ln u$

2. Find 2-dimensional first-order partial differential equation that satisfies each of the following functions:

   i. $u(t,x) = e^t \cos x$
   ii. $u(x,y) = x^2 + y^2$
   iii. $u(t,x) = x^2 t$
   iv. $u(x,y) = e^{-x^2}$
   v. $u(x,y) = \ln(x^2 + y^2)$

3. Find two-dimensional second-order partial differential equation that are satisfied by each of the following functions in question (2).

4. Find a partial differential equation by eliminating a and b from the following equations.

   (a) $u(x,y) = ax + by + a^2 + b^2$
   (b) $u(x,y) = ax + (1-a)y + b$

Appendix I(e): Test 2 for the Study

Instruction: Answer all questions

1. Use the method of characteristics to find the solution of the following PDEs. In each case, sketch the paths of the characteristics and the solution using any graphical tool at your disposal.

   (a) $u_t - u_x = 0; u(0,x) = \cos x$
   (b) $tu_t - u_x = 1; u(t,0) = e^{-t^2}$
   (c) $xu_t + tu_x = -xu; u(0,x) = 1 - x$ for $x \geq 0$

2. Solve the IVP for $u(t,x)$ at times $t \geq 0$ in terms of t and a characteristic variable:

   (a) $u_t + \frac{u_x}{u} = 0$ with $u(0,x) = u_0(x) = \dfrac{1+x^2}{2+x^2}$
   Do the characteristics cross for any $t \geq 0$ and if so where and when?

   (b) $u_t + uu_x = 1$ with $u(0,x) = u_0(x) = 1 - \frac{1}{2}\tanh(x)$

3. Consider a one-dimensional road with cars moving along it in one direction only. If $\rho(t,x)$ denotes the concentration or density of the cars at position x and time t (in mass/unit length), derive the transport equation for the problem.

Appendix I(f): Test 3 for the Study

Instruction: Answer all questions

1. Consider the one-dimensional heat problem

$$u_t - u_{xx} = e^{-t}\sin(3x); 0 < x < \pi; t > 0$$
$$u(t,0) = u(t,\pi) = 0, t > 0$$
$$u(0,x) = x\sin x; 0 \leq x \leq \pi$$

   (a) Solve the problem using the Laplace transform.

   (b) Show that the solution $u(t,x)$ is indeed a solution of the equation

$$u_t - u_{xx} = e^{-t}\sin(3x); 0 < x < \pi; t > 0$$

2. A flexible string of length 2cm is stretched horizontally so that one end is at $x = 0$ and the other at $x = 2$. While the ends are held fixed, the string is moved vertically from equilibrium so that the point x is displaced by the function $f(x) = x - 1$ (with $f(x) > 0$ or $f(x) < 0$ depending on whether the point x is moved above or below its equilibrium position). Hence the shape of the string, is given by the graph of $f(x)$. Let $u(x,t)$ denotes the (vertical) displacement at time t at point x. The IVBP (initial-value boundary problem) of the problem is given by

$$\begin{cases} u_{tt} = 9u_{xx}, 0 < x < 2, t > 0 \\ u(x,0) = x - 1, 0 < x < 2, t > 0 \\ u_t(0,t) = \cos(\pi x), 0 < x < 2, t > 0 \\ u_x(0,t) = 0, t > 0 \\ u_x(2,t) = 0, t > 0 \end{cases}$$

   (a) What does the initial conditions $u(x,0) = x - 1$ and $u_t(x,0) = \cos(\pi x)$ means?

   (b) What type of boundary condition is given in the IVBP and what does it mean?

   (c) Compute the Laplace transform of the IVBP.

   (d) Using the Laplace transform from (c), find the solution $u(x,t)$. (You may referred to the Laplace transform table provided at the last page of this paper).

   (e) What is the displacement of the string at $x = 0.6$ after 20 seconds? Describe the position of the string.

Appendix II: Activities for the Study

Appendix II(a): The Prediction Task: The Traffic Flow

Imagine a number of cars spread over a one-lane road. The road may not necessarily be straight. Given that, the traffic density, $\rho(x,t)$, on the road which is associated with a given position x and time t, is the average number of vehicles per unit length of road at the specified position and time.

Question: If the one-lane road have traffic with traffic density $\rho(x,t)$ and velocity field $u(x,t)$, will the traffic density affect the rate at which the cars moved?

The traffic density will definitely affect the rate of movement of the cars. The rate at which the cars moved is referred to as traffic flow. It is defined as the number of cars per unit time which cross a given point on the road. The flow rate or traffic flow can also be referred to as traffic flux. The traffic flux is given by

$$\text{flux} = \text{density} \times \text{velocity} \tag{6.1}$$

Task 1: If there are 30 cars per kilometer on a road and each car is moving at a speed of $80km/h$, how many cars will pass over an observer at the side of the road in an hour?

Answer: The number of cars that will pass the observer in an hour is the flux so

$$\text{The flux}, q = \rho u = 30 \times 80 = 2400 \text{ cars per hour}$$

Task 2: Let q be the flux. What are the variables that the flux depends on? Write the equation.

Explanation: The flux depends on the density, ρ, the velocity, u, the point x on the road and the time, t at that point on the road. The equation is given by

$$q(x,t) = \rho(x,t)u(x,t) \tag{6.2}$$

Task 3: Suppose cars cannot enter or leave a one-way of a two-way thoroughfare because one lane was closed at some point which resulted in confusion which was later settled down to a steady state. The condition before the closing of the lane was assumed to be constant and uniform an as such both lanes were having the same density ρ_1 and velocity u_1. The condition after the closing of the lane reached a steady state and so was again uniform on the single lane with $u = u_2$ and $\rho = \rho_2$.

Question: State the relationship that exist between ρ_1, ρ_2, u_1 and u_2.

Answer: Since there were two lanes, the flux density before the closing of the lane is given by

$$q_1 = 2\rho_1 u_1 \tag{6.3}$$

and the flux density after the closing of the lane is also given by

$$q_2 = \rho_2 u_2 \tag{6.4}$$

Since no car can be created nor destroyed, in other words, no car leaves or enter the region, the conservation law can be applied. So the flux into the closing region must be equal to the flux out of the closing region and so we have

$$2\rho_1 u_1 = \rho_2 u_2 \tag{6.5}$$

If the traffic speed stay the same due to the assumption in equation (6.5), $(u_1 = u_2)$, the density must double. Note that, the higher the density, the smaller the traffic speed.

Question: How does flux and density relate to the conservation of the number of cars in the road when no car leaves or enter the region (the other assumption is that the number of cars entering the region is equal to the number of cars leaving the region).

Explanation: Let us imagine a number of cars on a particular one-lane stretch with the assumption that no cars enters or leaves that region. Let x be the distance along the stretch. If we consider the number of cars on the road from a point $x = a$ to $x = b$ such that $a < x < b$ at time t, we will be able to find the number of cars within the stretch at any time t. We can therefore calculate the flux at a and b. That is, the rate of change of the number of cars in the segment with respect to time. The number of cars is equal to the difference in the flow rate or flux. If N_{ab} is the number of cars at time t in the region $a < x < b$, then the rate of change of the number of cars is given by

$$\frac{dN_{ab}(t)}{dt} = -q(b,t) + q(a,t) \tag{6.6}$$

Again, the number of cars at time t in the region $a < x < b$ can be express in terms of the density function as

$$N_{ab}(t) = \int_a^b \rho(x,t)dx \tag{6.7}$$

From equations (6.6) and (6.7), we can write the relation

$$\frac{d}{dt}\int_a^b \rho(x,t)dx = q(a,t) - q(b,t) \tag{6.8}$$

Equation (6.8) is know as the global conservation law for the vehicles on the road.

Question: What does it mean to say $q(b,t) > q(a,t)$ and what happens?

Explanation: If $q(b,t) > q(a,t)$, it means more cars are leaving the region than those who entering the region and this will result in the number of cars within the region in time.

We realized that, the fluxes were the same when the road ws in a steady state. Note also that

$$\frac{d}{dt}\int_a^b \rho(x,t)dx = \int_a^b \frac{\partial\rho}{\partial t}dx \tag{6.9}$$

since a and b are constants.

Question: Why was partial derivative used on the right side of equation (6.9)?

Explanation: It was used because ρ also depends on x. But from the fundamental theorem of calculus

$$\int_a^b \frac{\partial q(x,t)}{\partial x} dx = q(b,t) - q(a,t) \tag{6.10}$$

This means that from equations (6.8) to (6.10), we have

$$\int_a^b \frac{\partial \rho}{\partial t} dx = -(q(b,t) - q(a,t)) = -\int_a^b \frac{\partial q(x,t)}{\partial x} dx$$

$$\Longrightarrow \int_a^b \left[\frac{\partial \rho(x,t)}{\partial t} + \frac{\partial q(x,t)}{\partial x}\right] dx = 0 \tag{6.11}$$

But from our knowledge in Calculus,if $\int_a^b f dx = 0$ then $f = 0$ and so

$$\frac{\partial \rho(x,t)}{\partial t} + \frac{\partial q(x,t)}{\partial x} = 0 \tag{6.12}$$

But the flux $q(x,t) = \rho(x,t)u(x,t)$ where $\rho(x,t)$ is the density and $u(x,t)$ is the velocity, so equation (6.12) can be written as

$$0 = \frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} \tag{6.13}$$

$$= \rho_t + \rho_x u + \rho u_x \tag{6.14}$$

But equation (6.14) is a first-order PDE involving two dependent variables ρ and u.

Question: How do we re-write equation (6.14) as equation with one dependent variable?

Explanation: In order to close the model or write the equation as equation with one dependent variable, there is the need to close the model by the introduction of another model. One of the major assumptions made by the modelers of traffic flow stated that "the velocity may be reasonably assumed to be a function of the density alone. And so instead of $u = u(x,t)$ we have

$$u = u(\rho) = u(\rho(x,t))$$

So from equation (6.13), we have

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u(\rho))}{\partial x} = 0 \tag{6.15}$$

I we let $f(\rho) = \rho u(\rho)$ then

$$\frac{\partial(\rho u(\rho))}{\partial x} = \frac{\partial}{\partial x} f(\rho) = f'(\rho)\rho_x$$

hence equation (6.15) becomes

$$\rho_t + f'(\rho)\rho_x = 0 \tag{6.16}$$

Hence the traffic density of a traffic flow is a first-order partial differential equation.

Appendix II(b): Investigating the Geometric Method of Solving First-Order PDEs

Activity 1

Materials Required: Graph Sheet and rule.

Step 1: Plot the following points on the graph sheet: $A(0,1), B(2,1), C(4,1), D(6,1), E(6,3), F(6,5)$.

Step 2: Join the points A to B, B to C, C to D, D to E and E to F with arrows.

Step 3: Join A to F with arrow as well.

Step 4: Describe the figure obtained

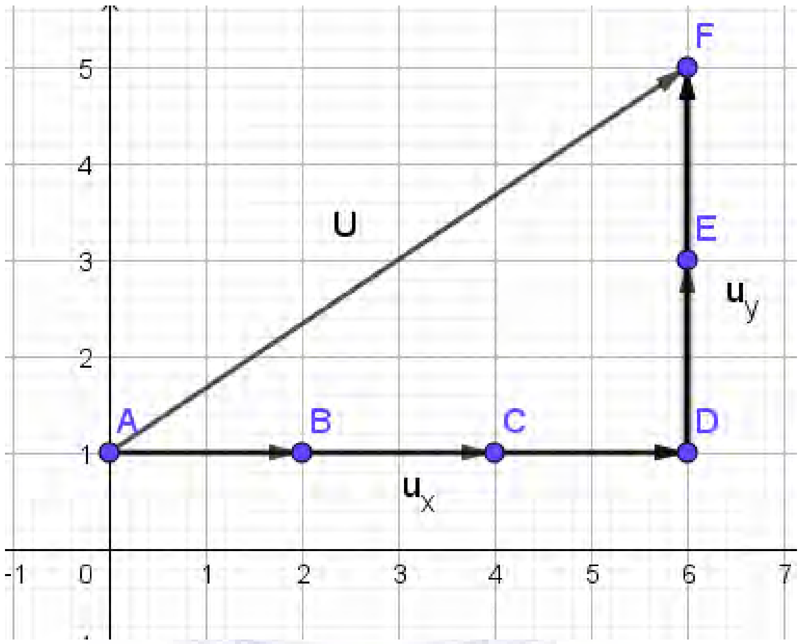

Figure 6.1: Vector AF

Step 5: Which of the variables x and y changes as you move from A to D and which of them changes when you move along D to F.
Since x changes along the vector $\overrightarrow{AF}$, if the function that represents the movement of an object from the point A to the point F along AD and Df lines is $u(x,y)$ then the movement along the vector line $\overrightarrow{AD}$ can be represented by u_x. The vector along DF can be represented by u_y.

