UNIVERSITY OF EDUCATION, WINNEBA

SENIOR HIGH SCHOOL STUDENTS' POOR PERFORMANCE, ERRORS AND MISCONCEPTIONS IN DETERMINING MEASURES, OF VARIATION IN STATISTICS

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MASTER OF PHILOSOPHY

2024

UNIVERSITY OF EDUCATION, WINNEBA

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A thesis in the Department of Mathematics Education,
Faculty of Science Education,
submitted to the School of
Graduate Studies, in partial fulfillment
of the requirements for the award of the degree of
Master of Philosophy
(Mathematics Education)
in the University of Education, Winneba

NOVEMBER, 2024

DECLARATION

STUDENT'S DECLARATION

I, Gifty Dzifa Amewu, declare that this thesis, with the exception of quotations and references contained in published works which have all been identified and duly acknowledged, is entirely my own original work, and it has not been submitted, either in part or whole, for another degree elsewhere.

Signature**: ……………………………….......**

Date: **………………………………………………**

SUPERVISOR'S DECLARATION

I hereby declare that the preparation and presentation of this work was supervised in accordance with the guidelines for supervision of thesis as laid down by the University of Education, Winneba.

Dr. Sylvester Ali Frimpong (Supervisor)

Signature**: ……………………………….……………**

Date**: ……………………………….. ………………**

DEDICATION

To my lovely daughter, Dacosta Rose Mirabel Klenam

ACKNOWLEDGEMENT

To you my sovereign God, I deem it very necessary to thank you for good health, wisdom and grace to undertake and accomplish this study.

My deepest gratitude and admiration go to my cherished mentor and supervisor, Dr. Sylvester A. Frimpong. I genuinely want to thank you for everything you have done and taught me.

My profound thanks goes to my father and mentor Damian Kofi Mereku (prof) whose tuition and great thought has brought me this far in my scholarly development. To Mr. Ofori Isaac, I say a very big thank your support and guidance without which this work piece would have been mirage.

I must express my heartfelt gratitude to my wonderful daughter Dacosta Rose Mirabel Klenam for your moral support throughout my studies.

I acknowledge my course-mates for their motivation and my coworkers for their moral and emotional support throughout my journey.

God bless you all.

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ABSTRACT

This study examines Senior High School students' errors and misconceptions in understanding statistical measures of variation, focusing on students at Winneba Senior High School, Ghana. Statistics, a critical discipline for data interpretation and informed decision-making, often presents conceptual and procedural challenges, particularly regarding measures such as range, variance, and standard deviation. The research employed a mixed-methods approach within a pragmatist paradigm, utilising an exploratory sequential design. Quantitative data were collected through structured assessments administered to 140 final-year students to identify their errors and misconceptions in dealing with tasks on measures of variation. Subsequently, semi-structured interviews with a purposively selected subset of 10 students provided qualitative insights into their reasoning and cognitive challenges. The findings revealed that students frequently confuse measures of central tendency with variation, misuse statistical formulas, and struggle with abstract concepts like deviation and distribution. Common errors included conflating range and standard deviation, misinterpreting variability, and relying heavily on rote memorisation of formulas without understanding their applications. Pedagogical approaches emphasizing procedural skills over conceptual understanding, insufficient use of real-world data, and linguistic barriers in statistical terminology were identified as key contributors to these challenges. The study concluded that traditional teaching methods inadequately address the cognitive demands of statistical reasoning. It recommended integrating experiential learning strategies, real-world data applications, and enhanced teacher training programs focused on fostering statistical reasoning and interpretation.

CHAPTER ONE

INTRODUCTION

1.0 Overview

This chapter discusses the background of the study, the statement of the problem, the purpose of the study, the objective of the study, the research questions, the significance of the study, the scope of the study and the organization of the study.

1.1 Background to the study

Statistics is a fundamental science utilized in several domains, equipping individuals with the necessary tools to analyze data, comprehend trends, and make informed judgments. A fundamental aspect of statistics is comprehending variability how data points within a set differ from one another or the mean. Measures of variation, including range, variance, standard deviation, and interquartile range, are essential for comprehending the dispersion and distribution of data. Notwithstanding the significance of these notions, research constantly indicates that students encounter difficulties in comprehending and accurately utilizing measures of variance. Acquiring statistics needs both procedural and conceptual understanding, with the latter presenting greater difficulties. Research indicates that students often emphasize procedural knowledge, such as the application of formulas, rather than a profound conceptual comprehension of the implications of statistical measurements (Batanero, Burrill, & Reading, 2011). Measures of variation provide significant challenges for students as they require comprehension of abstract concepts such as deviation, distribution, and spread, which are frequently less intuitive than measures of central tendency like the mean (delMas & Liu, 2005).

Students frequently err in their comprehension of measures of variation due to an inadequate grasp of data variability (Garfield and Ben-Zvi 2008). Students occasionally conflate variability with central tendency, erroneously presuming that a higher mean signifies greater variability or that data sets sharing the same mean must exhibit comparable levels of dispersion (Shaughnessy, 2007). Furthermore, students frequently find it challenging to differentiate between other measures of variation, such as range and standard deviation, and may employ them indiscriminately without comprehending their unique functions and implications (Groth, 2005). Mevarech and Kramarski's (2003) research suggests that these misconceptions stem, in part, from students' challenges with statistical reasoning. The issue is exacerbated by the absence of a robust basis in mathematical reasoning among many students, which is crucial for understanding more intricate statistical concepts. Garfield and Ben-Zvi (2008) assert that statistical reasoning necessitates both the capacity to compute measures of variation and the ability to evaluate them in relation to the data under examination. The disparity between computation and interpretation is where numerous misconceptions and errors arise.

There is growing evidence worldwide that students in diverse educational systems face considerable challenges in comprehending statistical measures of variation. Although statistics are crucial for data literacy in the 21st century, numerous learners exhibit inadequate conceptual understanding and elevated error rates when engaging with measures such as range, standard deviation, and variance. Research in developed contexts, such as the United States, Australia, and Europe, indicates that students tend to emphasise procedural execution rather than deep understanding, frequently utilising formulas without comprehending their significance or application (Garfield & Ben-Zvi, 2008; delMas & Liu, 2005). Watson et al. (2003) observed that, in Australia, students were able to compute averages but demonstrated a deficiency in comprehending variability, especially regarding standard deviation and its relevance in practical situations. Chan and Ismail (2013) found that more than 40% of students in Malaysia incorrectly answered standard deviation tasks, highlighting enduring misconceptions among upper-secondary learners. This global trend highlights a significant educational issue: the excessive focus on computation at the expense of interpretation. This phenomenon indicates that insufficient instructional strategies and a neglect of statistical reasoning hinder the development of strong statistical literacy among students. The global insights underscore the widespread nature of the issue and affirm the need for reforms in teaching measures of variation as a fundamental component of statistical education.

In West Africa and across the African continent, students demonstrate significantly low comprehension of statistical measures of variation, which indicates persistent pedagogical and systemic issues in mathematics education. Instruction in statistics in several West African nations, such as Nigeria and Ghana, often prioritises procedural fluency over conceptual understanding, leading to students being inadequately prepared to interpret or apply statistical concepts effectively (Ndemo & Ndemo, 2018). This imbalance is particularly pronounced in the teaching of variation measures, where students are instructed to mechanically use formulas for range or standard deviation, lacking a foundational understanding of variability or distribution (Armah & Asiedu-Addo, 2014). The situation is further exacerbated by teacher-centred instructional models and curricula that rarely incorporate real-world data or context-based learning, which are essential for fostering statistical thinking. Language plays a crucial role; the specialised vocabulary of statistics, which often differs from everyday language, presents an additional cognitive barrier for many learners (Dunn et al., 2003). The lack of specialised teacher training in statistical pedagogy across the region limits educators' ability to effectively address students' misconceptions. The patterns observed in various regions of West Africa underscore the urgent need for educational reforms that emphasise conceptual understanding and contextual learning in statistics education.

Moreover, authorities in statistical education, such as Watson and Moritz (2000), have discovered that students frequently possess entrenched attitudes regarding randomness and variability, resulting in erroneous evaluations when analyzing data sets. Students might anticipate that all data sets exhibit symmetrical distributions or may be taken aback by the variability in tiny samples, presuming that each sample should accurately reflect the population. These assumptions may lead to erroneous evaluations of variation metrics, since students could neglect or misinterpret the dispersion of data points within a dataset. Numerous particular misconceptions about variance measures have been recorded in scholarly literature. A recurring problem is the conflation of range and standard deviation. The range is a basic metric indicating the difference between the maximum and minimum values in a dataset, whereas the standard deviation offers a more nuanced perspective by quantifying the average deviation of each data point from the mean. Many students, however, fail to recognize the necessity of more sophisticated metrics such as standard deviation, erroneously assuming that the range adequately captures variability (Lem, Onghena, Verschaffel, & Van Dooren, 2013).