Step 6: How can you write the vector $\vec{U}$ in terms of the vectors $u_x = \overrightarrow{AD}$ and $u_y = \overrightarrow{DF}$?

$$\vec{U} = u_x + u_y$$

Activity 2

Step 1: Given that $u(x,y) = y^2$, find u_x.

Step 2: If $u(x,y) = x^3$, find u_y.

Activity 3

Use your knowledge in integral calculus and ordinary differential equations to find the general solution of the following equations while reflecting on the results you obtained in activity 2.

Step 1: If $u_x = 0$, find u.

Step 2: Given that $u_y = 0$ find u.

Activity 4

Revise the results and findings of activity 1 to 3 and use it to answer the following questions. You may share your findings among yourself. Give reasons for your answer.

Q. 1: What is meant by the equation $u_x = 0$
Expected answer: *It means the rate of change of u in the direction of the vector* $(1,0)$ *which is parallel to the x-axis is zero. That is* $(1,0).(u_x,u_y) = 0$

Q. 2: What is the meaning of the equation $u_y = 0$?
Expected answer: It means the rate of change of u in the direction of the vector $(0,1)$ which is parallel to the y-axis is zero. That is, $(0,1).(u_x,u_y) = 0$.

Q. 3: Given the first-order linear homogeneous PDE with constant coefficients

$$au_x + bu_y = 0$$

where a and b are not both zero. That is $a^2 + b^2 \neq 0$, explain the meaning of the equation and its expected solution.
Expected answer: the quantity $au_x + bu_y = 0$ is an equation obtained from directional derivative of the function u in the direction of the vector

$$\vec{V} = (a,b) = a\mathbf{i} + b\mathbf{j}$$

And it must be zero. This means the solution $u(x,y)$ must be constant in the direction of the vector $\vec{V}$.

Activity 5

For question (1) and (2), find the general solution of the PDEs.

1. $u_x + 3u_y = 0$
2. $3u_x - 7u_y = 0$
3. Solve the first-order equation $2u_t + 3u_x = 0$ with the auxiliary condition $u(x,t) = \sin x$ when $t = 0$. Using the Geogebra, sketch the solution $u(x,0)$.

Appendix II(c): Investigating the Geometrical Content of First-Order Quasilinear PDEs

The first-order quasilinear PDEs are of the form

$$a(x,y,u)u_x + b(x,y,u)u_y - c(x,y,u) = 0 \tag{6.17}$$

and has a solution of the form

$$u = u(x,y) \tag{6.18}$$

or in an implicit form

$$f(x,y,u) \equiv u(x,y) - u = 0 \tag{6.19}$$

This represents a possible solution surface in the (x,y,u) space.

Objectives

We are to investigate how to solve such equations and use it to obtain the solution to the PDE $u_x + xu_y = 0$ which satisfies the initial condition $u(0,y) = \sin y$ on the interval $-\infty \leq x \leq \infty$.

Activity 1

From our previous knowledge of functions of two variables, a function of the form $f(x,y,u) = 0$ represents a surface called integral surface of equation (6.17).

1. At any point (x,y,u) on the solution surface, what is the gradient vector of the solution (6.19)?
   Expected Answer:The gradient vector to the solution is given by

$$\nabla f = f_x\mathbf{i} + f_y\mathbf{j} + f_u\mathbf{k} = (f_x, f_y, f_u) = (u_x, u_y, -1) \tag{6.20}$$

2. Write equation (6.18) as a dot product of two vectors.
   Expected answers:

$$au_x + bu_y - c = \langle a,b,c\rangle \cdot \langle u_x, u_y, -1\rangle = 0 \tag{6.21}$$

3. If the dot product of the two vectors in equation (6.21) is zero, then describe the vector $\langle a,b,c\rangle$.
   Expected Answer: It is a tangent vector of the integral surface (6.19) at the point (x,y,u).

4. What does the vector $\langle a,b,c\rangle$ determines?
   Expected answer: It determines the direction of the vector $\langle u_x, u_y, -1\rangle$.

5. Considering equation

$$u_x + xu_y = 0 \tag{6.22}$$

   identify the direction vector and hence write it as a dot product of two vectors.
   Expected answer: The direction vector is $\langle 1,x,0\rangle$ and

$$u_x + xu_y = \langle 1,x,0\rangle \cdot \langle u_x, u_y, 0\rangle = 0$$

6. A curve in (x, y, u)-space whose tangent at every point coincides with the characteristic direction field $\langle a, b, c\rangle$, is called a characteristic curve. If the parametric equations of this characteristic curve are

$$x = x(t), y = y(t), u = u(t) \tag{6.23}$$

what is the tangent vector to this curve and what is its relationship to the direction vector $\langle a, b, c\rangle$?

Expected answer: The tangent vector is $\left\langle \frac{dx}{dt}, \frac{dy}{dt}, \frac{du}{dt} \right\rangle$ and it must be equal to $\langle a, b, c\rangle$.

7. Using equation (6.22), write the system of ordinary differential equations for the characteristic curve of the equation $u_x + xu_y = 0$.
Expected answer: $\frac{dx}{dt} = 1, \frac{dy}{dt} = y$.

8. Find $\frac{dy}{dx}$ and combining the idea of solving first-order PDEs you learnt in the geometric method, find the solution $u = u(x, y)$ of the equation subject to the initial condition $u(0, y) = \sin y$ and sketch the solution using Geogebra or MATLAB.
Expected answer:

$$\frac{dx}{dt} = 1$$
$$\frac{dy}{dt} = x$$
$$\frac{dy}{dx} = x$$
$$\therefore y = \frac{x^2}{2} + c$$

But $f(c) = u(x, y)$

$$u(x, y) = f(y - \frac{x^2}{2})$$

Using the initial condition $u(0, y) = \sin y$ we have

$$u(0, y) = f(y) = \sin y$$
$$\therefore u(x, y) = \sin\left(\frac{2y - x^2}{2}\right)$$

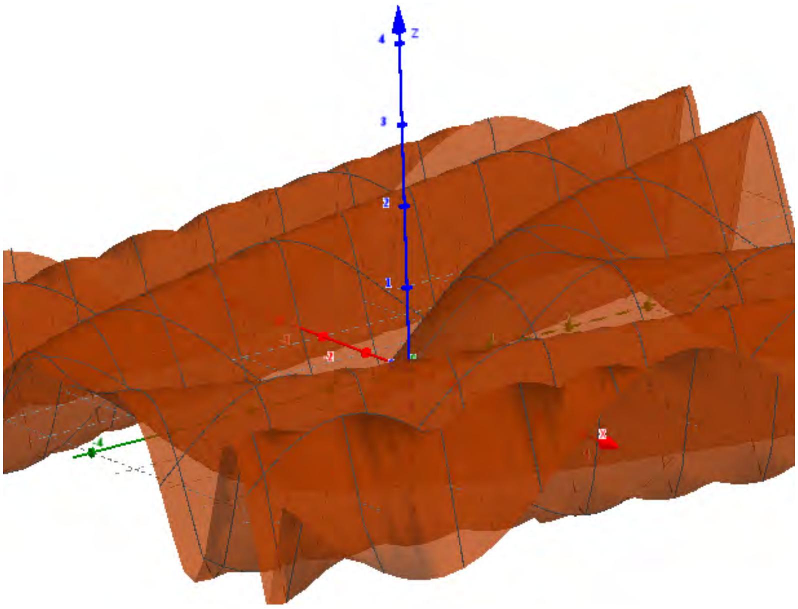

Figure 6.2: Surface Curve of the solution of $u_x + xu_y = 0 \quad (u(x,y) = \sin(\frac{2y-x^2}{2}))$

Instructors Explanation

The vector $\langle a,b,c\rangle$ determines the direction field called the characteristic direction or Monge axis. It is of fundamental importance in the determination of a solution to the PDEs (6.17). In summary, we have shown that $f(x,y,u) = u(x,y) - u = 0$, as a surface in the (x,y,u)-space, is a solution of equation (6.17) if and only if the direction vector field $\langle a,b,c\rangle$ lies in the tangent plane of the integral surface $f(x,y,u) = 0$ at each point (x,y,u) where $\nabla f \neq 0$ as shown below in figure 6.3.

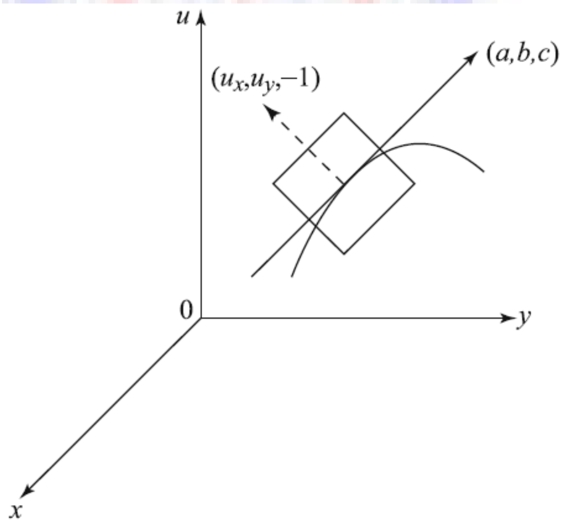

Figure 6.3: The Tangent and Normal Vector fields of solution surface at a point (x,y,u)

A curve in the (x,y,u)-space, whose tangent at every point coincides with the characteristic direction field $\langle a,b,c\rangle$ is called a characteristic curve. If the parametric equations of this characteristic curve are $x = x(t), y = y(t), u = u(t)$, then the tangent vector to the curve is $\left\langle \frac{dx}{dt}, \frac{dy}{dt}, \frac{du}{dt} \right\rangle$ and is equal to $\langle a,b,c\rangle$. The system of ordinary differential equations of the characteristic curve is given by

$$\frac{dx}{dt} = a(x,y,u), \frac{dy}{dt} = b(x,y,u), \frac{du}{dt} = c(x,y,u) \tag{6.24}$$

Equation (6.24) is called the characteristic equations of the quasilinear equation since there are only two independent ordinary differential equation equations in the system (6.24), the solution consist of two parametric family of curves in (x,y,u)-space. The projection on $u = 0$ of a characteristic curve on the (x,t)-plane is called a characteristic base curve or simply characteristic and they are equal. So the characteristic equations (6.24) in the non-parametric forms are

$$\frac{dx}{a} = \frac{dy}{b} = \frac{du}{c} \tag{6.25}$$

An important observation regarding the nature of the characteristic equation in the (x,t)-plane is that, the characteristics are determined by the first two equations in (6.24) with their slopes

$$\frac{dy}{dx} = \frac{a(x,y,u)}{b(x,y,u)} \tag{6.26}$$

If equation (6.17) is a linear equation, that is, a and b are independent of u, the characteristic equation of (6.24) are plane curves with slopes

$$\frac{dy}{dx} = \frac{a(x,y)}{b(x,y)} \tag{6.27}$$

After integration, we can identify the characteristics which represents a one-parameter family of curves in the (x,t)-plane. If a and b are constants, the characteristic equation of equation (6.17) are straight lines.

Appendix II(d): Investigating the Method of Characteristics Using Cramer's Rule

This investigation was in the form interaction between the instructor and the students based on their previous knowledge on geometrical interpretation of functions of two variables, method of solving system of equations using Cramer's rule. The summary of the whole activity is presented below.

First-Order PDEs

Let's consider the general first-order partial differential equations (PDEs).

$$a(x,y,u)u_x + b(x,y,u)u_y = c(x,y,u) \tag{6.28}$$

where a,b and c may be constants or functions of x,y and u and u is a function of x and y. Note that u is not a function of u_x and u_y. Since u is a function of x and y, that is, $u = u(x,y)$, the it implies from our previous activities that, u is a surface s and it is the solution of equation (6.28) denoted by Figure 6.4. For simplicity, let's write equation (6.28) as

$$au_x + bu_y = c \tag{6.29}$$

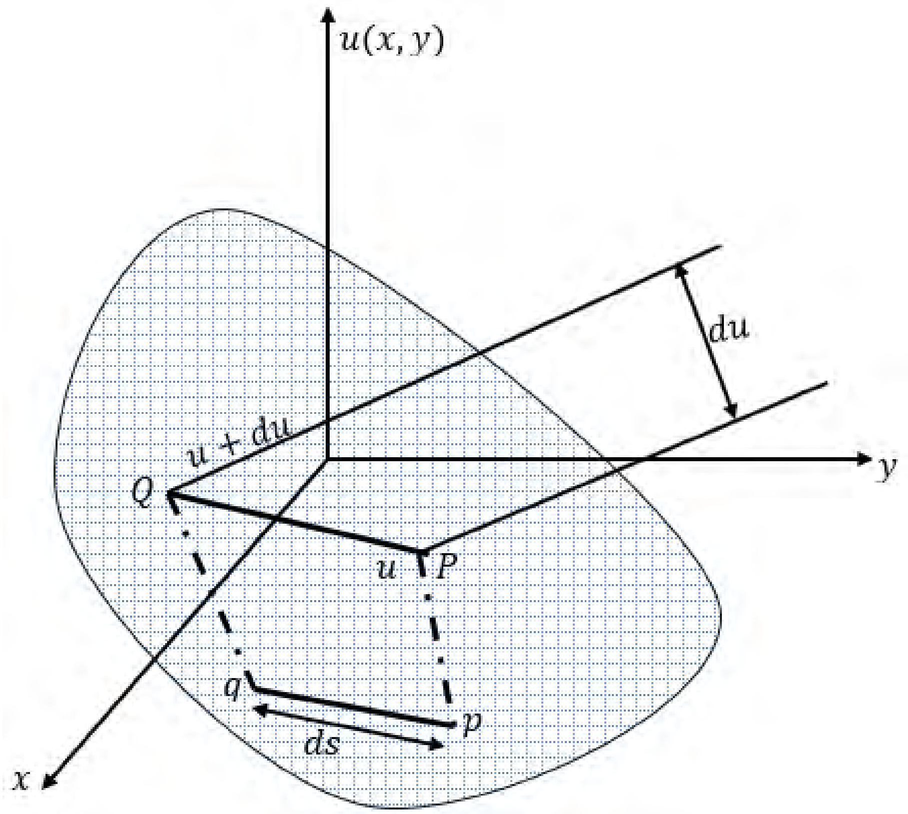

Figure 6.4: Surface denoting the solution of equation (6.28)

1. **Question**: Since u is the solution which is represented by the surface, then the change in the value of u between two neigbhouring points on the surface, P and Q is given by?
   Expected Answer: The change in the value of u between two neigbhouring points on the surface, P and Q is assumed to be du.
   Question: Using the rate of change, u_x and u_y, find du.
   Expected Answer: Using the rate of change we have

$$du = u_x dx + u_y dy \tag{6.30}$$

2. Equations (6.29) and (6.30) gives the system of equations

$$\begin{cases} au_x + bu_y = c \\ u_x dx + u_y dy = du \end{cases} \tag{6.31}$$

3. **Question**: Solve the system (6.31) using Cramer's rule.
   Expected Answer: Expressing the system (6.31) in matrix equation form, we have

$$\begin{bmatrix} a & b \\ dx & dy \end{bmatrix} \begin{bmatrix} u_x \\ u_y \end{bmatrix} = \begin{bmatrix} c \\ du \end{bmatrix} \tag{6.32}$$

   Letting $D = \begin{bmatrix} a & b \\ dx & dy \end{bmatrix}$, the system will have a unique solution if the determinant of the matrix D is non-zero.

Question: When will the system have infinite number solutions (i.e., infinite number of surfaces containing the line PQ).
Expected Answer: The system will have infinite number of solutions when the determinant of D is zero. That is

$$\begin{bmatrix} a & b \\ dx & dy \end{bmatrix} = 0 \tag{6.33}$$

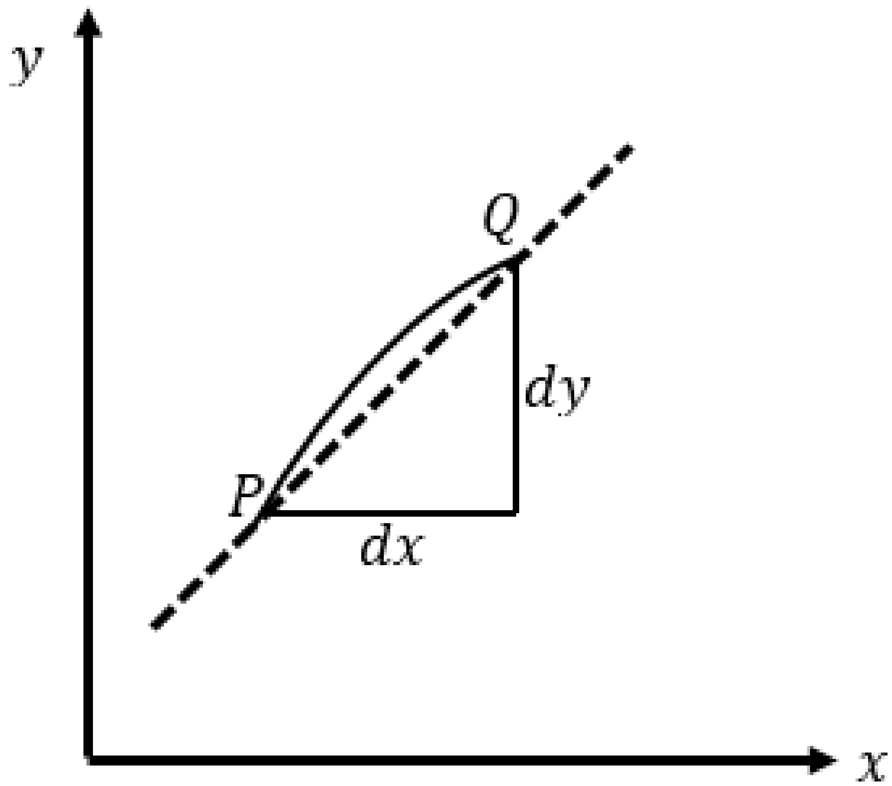

Figure 6.5: Slope of the Characteristic

Note: A characteristic curve is defined as a curve along which the determinant of the matrix D is zero.

4. **Question**: From the definition above, deduce the characteristic equation of the equation (6.28).
Expected Answer: The direction of the characteristic is

$$ady - bdx = 0 \tag{6.34}$$

This means that

$$ady = bdx$$
$$\frac{dy}{dx} = \frac{b}{a}$$

From which we can also write

$$\frac{dy}{b} = \frac{dx}{a} \tag{6.35}$$

From Cramer's rule, the solution of equation (6.33) can be found if

$$\begin{vmatrix} c & b \\ du & dy \end{vmatrix} = 0 \text{ or } cdy = bdu \tag{6.36}$$

and

$$\begin{vmatrix} a & c \\ dx & du \end{vmatrix} = 0 \text{ or } adu = cdx \tag{6.37}$$

5. From equations (6.36) and (6.37), we have

$$\frac{dy}{b} = \frac{du}{c} \tag{6.38}$$

and

$$\frac{du}{c} = \frac{dx}{a} \tag{6.39}$$

From equations (6.35), (6.38) and (6.39), we have

$$\frac{dx}{a} = \frac{dy}{b} = \frac{du}{c} \tag{6.40}$$

Equation (6.40) is called the characteristic equation of equation (6.28). So in order to solve equation (6.28), we just need to solve the characteristic equation.

Second-Order Equations

1. Consider the second-order quasilinear hyperbolic differential equation

$$au_{xx} + bu_{xy} + cu_{yy} = f \tag{6.41}$$

where a, b, c and f are constants or functions of x, y, u, u_x and u_y, the differentials $d(u_x)$ and $d(u_y)$ can be expressed as

$$d(u_x) = \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right) dx + \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}\right) dy = u_{xx}dx + u_{xy}dy \tag{6.42}$$

$$d(u_y) = \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y}\right) dx + \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right) dy = u_{xy}dx + u_{yy}dy \tag{6.43}$$

2. Equations (6.41), (6.42) and (6.43) can be expressed in matrix form as

$$\begin{bmatrix} a & b & c \\ dx & dy & 0 \\ 0 & dx & dy \end{bmatrix} \begin{bmatrix} u_{xx} \\ u_{xy} \\ u_{yy} \end{bmatrix} = \begin{bmatrix} f \\ d(u_x) \\ d(u_y) \end{bmatrix} \tag{6.44}$$

3. As in the case of the first-order, the equation (6.44) can be solved uniquely except when the determinant of the coefficient matrix is zero. That is

$$\begin{vmatrix} a & b & c \\ dx & dy & 0 \\ 0 & dx & dy \end{vmatrix} = a(dy)^2 - b(dxdy) + c(dx)^2 = 0 \tag{6.45}$$

4. Dividing equation (6.45) by $(dx)^2$, we have

$$a\left(\frac{dy}{dx}\right)^2 - b\left(\frac{dy}{dx}\right) + c = 0$$

Solving the equation using the quadratic formula, we have

$$\frac{dy}{dx} = \frac{b \pm \sqrt{b^2 - 4ac}}{2a} \tag{6.46}$$

This is called the characteristic equation of the second-order non-homogeneous equation (6.41).

Appendix III: Mathematical Modelling and Numerical Approaches

Appendix III(a): Numerical Approach in Solving First-Order and Second-Order PDEs

Topic: Discretization of first-order and second-order partial differential equations
Objectives: To introduce students to

i. two dimensional grid for partial derivatives of functions of two independent variables.

ii. forward, backward and central difference of partial derivatives of functions involving two independent variables.

R.P.K: Numerical Integration (Trapezoidal rule, Simpson's rule, integration with unequal/equal segments, integration in two- and three-dimensional domain).

Finite Difference Approximations

Recall that the partial derivative $\frac{\partial u}{\partial x} = u_x$ and $\frac{\partial u}{\partial y} = u_y$ of function of two independent variables x and y is given by

$$u_x = \frac{\partial u}{\partial x} = \lim_{\Delta x \to \infty} \frac{u(x+\Delta x, y) - u(x,y)}{\Delta x}$$
$$u_y = \frac{\partial u}{\partial y} = \lim_{\Delta y \to \infty} \frac{u(x, y+\Delta y) - u(x,y)}{\Delta y}$$

We can therefore use the approximations

$$\frac{\partial u}{\partial x} \approx \frac{u(x+\Delta x, y) - u(x,y)}{\Delta x}$$
$$\frac{\partial u}{\partial y} \approx \frac{u(x, y+\Delta y) - u(x,y)}{\Delta y}$$

In partial differential equations of an unknown function u involving two independent variables x and y, we use a two-dimensional finite difference grid as shown in Figure 6.6.

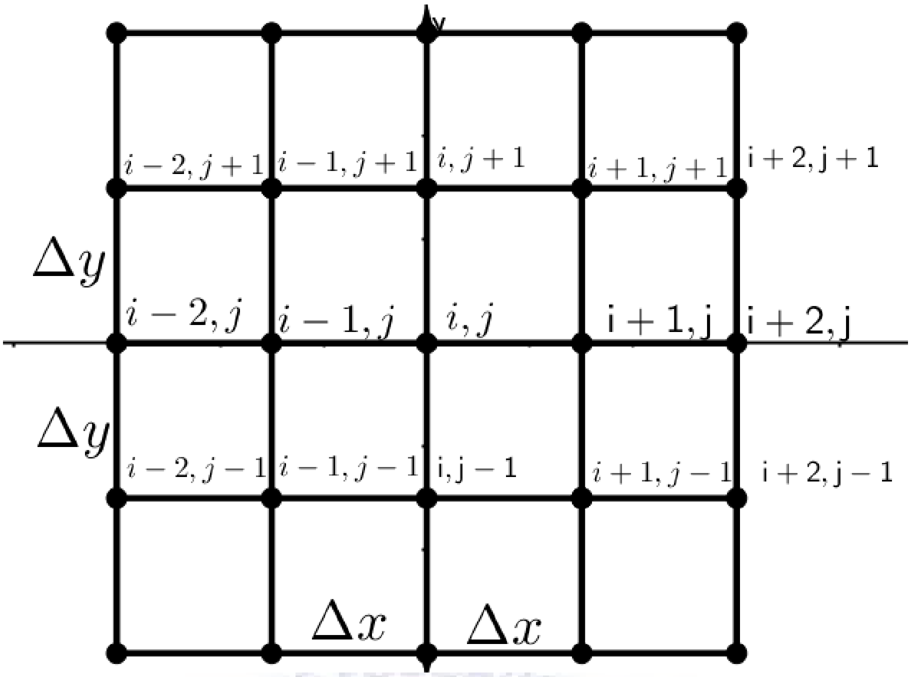

Figure 6.6: Two-Dimensional Grid for Finite Difference

For a function $u(x,y)$, the finite difference approximation for the first-order partial derivatives $u_x = \dfrac{\partial u}{\partial x}$ at $x = x_i$, $y = y_j$ can be found by fixing the value of y at y_j and treating $u(x,y_j)$ as a one-variable function. In this study, we will ignore the errors due to discretization and approximations.

The forward, backward and central difference of u_x can be express as

$$\left.\frac{\partial u}{\partial x}\right|_{i,j} \approx \frac{u(x_i+\Delta x, y_j) * u(x_i, y_j)}{\Delta x} \tag{6.47}$$

$$\left.\frac{\partial u}{\partial x}\right|_{i,j} \approx \frac{u(x_i, y_j) - u(x_i-\Delta x, y_j)}{\Delta x} \tag{6.48}$$

$$\left.\frac{\partial u}{\partial x}\right|_{i,j} \approx \frac{u(x_i+\Delta x, y_j) - u(x_i-\Delta x, y_j)}{2\Delta x} \tag{6.49}$$

The central difference approximation of the second-order partial derivatives at (x_i, y_j) can be derived as

$$\left.\frac{\partial^2 u}{\partial x^2}\right|_{i,j} \approx \frac{u(x_i+\Delta x, y_j) - 2u(x_i, y_j) + f(x_i-\Delta x, y_j)}{(\Delta x)^2} \tag{6.50}$$

$$\left.\frac{\partial^2 u}{\partial y^2}\right|_{i,j} \approx \frac{u(x_i, y_j+\Delta y) - 2u(x_i, y_j) + f(x_i, y_j-\Delta y)}{(\Delta y)^2} \tag{6.51}$$

$$\left.\frac{\partial^2 u}{\partial x \partial y}\right|_{i,j} \approx \frac{u(x_i+\Delta x, y_j+\Delta y) - u(x_i+\Delta x, y_j-\Delta y) - u(x_i-\Delta x, y_j+\Delta y) + u(x_i-\Delta x, y_j-\Delta y)}{(4\Delta x \Delta y)} \tag{6.52}$$

The region R for which the partial derivatives of the function u is taken is assumed to be rectangular. The domain for the integration is is divided into m equal parts along the x-direction and n equal parts along the y-direction. So in the discretization, $i = 0, 1, 2, ..., n$ and $j = 0, 1, 2, ..., m$.

Appendix III(b): Modelling Heat Conduction in a Thin Rod

Teaching Learning Materials: Pieces of thin insulated copper wires of length 3 5cm each, lighter and stop-watch.

In a metal rod with non-uniform temperature, heat (thermal energy) is transferred from regions of higher temperature to regions of lower temperature. Three physical principles are used here.

1. Heat (or thermal) energy of a body with uniform properties is given by

$$\text{Heat Energy} = cmu \tag{6.53}$$

   where m is the mass of the body, u is the temperature and c is the specific heat capacity and is the energy required to raise a unit mass of the substance 1 unit in temperature.

2. Fourier law of heat transfer: The rate at which heat energy transfer is proportional to negative temperature gradient at the surface. That is,

$$\frac{\text{Rate of heat transfer}}{\text{area}} = -K_0 \frac{\partial u}{\partial x} \tag{6.54}$$

   where K_0 is the thermal conductivity of the material.

3. The law of conservation of energy: Energy is neither created nor destroyed.

With these three physical properties, we deduce the heat equation.

Consider a uniform rod of length l with non-uniform temperature lying on the x-axis from $x = 0$ to $x = l$. By uniform rod, we mean the density ρ, specific heat c, thermal conductivity K_0, cross-sectional area A of the metal rod are ***all*** constant. Assume the sides of the rod are insulated and only the ends may be exposed. Also assume there is no heat source within the rod.

At this stage, students were directed to apply heat to one end of their thin insulated copper wire and measure the time it will take for them to feel the heat.

Even though there was no measuring instrument to measure the amount of heat as the heat transfer from one region to another along the rod and the heat energy at the end of the rod.

Consider an arbitrary thin slice of the rod of width Δx between x and $x + \Delta x$ as indicted in Figure 6.7.

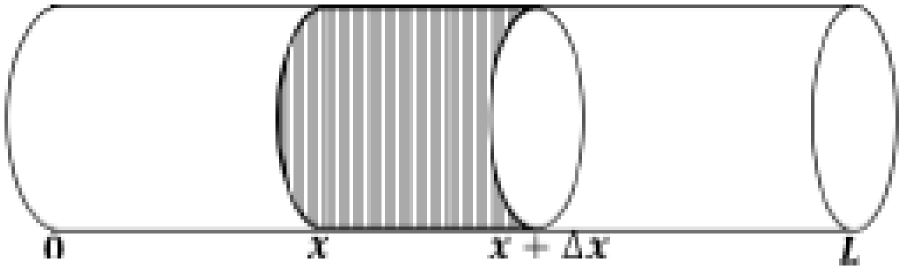

Figure 6.7: Thin Metal Rod

The slice is so thin that the temperature throughout the slice is $u(x,t)$. Thus,

$$\text{Heat energy of segment} = c \times \rho A \Delta x u(t,x) \tag{6.55}$$

By the law of conservation of energy

$$H_{LR} = H_L - H_R \tag{6.56}$$

where H_{LR} represent change of heat energy of the segment in time Δt, H_L is heat in from left boundary and H_R is the heat out from right boundary. From Fourier's law,

$$c\rho A \Delta x u(x,t+\Delta t) - c\rho A \Delta x u(x,t) = \Delta t A \left(-K_0 \frac{\partial u}{\partial t}\right)_x - \Delta t A \left(-K_0 \frac{\partial u}{\partial x}\right)_{x+\Delta x} \tag{6.57}$$

Recall that ρ, c, A, K_0 are constants. Rearranging equation (6.57) yields

$$\frac{u(x,t+\Delta t) - u(x,t)}{\Delta t} = \frac{K_0}{c\rho}\left(\frac{\left(\frac{\partial u}{\partial x}\right)_{x+\Delta x} - \left(\frac{\partial u}{\partial x}\right)_x}{\Delta t}\right) \tag{6.58}$$

Taking the limits as $\Delta t, \Delta x \to 0$ gives the heat equation

$$\frac{\partial u}{\partial t} = \kappa \frac{\partial^2 u}{\partial x^2} \tag{6.59}$$

where

$$\kappa = \frac{K_0}{c\rho} \tag{6.60}$$

is called the thermal diffusivity.

Appendix III(c): Modelling the Wave Equation in a String

Teaching/Learning Materials: An elastic strings, nails, a pieces of sticks.

Suppose we stretched a perfectly flexible or elastic string to a length L and fix the ends. If the string is plugged at time $t = 0$ and release so that it vibrates, we can describe the motion of the string by determining the deflection $u(x,t)$ at any point x and time $t > 0$. That is, we would like to be able to determine the shapes of these vibrations and for that matter derive a PDE, the wave equation, which must be satisfied by the position of the string. But before we go to the derivation of the wave equation, let's consider some physical assumptions that must be considered.

Assumptions

Physical assumptions leading to the model are

1. Our string is homogeneous: The mass per unit length being constant. That is, the string is made of the same material all the way through.

2. Our string does not furnish any resistance to bending (i.e. perfect elasticity).

3. The gravitational force acting on the string can be ignored due to the large tension caused by the string prior to clamping it at the end points.

4. Each particle of the string moves vertically only, exhibiting small motions so the deflection and the slope within each point of the string remains small. i.e. absolute value.

Suppose our string has length L and that it is nailed down at the endpoints $(0,0)$ and $(L,0)$ as shown in Figure 6.8. The string's motion is described by its position function

$$u(x,t) = height\ of\ string\ at\ point\ x\ at\ time\ t \tag{6.61}$$

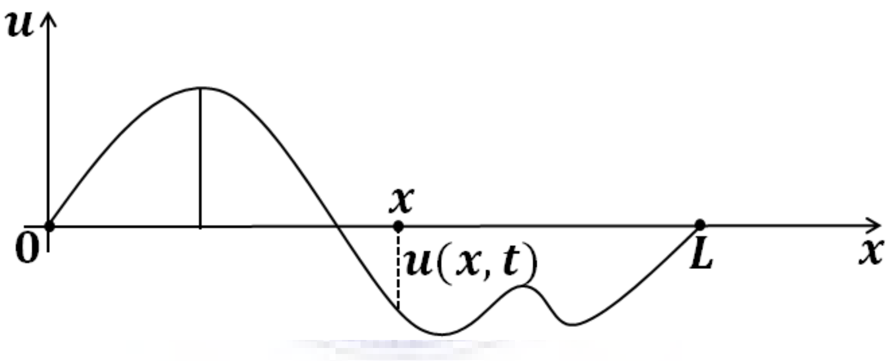

Figure 6.8: Vibrating String

Let's consider the force acting on a small portion of our string as shown in Figure 6.9

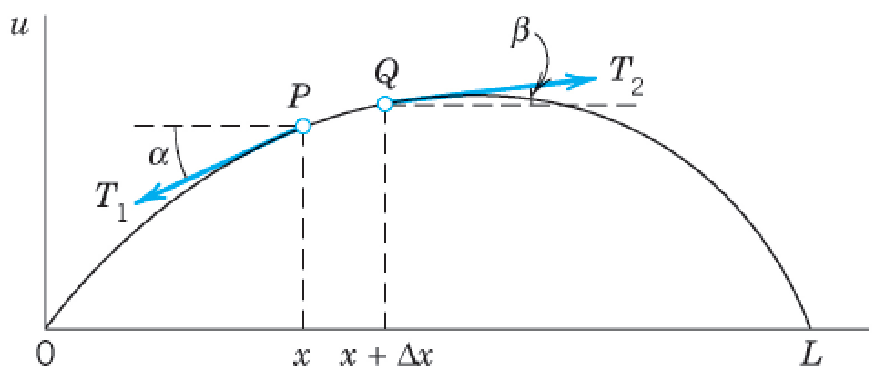

Figure 6.9: Portion of the String

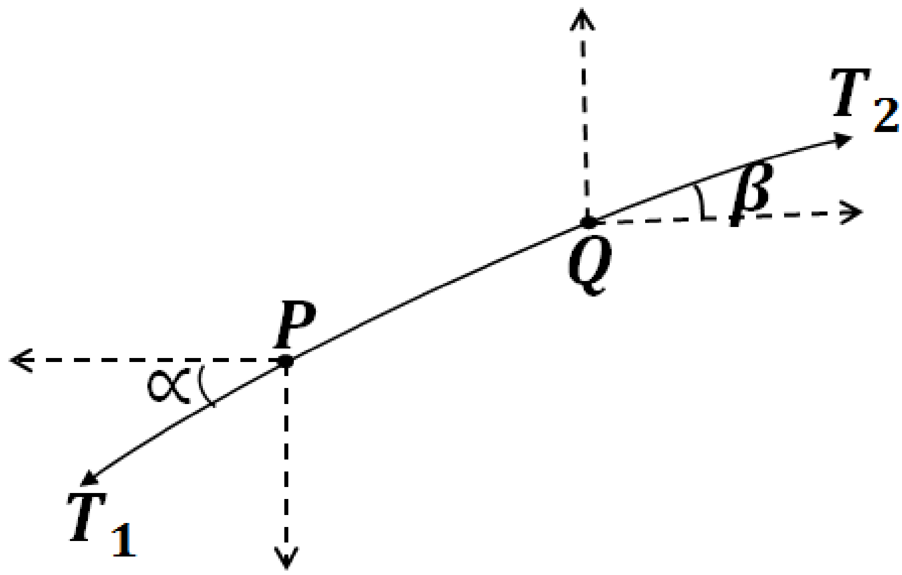

Figure 6.10: Tensions on the String

- Since we have assumed that our string offers no resistance to bending, the tension is tangential to the curve of the string at every point.
- Let T_1 and T_2 denote the tensions (respectively) at the points P and Q or our small portion.
- Due to our assumption, there is no movement in the horizontal direction, the horizontal components of the tension must be constant. That is, each point on the string moves only in u-direction and the motion is restricted to the x-u plane. Thus each point on the string occupies the same x-coordinate at all times.

To obtain the equation of vibrating string, we consider the forces acting on a small portion of the string as shown in Figure 6.9. Since the string offers no resistance to bending, the tension is tangential to the curve of the string at each point. Let T_1 and T_2 be the tension at the endpoints P and Q of that portion. Since the points of the string move vertically, there is no motion in the horizontal direction. Hence the horizontal components of the tension must be constant. Then we have

$$\vec{T}_1 \cos\alpha = \vec{T}_2 \cos\beta = T = Constant \tag{6.62}$$

Since the string vibrate freely in the vertical direction, there are only two forces, namely, the vertical components - $-T_1 \sin\alpha$ of T_1 at P and $T_2 \sin\beta$ of T_2 at Q. Here, the minus sign appears because the component at P points to the opposite direction of u and $\sin\alpha$ is positive. That is, the minus sign occurs due to the components at P pointing downwards. Applying Newton's law $(F = ma)$, we know that the resultant to the 2 forces is equal to the mass of the string $(\rho\Delta x)$ times the acceleration $\frac{\partial^2 u}{\partial t^2}$ evaluated at some point between x and $x + \Delta x$ where ρ is the mass of the undeflected string per unit length (the density function) and Δx is the length of the portion of the undeflected string.

$$\text{Thus } T_2 \sin\theta_2 - T_1 \sin\theta_1 = \rho\Delta x \frac{\partial^2 u}{\partial t^2} \tag{6.63}$$

Dividing through by equation (6.62), we have

$$\frac{T_2 \sin\theta_2}{T_2 \cos\theta_2} - \frac{T_1 \sin\theta_1}{T_1 \cos\theta_1} = \frac{\rho\Delta x}{T}\frac{\partial^2 u}{\partial t^2} = \tan\theta_2 - \tan\theta_1 \tag{6.64}$$

Note that, the tangent values are the corresponding slopes of the string at x and $x+\Delta x$, that is

$$\tan\theta_1 = u_x(x,t) \ and \ \tan\theta_2 = u_x(x+\Delta x,t) \tag{6.65}$$

Substituting equation (6.65) into (6.64), we have

$$\frac{1}{\Delta x}[u_x(x+\Delta x,t) - u_x(x,t)] = \frac{\rho\partial^2 u}{T\partial t^2} \tag{6.66}$$

The limit as $\Delta x \to 0$

$$\frac{T}{\rho}u_{xx} = u_{tt} \tag{6.67}$$

Let $c^2 = \frac{T}{\rho}$

$$\text{Therefore } u_{tt} = c^2 u_{xx} \tag{6.68}$$

Equation (6.68) is known as the **one-dimensional wave equation** where $c^2 = \frac{T}{\rho}$ is called the wave speed.

Appendix III(d): Microsoft Excel Implementation of Solutions to Heat Equation

Teaching Learning Materials: Computer with Microsoft Excel application installed.

Introduction: Microsoft Excel was used to implement a simple approach in solving initial-value boundary problems of partial differential equations. In this excel implementation, finite difference method was used for both the heat equation and wave equation. This approach was adapted from Radwan (2001) and Ketkar and Reddy (2003) and modified.

Discretization of Heat Equation

The finite difference approach was used to find the solution to one-dimensional heat equation of the form

$$u_t = \alpha^2 u_{xx} \text{ for } 0 < x < L, 0 < t \leq T \tag{6.69}$$

with initial conditions

$$u(x,0) = f(x), 0 < x < L \tag{6.70}$$

and boundary conditions

$$u(0,t) = g_1(t), u(L,t) = g_2(t), 0 < t \leq T \tag{6.71}$$

The following steps were followed:

1. The interval $[0,L]$ was divided into n pieces, each of length $h = \Delta x = L/n$.

2. The corresponding points (on the two-dimensional grid) were denoted by x_i for $i = 0,1,2,\ldots,n$. The ends of the interval were at $x_0 = 0$ and $x_n = L$.

3. The interior points were $x_i = ih$, for $i = 1, 2, ..., n-1$.
4. Similarly, the mesh for the time interval $[0, T]$ was computed by dividing the intervals by m with $k = \Delta t = T/m$ and $t_j = 0, 1, 2, ..., m$.
5. The ends of the time interval were $t_0 = 0$ and $t_m = T$.
6. The solution at the grid points $u(x,t)$ was denoted by $u_{i,j}$.
7. The finite difference techniques was used to replace the partial derivatives with difference quotients.
8. The forward difference formula was used for the heat equation for u_t. That is

$$u_t = \frac{1}{k}[u_{i,j+1} - u_{i,j}] \tag{6.72}$$

and the u_{xx} was replaced by

$$u_{xx} = \frac{\alpha^2}{h^2}[u_{i-1,j} - 2u_{i,j} + u_{i+1,j}] \tag{6.73}$$

9. A linear system of equation was obtained after the space derivative was replaced by the difference formula at the jth time step and the time derivative by a forward difference formula.
10. The temperature u at the grid points wa given by

$$\frac{1}{k}\left[u_{i,j+1} - u_{i,j}\right] = \frac{\alpha^2}{h^2}\left[u_{i-1,j} - 2u_{i,j} + u_{i+1,j}\right] \tag{6.74}$$

11. We let $r = \dfrac{\alpha^2 k}{h^2}$ and then the solution $u_{i,j+1}$ is found.
12. Because the solution is known at $t = 0$, the first step was explicitly solved and was then followed by step by step approach.
13. The condition for the stability of the finite difference scheme is $\left|\dfrac{2\alpha^2 \Delta t}{(\Delta x)^2}\right| \leq 1$ and so $\Delta t \leq \dfrac{(\Delta x)^2}{2\alpha^2}$ and so $0 < r < 0.5$.
14. We therefore solve numerically for the temperature using

$$u_{i,j+1} = r[u_{i+1,j} + u_{i-1,j}] + (1-2r)u_{i,j}$$

Microsoft Excel Implementation Steps

We take steps to find the temperature distribution of a metal rod of length $2cm$ which was kept under room temperature (30^oC) with boundary conditions $u(0,t) = 80^oC, u(2,t) = 30^oC$ given that the heat equation is $u_t = 0.2u_{xx}$. We will look at this distribution by dividing the rod into 5 equal parts. We also assume that there is no heat loss to the surroundings. We go through the following steps to find out how the temperature varies with respect to time and with respect to space:

1. Computing $\Delta x = \frac{L}{m}$ where m is the number of divisions of the rod and L is the length of the rod, we have $\Delta x = \frac{2}{5} = 0.4$

2. Since we want to we want to verify the temperature distribution with respect to time and space, we have our grid in the form shown in Figure 6.11.

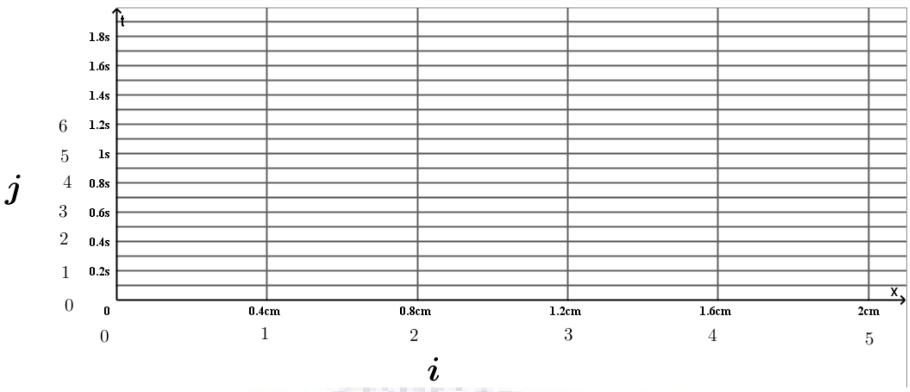

Figure 6.11: The Grid

3. Note that, $\alpha^2 = 0.2$ and so Δt should in such a way that $\Delta t \leq \frac{0.2^2}{2(0.2)} = 0.4$. In this case we take $\Delta t = 0.1$.

4. Launch Microsoft Excel.

5. In cell **A1**, type "**alpha_sq**" and type **0.2** in cell **B1**. This is to say that $\alpha^2 = 0.2$.

6. In cell **F1** type **L** and type **2** in cell **G1**. This is to indicate that the length $L = 2cm$.

7. In cell **F2**, type Division and type **5** in cell **G2** indicating that the number of divisions $n = 5$.

8. In cell **C1** type **Delta x** and type **=G1/G2** in cell **D1** to represent $\Delta x = \frac{L}{n}$.

9. In cell **D2** type **Delta t** and type 0.1 to represent $\Delta t = 0.1$. Note that, $0 \leq \Delta t \leq 0.4$.

10. In cell **A3**, type **r** and type **=B1*E2/E1∧2** in cell **B3** to compute the value of **r**. That is $r = \frac{\alpha^2 \Delta t}{(\Delta x)^2}$.

11. Your Excel screen should look like as illustrated in Figure 6.12.

	A	B	C	D	E	F	G
1	alpha_sq	0.2	Delta x	0.4		L	2
2			Dt	0.1		Divisions	5
3	r	0.125				DT<=	0.4

Figure 6.12: Excel Implementation 1

12. In cell **B5**, type **i** and enter **1** to **6** from cell **C5** to **H5** as illustrated in Figure 6.13. This represents the **ith** position of the rod in space, that is, along the x-axis.

13. In cell **A6**, type **j** and type **1** in cell **A7**. In cell **A8**, type **=A7+1**. Click in cell **A8** and drag the handler to generate the numbers from **1** to **51** in cells **A7** to **A57** as illustrated in Figure 6.13. This illustrates the **jth** position in time along the vertical axis.

14. In cell **B6**, type **t/x** for time in column and space in row **5**. In cell **C**, enter the number 0 to indicate the beginning of the rod.

15. Click in cell **D6** and type **=C6+$D$1** and press enter. Click inside cell **D6** and use the handle to fill from cell **D6** to **H6** as indicated in Figure 6.13.

16. In cell **B7**, type **0**. Click in cell **B8** and type **=B7+$D$2**. Click in cell **B8** and drag the handler to fill cells **B7** to **B57** as indicated in Figure 6.13. This represents the changes in time.

	A	B	C	D	E	F	G	H	I	J	K
1	alpha_sq	0.2	Delta x	0.4		L	2				
2			Dt	0.1		Divisions	5				
3	r	0.125				DT<=	0.4				
4											
5		i	1	2	3	4	5	6			
6	j	t/x	0	0.4	0.8	1.2	1.6	2			
7	1	0	30	30	30	30	30	30		Initial condition	
8	2	0.1	80					30			
9	3	0.2	80					30			
10	4	0.3	80					30			
11	5	0.4	80					30			
12	6	0.5	80					30			
13	7	0.6	80					30			
14	8	0.7	80					30			
15	9	0.8	80					30			
16	10	0.9	80					30			
17	11	1	80					30			
18	12	1.1	80					30			
19	13	1.2	80					30			
20	14	1.3	80					30			
21	15	1.4	80					30			
22	16	1.5	80					30			
23	17	1.6	80					30			

Figure 6.13: Excel Implementation 2

17. In cells **C7** to **H7**, enter 30 to as shown in Figure 6.13. This indicates the initial condition $u(x,0) = 30$.

18. In cells **C8** to **C57**, enter **80** to indicate the left boundary condition $u(0,t) = 80$ and **30** in cells **H8** to **H57** to indicate the right boundary condition $u(2,t) = 30$ as shown in Figure 6.13.

19. We now compute the temperature distribution $u_{i+1,j}$ along the rod in time (with respect to time) and in space (with respect to the length) using

$$u_{i,j+1} = r[u_{i+1,j} + u_{i-1,j}] + (1-2r)u_{i,j}$$

20. In cell **D8**, type "**=$B$3*(E7+C7)+(1-2*$B$3)*D7**". Click inside cell D8 and drag the handler from **D8** to **G8** to fill in the time steps.

21. Select cells D8 to G8 and use the handler to fill cells D8 to G57 as shown in Figure 6.14.

	A	B	C	D	E	F	G	H	I	J	K
1	alpha_sq	0.2	Delta x	0.4		L	2				
2			Dt	0.1		Divisions	5				
3	r	0.125				DT<=	0.4				
4											
5		i	1	2	3	4	5	6			
6	j	t/x	0	0.4	0.8	1.2	1.6	2			
7	1	0	30	30	30	30	30	30		Initial condition	
8	2	0.1	80	30	30	30	30	30			
9	3	0.2	80	36.25	30	30	30	30			
10	4	0.3	80	40.9375	30.7813	30	30	30			
11	5	0.4	80	44.5508	31.9531	30.0977	30	30			
12	6	0.5	80	47.4072	33.2959	30.3174	30.0122	30			
13	7	0.6	80	49.7174	34.6875	30.6516	30.0488	30			
14	8	0.7	80	51.624	36.0617	31.0807	30.1181	30			
15	9	0.8	80	53.2257	37.3844	31.583	30.2236	30			
16	10	0.9	80	54.5923	38.6394	32.1383	30.3656	30			
17	11	1	80	55.7742	39.8209	32.7293	30.5415	30			
18	12	1.1	80	56.8082	40.9286	33.3423	30.7473	30			
19	13	1.2	80	57.7223	41.9653	33.9662	30.9782	30			
20	14	1.3	80	58.5373	42.935	34.5926	31.2295	30			
21	15	1.4	80	59.2699	43.8425	35.215	31.4962	30			
22	16	1.5	80	59.9327	44.6925	35.8286	31.774	30			
23	17	1.6	80	60.5361	45.4895	36.4297	32.0591	30			

Sheet1 Sheet4

Figure 6.14: Excel Implementation of the temperature Distribution in the rod

22. We will now plot the graph of the temperature distributions as 0, 1, 2, 3, 4 and 5 seconds using scatter plot.

23. Select cells **C6** to **H7** and go to **Insert** → **Charts** and select "**Scatter with Smooth Lines and Markers**". You will obtain the first scatter plot with the initial conditions as indicated in Figure 6.15.

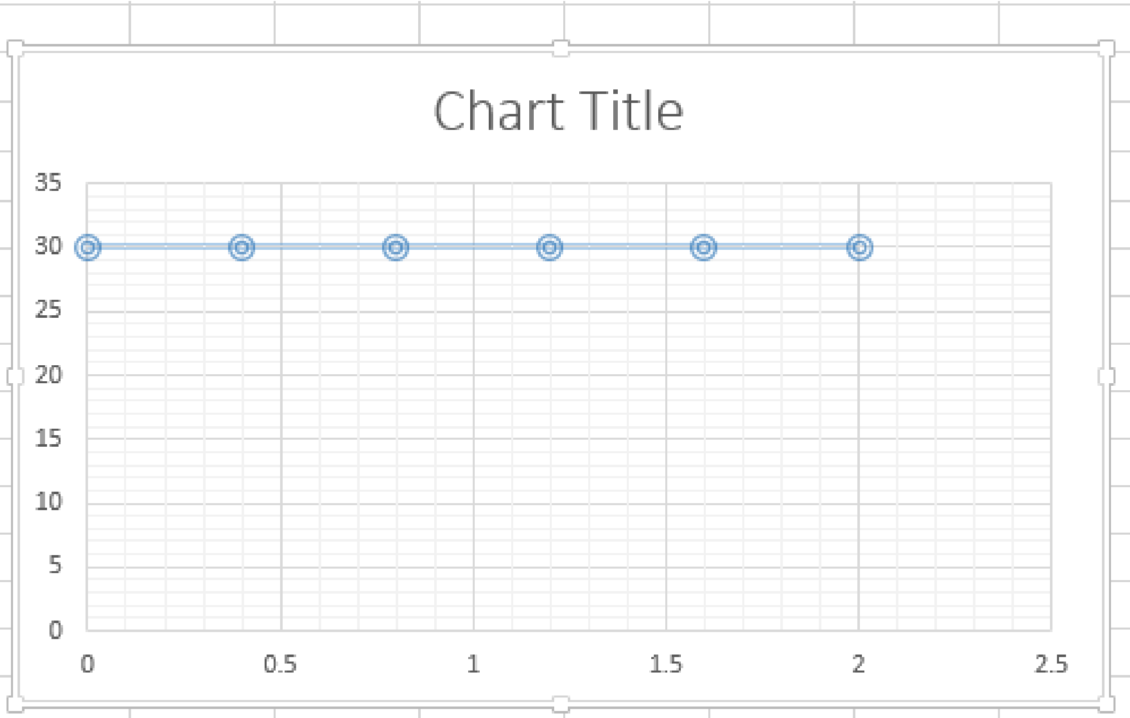

Figure 6.15: Scatter Plot

24. Right-click in the plot area and choose ”Select Data”. The Select Data Source dialog box will display as shown in Figure 6.16.

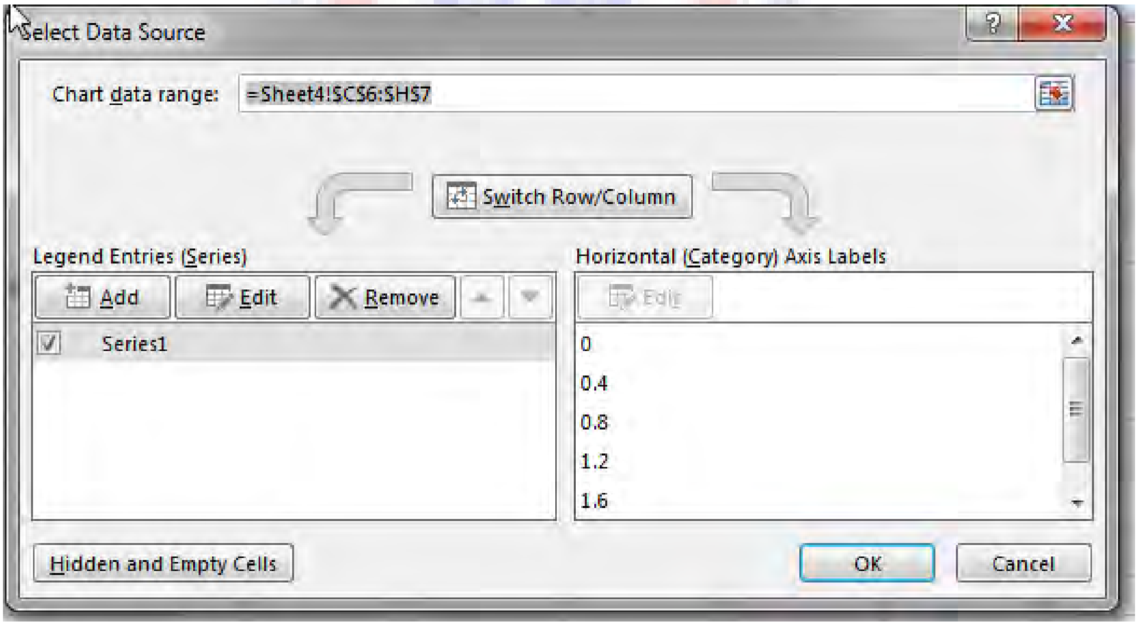

Figure 6.16: Data Source Selection

25. Click Add. The Edit Series dialog box will display. In the edit series dialog box, type ”=Sheet4!$C$6:$H$6” for ”Series X Values” and type ”=Sheet4!$C$17:$H$17” for ”Series Y Values” as indicated in Figure 6.17 and click OK. Note that, Yours may be Sheet1!, Sheet2!, etc depending on the name of the sheet you are working in. I am working in Sheet4.

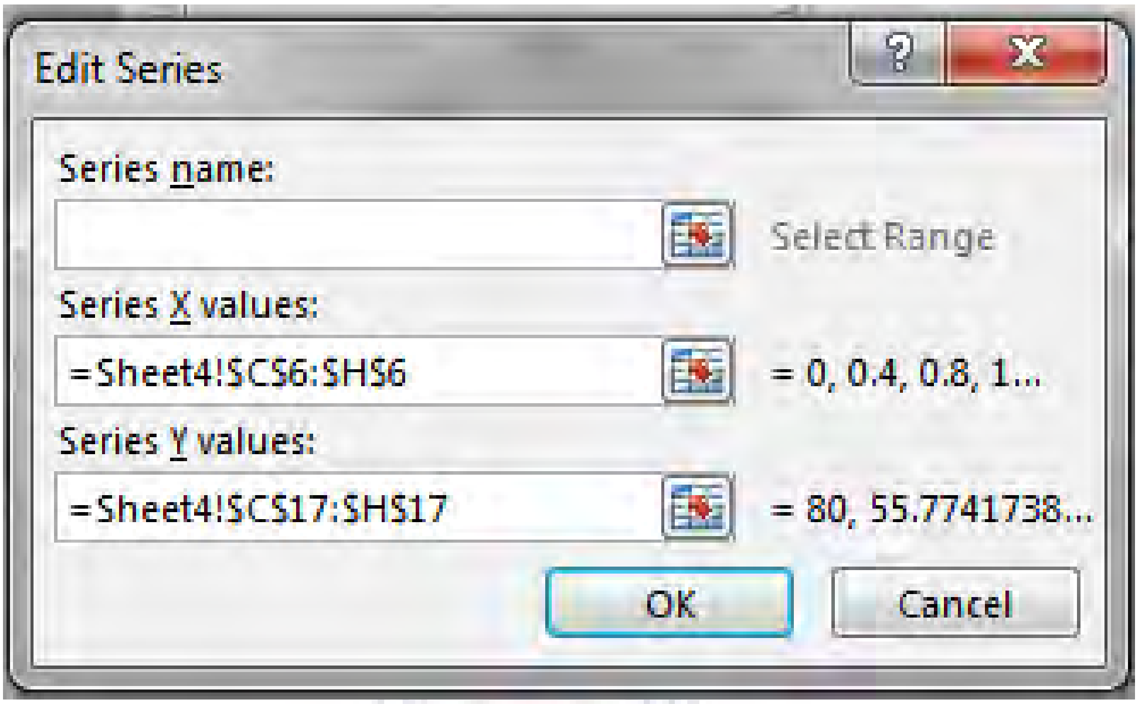

Figure 6.17: Edit Series Dialog

26. Repeat step (25) again but repeat the same as range for Series with X values but **=Sheet4!$B$27:$H$27** for series with Y values and click Ok. The will be for the time $t = 1s$.

27. Repeat the process for the times $t = 2, 3, 4, 5$. When done, your window will look as shown in Figure 6.18.

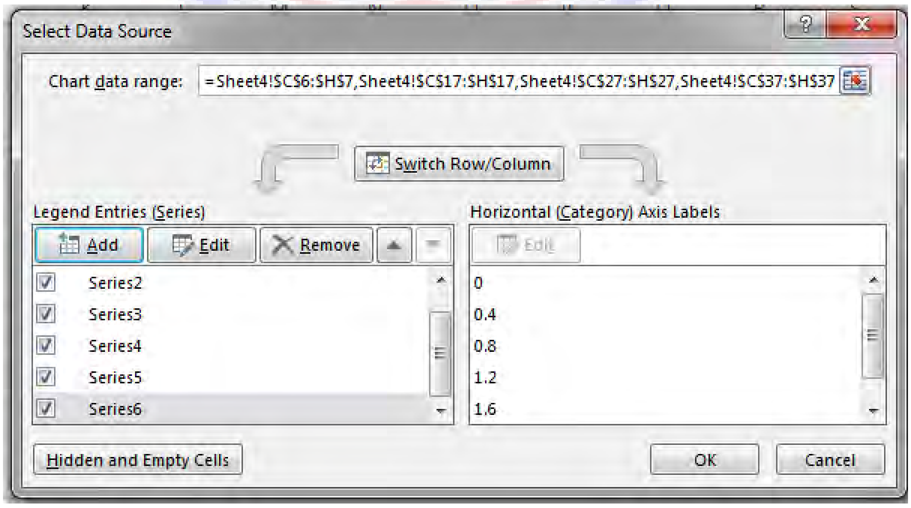

Figure 6.18: Final Data Source

28. Click OK again and the graph will display as shown in Figure 6.19.

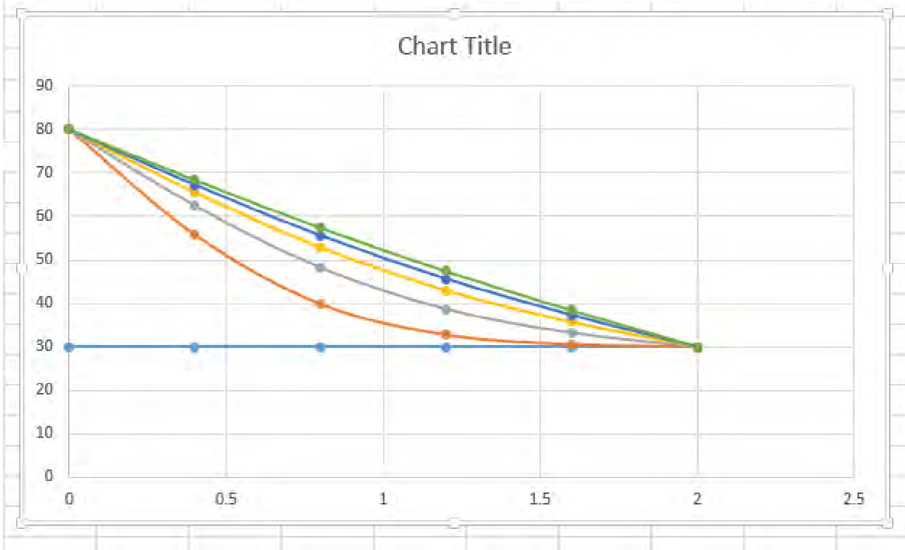

Figure 6.19: Graph of Heat Distribution in the Rod

29. To check the stability, change the value of Δt in cell D2 to 1. Write your comment. Read both the graph and the results and write your comment.

30. Change it to 0.3 and write your comment.

31. Change it 0.1 and change the length L to different values to verify the heat distribution in different length of the rod.

32. Change the number of divisions and write your comment. In all cases, look at the value of Δt in cell **D2** and compare to the condition in cell **G3**. Adjust the value of Δt in cell **D2** to meet the condition and write your comments.

Appendix VI: Interview Guide

Length: 10 minutes
Primary goal: To determine the challenges students encounter during teaching and learning of PDEs and the factors that lead to these challenges students faces in learning partial differential equations.
Verbal Consent: You have been selected among 4 other students for this interview. The main purpose of this interview is to solicit information about some of the challenges you face during instructional delivery for partial differential equations. I am actually interested in the factors that contributed to these challenges and anything you say is for the purpose of studies. So feel free to tell me anything you believe or you think about my question. All informations provided will be confidential. Under no circumstance will your information be shared with anyone else and all information is on academic purpose only.

1. Tell me briefly about your interest in Mathematics in general.

2. How do you find teaching and learning of PDEs?

3. Are you affected by the following factors during teaching and learning of PDEs?

(a) Instructional Strategies and methods
(b) Your ability to recall what you have been taught previously that are pre-requisite to PDEs.
(c) Computational ability: Your ability to formulate mathematical formulates and algebraically manipulate the equations in order to be able to solve.
(d) Availability ICT tools.
(e) Comprehension. Your ability to read and understand the given problem or task.
(f) Motivation
(g) Computer applications for mathematics
(h) Gender
(i) Any other thing you can think of.

4. Comparing the traditional teaching method you were going through to this instructional approach, which of them will you prefer and why?

5. Briefly tell me about how you fill about the whole instructional approach and what you hope needs improvement.

Appendix V: Analysis of the Pilot Study Results

Appendix V(a): Reliability Analysis

Table 6.1 Case Processing Summary

		N	%
Cases	Valid	80	100.00
	Excluded[a]	0	0
	Total	80	100.00

a. Listwise deletion based on all variables in the procedure

Table 6.2 Reliability Statistics for Split-Half Method

Cronbach's Alpha	Part 1	Value	0.745
		No. of Items	3[a]
	Part 2	Value	0.791
		No. of Items	3[b]
	Total No. of Items		6
Correlation Between Forms			0.829
Spearman-Brown Coefficient	Equal Length		0.907
	Unequal Length		0.907
Gutman Split-Half Coefficient			0.903

a. The items are: Question 1, 3 and 5
b. The items are: Question 2, 4 and 6

Table 6.3 Scale Statistics

	Mean	Variance	Std. Deviation	No. of Items
Part 1	15.55	37.820	6.150	3[a]
Part 2	16.65	29.699	5.450	3[b]
Both Parts	32.20	123.095	11.095	6

a. The items are: Question 1, 3 and 5
b. The items are: Question 2, 4 and 6

Table 6.4 Cronbach's Alpha Reliability Statistics

Cronbach's Alpha	No. of Items
0.878	6

Appendix V(b): Analysis of Diagnostic Test

Appendix V(b)(i): Analysis of the Diagnostic Test for the pilot Study before regrouping

Table 6.5 Group Statistics

	Groups	N	Mean	Std. Deviation	Std. Error Mean
Scores	Control Group	40	7.450	1.9075	0.3016
	Experimental Group	40	6.025	1.7612	0.2785

Table 6.6 Independent Samples Test

			Scores	
			EVA	EVNA
Levene's Test for Equality of Variances	F		0.976	
	Sig.		0.326	
t-test for Equality of Means	t		3.471	3.471
	df		78	77.509
	Sig. (2-tailed)		0.001	0.001
	Mean Difference		1.4250	1.4250
	Std. Error Difference		0.4105	0.4105
	95% Confidence Interval of the Difference	Lower	0.6078	0.6077
		Upper	2.2422	2.2422

Keys: EVA-Equal Variances Assumed, EVNA-Equal Variances not Assumed

Analysis for the Diagnostic Study after Re-Grouping

Table 6.7 Group Statistics

	Groups	N	Mean	Std. Deviation	Std. Error Mean
Scores	Control Group	40	6.775	1.9675	0.3111
	Experimental Group	40	6.700	1.9768	0.3126

Table 6.8 Independent Samples Test

			Scores	
			EVA	EVNA
Levene's Test for Equality of Variances	F		0.018	
	Sig.		0.894	
t-test for Equality of Means	t		0.170	0.170
	df		78	77.998
	Sig. (2-tailed)		0.865	0.865
	Mean Difference		0.0750	0.0750
	Std. Error Difference		0.441	0.441
	95% Confidence Interval of the Difference	Lower	-0.8029	-0.8029
		Upper	0.9529	0.9529

Keys: EVA-Equal Variances Assumed, EVNA-Equal Variances not Assumed

Appendix V(c): Analysis of the Pilot Study

Appendix V(c)(i): Analysis of the Pilot Study Test 1

Table 6.9 Group Statistics

	Groups	N	Mean	Std. Deviation	Std. Error Mean
Scores	Control Group	40	7.225	1.7612	0.2785
	Experimental Group	40	7.975	1.6562	0.2619

Table 6.10 Independent Samples Test

			Scores	
			EVA	EVNA
Levene's Test for Equality of Variances	F		0.235	
	Sig.		0.629	
t-test for Equality of Means	t		-1.962	-1.962
	df		78	77.707
	Sig. (2-tailed)		0.053	0.053
	Mean Difference		-0.75	-0.75
	Std. Error Difference		0.441	0.441
	95% Confidence Interval of the Difference	Lower	-1.511	-1.5111
		Upper	0.011	0.0111

Keys: EVA-Equal Variances Assumed, EVNA-Equal Variances not Assumed

Appendix V(c)(ii): Analysis of the Pilot Study Test 2

Table 6.11 Group Statistics

	Groups	N	Mean	Std. Deviation	Std. Error Mean
Scores	Control Group	40	6.200	1.2850	0.2032
	Experimental Group	40	7.725	1.7539	0.2773

Table 6.12 Independent Samples Test

			Scores	
			EVA	EVNA
Levene's Test for Equality of Variances	F		3.798	
	Sig.		0.055	
t-test for Equality of Means	t		-4.436	-4.436
	df		78	71.503
	Sig. (2-tailed)		0.000	0.000
	Mean Difference		-1.5250	-1.5250
	Std. Error Difference		0.441	0.441
	95% Confidence Interval of the Difference	Lower	-2.2094	-2.2104
		Upper	-0.8406	-0.8396

Keys: EVA-Equal Variances Assumed, EVNA-Equal Variances not Assumed

Appendix V(c)(ii): Analysis of the Pilot Study Test 3

Table 6.13 Group Statistics

	Groups	N	Mean	Std. Deviation	Std. Error Mean
Scores	Control Group	40	4.600	2.4158	0.382
	Experimental Group	40	7.450	1.825	0.2886

Table 6.14 Independent Samples Test

			Scores	
			EVA	EVNA
Levene's Test for Equality of Variances	F		12.057	
	Sig.		0.001	
t-test for Equality of Means	t		-5.953	-5.953
	df		78	72.579
	Sig. (2-tailed)		0.000	0.000
	Mean Difference		-2.85	-2.85
	Std. Error Difference		0.4748	0.4748
	95% Confidence Interval of the Difference	Lower	-3.803	-3042
		Upper	-1.897	-1.8958

Keys: EVA-Equal Variances Assumed, EVNA-Equal Variances not Assumed

Appendix V(d): Test of Assumptions for Pilot Study

Appendix V(d)(i): Test for Normality

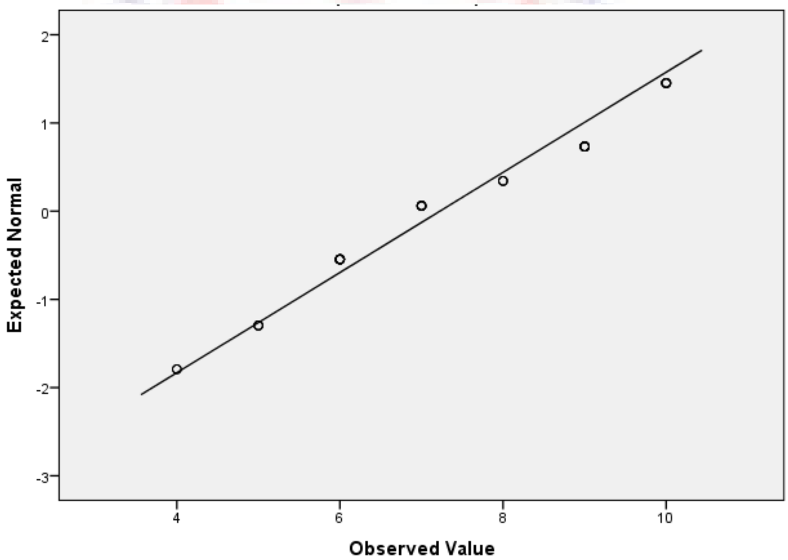

Figure 6.20: Q-Q Plot of the Control for Group Test 1 Scores

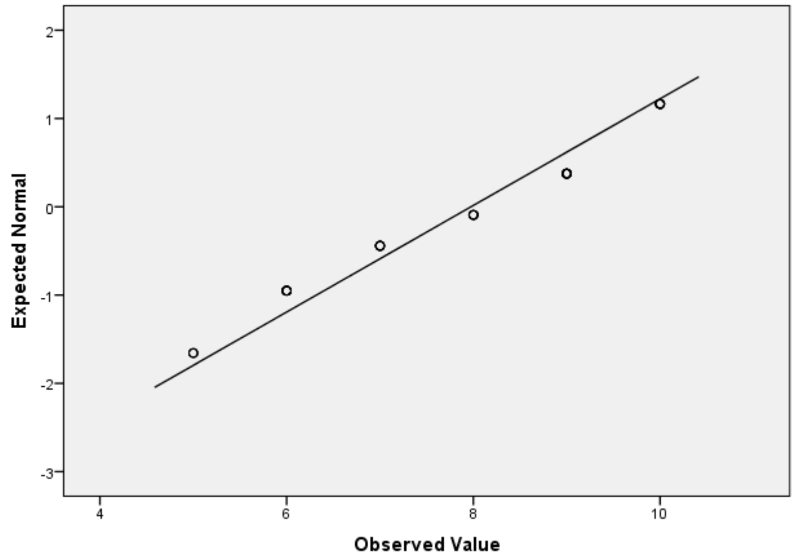

Figure 6.21: Q-Q Plot of the Experimental Group for Test 1 Scores

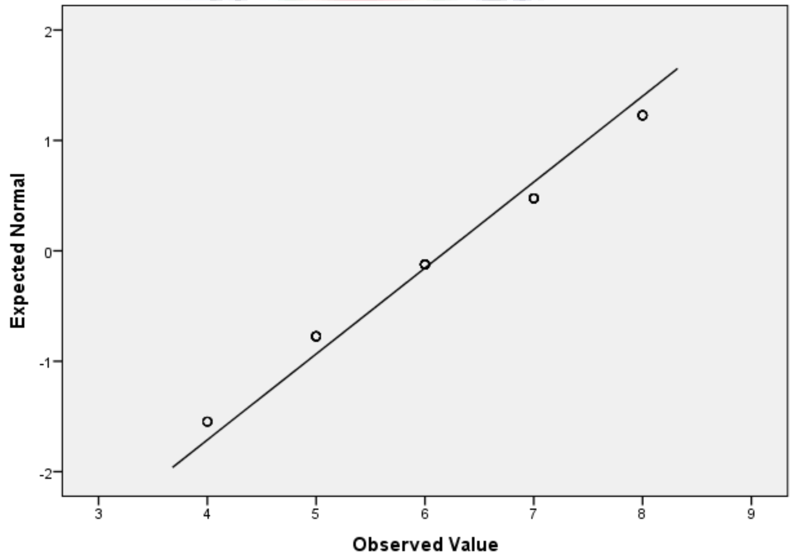

Figure 6.22: Q-Q Plot of the Control Group for Test 2 Scores

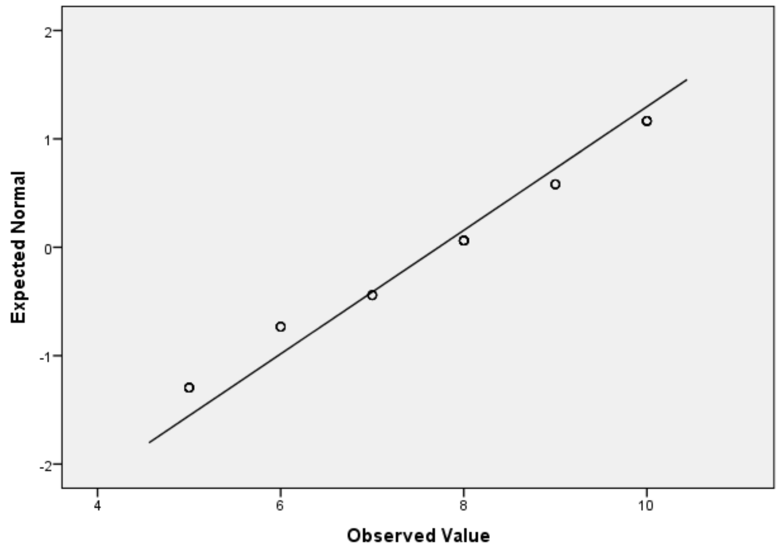

Figure 6.23: Q-Q Plot of the Experimental Group for Test 2 Scores

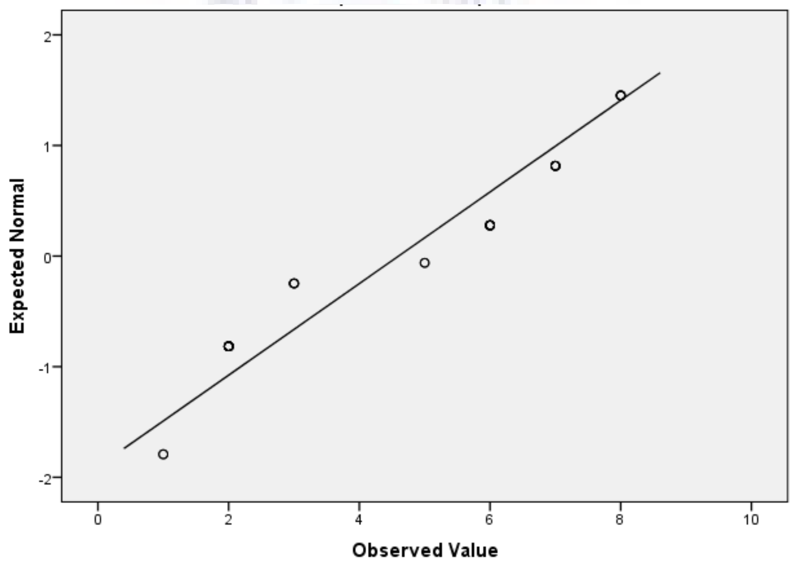

Figure 6.24: Q-Q Plot of the Control Group for Test 3 Scores

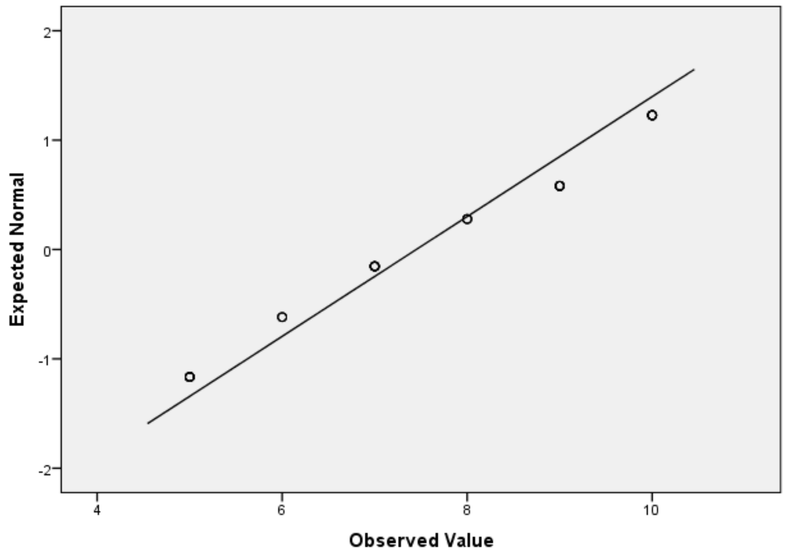

Figure 6.25: Q-Q Plot of the Experimental Group for Test 3 Scores

Appendix V(d)(ii): Box Plot for the Pilot Study

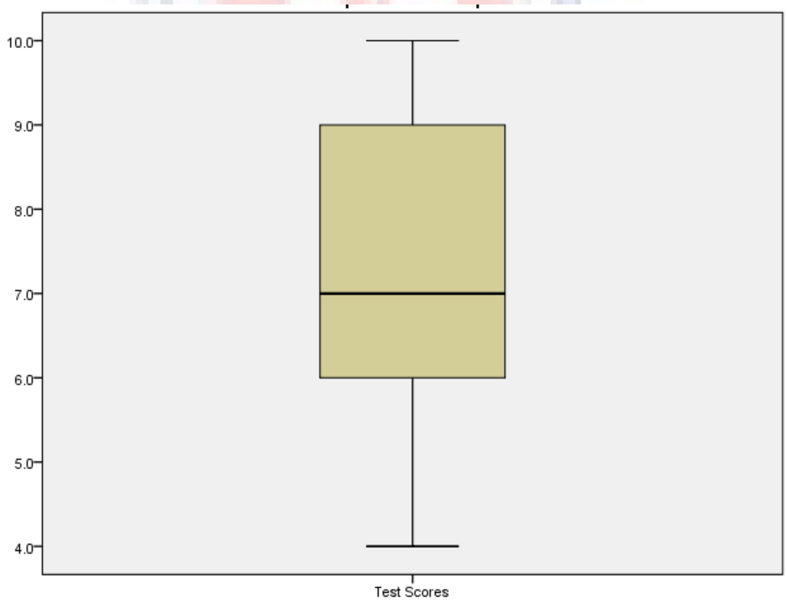

Figure 6.26: Box Plot of the Control Group for Test 1

Figure 6.27: Box Plot of the Experimental Group for Test 1

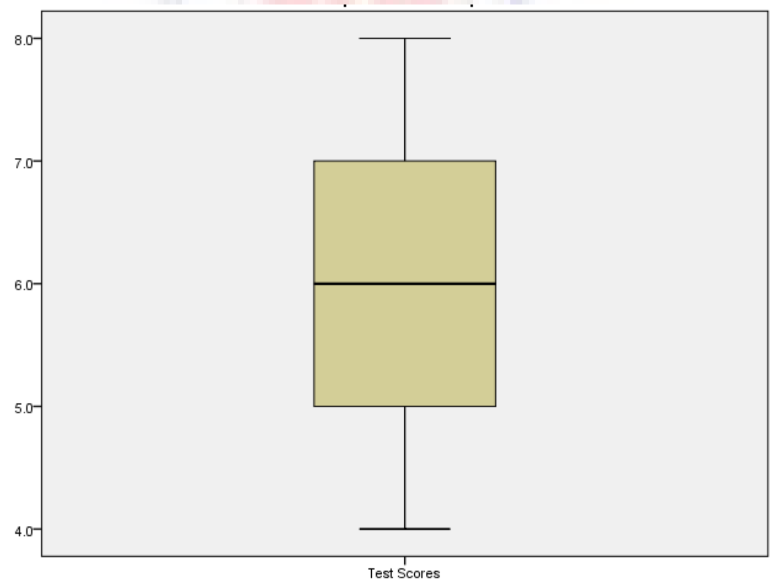

Figure 6.28: Box Plot of the Control Group for Test 2

Figure 6.29: Box Plot of the Experimental Group for Test 2

Figure 6.30: Box Plot of the Control Group for Test 3

Figure 6.31: Box Plot of the Experimental Group for Test 3

Table 6.15 Test for Normality of Pilot Study

			Kolmogorov-Smirnov			Shapiro-Wilk		
	Tests	Groups	Statistic	df	Sig.	Statistic	df	Sig.
Test Scores	Test 1	Control Gp	0.207	40	0.000	0.911	40	0.004
		Experimental Gp	0.207	40	0.000	0.893	40	0.001
	Test 2	Control Gp	0.158	40	0.013	0.907	40	0.003
		Experimental Gp	0.187	40	0.001	0.888	40	0.001
	Test 3	Control Gp	0.219	40	0.000	0.856	40	0.000
		Experimental Gp	0.172	40	0.004	0.881	40	0.001