A common mistake is the standard deviation calculation, which students frequently see as convoluted due to the steps of squaring variances, averaging them, and subsequently extracting the square root. This intricacy may result in computational errors or a failure to comprehend the rationale behind the formula's structure. Baker and Czarnocha (2016) indicated that students frequently employ simplistic and erroneous techniques for estimating standard deviation, such as averaging absolute variances, leading to additional confusion. Moreover, numerous students encounter difficulties in interpreting the standard deviation. For some individuals, the standard deviation is perceived as an abstract concept lacking evident practical significance (delMas, Garfield, & Chance, 1999). Students may find it challenging to correlate the magnitude of the standard deviation with the dispersion of the data set, and they frequently lack comprehension of its connection to other statistical metrics such as the mean or its utility in contrasting variability among data sets. The deficiency in comprehension is especially alarming when students must evaluate the diversity of empirical data, as encountered in scientific studies or social science research (Garfield & Ben-Zvi, 2008). The pedagogical approaches employed in teaching statistics in educational institutions profoundly influence students' inaccuracies and misunderstandings related to measures of variation. Research indicated that conventional pedagogical methods, emphasizing procedural fluency and formulaic computations, frequently do not equip students with a profound, conceptual grasp of statistics (Chance, 2002). When educators prioritize the memorization of formulas without elucidating the foundational concepts, pupils often resort to rote learning, resulting in mistakes when they are required to apply these concepts in novel situations (Shaughnessy, 2007). Instruction on measures of variation often emphasizes computations, such as determining the range or standard deviation, while insufficiently addressing the fundamental concepts of data dispersion and distribution (Bakker & Gravemeijer, 2004). This method may result in pupils possessing a cursory comprehension of variability and its significance. Watson and Moritz (2000) contended that students require opportunities to interact with real-world data and investigate how measures of variation can reveal patterns and anomalies within that data. In the absence of these experiences, students are prone to retain misconceptions regarding variance, even after acquiring the correct processes.

The difficulties encountered by high school pupils in comprehending measures of variance are particularly significant. These students frequently confront intricate statistical concepts for the first time, necessitating a transition from fundamental arithmetic to the examination of abstract notions such as data distribution, variability, and inference. However, several students at this level lack the requisite skills to comprehend these concepts, leading to numerous errors and misconceptions (Sorto, White, & Lesser, 2011). Studies indicated that high school pupils frequently have difficulties distinguishing between data types and comprehending how these categories influence the selection of statistical measures (Sorto, 2012). This may result in errors when students compute measures of variation for various datasets since they can employ incorrect formulas or utilize unsuitable measures. Furthermore, high school students may struggle to comprehend sample variability, mistakenly believing that a small sample may reliably represent the greater population (Watson & Kelly, 2002). Furthermore, students' comprehension of measures of variation is frequently constrained by their interaction with real-world data. Numerous students lack familiarity with the chaotic, flawed datasets that statisticians often face, and they may be unaware of how to interpret variance measurements in these contexts (Groth, 2013). This may result in misconceptions regarding the significance of variability and inaccuracies when making inferences from their computations. Considering the pivotal importance of statistics in contemporary life, including scientific inquiry and daily decision-making, students must cultivate a robust comprehension of statistical concepts, particularly measures of variance. This study, therefore, examined students' errors and misconceptions in calculating measures of variation at Winneba Senior High School in the Central Region of Ghana.

1.2 Statement of the problem

Statistics is essential in mathematics education, allowing students to analyze data, make educated judgments, and enhance critical thinking abilities in a data-driven environment (Moore, 1997). Fundamental statistical concepts, including measures of variation such as range, variance, standard deviation, and inter-quartile range, are essential for aiding students in comprehending data variability and the correlation of data points to the mean (Frankfort-Nachmias & Leon-Guerrero, 2000). However, research indicated that students encounter considerable difficulties with these concepts, leading to widespread inaccuracies and misconceptions (Ndemo & Ndemo, 2018). Such misunderstandings impede students' overall achievement in statistics and restrict their ability to utilize statistical reasoning in practical situations (Wild, Pfannkuch, Regan, & Horton, 2011). Research in statistics education indicates that students across all academic tiers encounter significant challenges in understanding measures of variation, mostly due to the abstract character of these concepts. In contrast to measures of central tendency like the mean, measures of variation require students to grapple with concepts such as deviation, distribution, and spread, which are fundamentally less intuitive (Garfield et al., 2008). Misunderstandings are prevalent; for example, students frequently confuse variability with central tendency, erroneously supposing that a higher mean signifies greater variability or that data sets with equivalent means must have comparable dispersion levels (Shaughnessy, 2007). Moreover, students sometimes conflate range and standard deviation, frequently employing these metrics interchangeably without comprehending their unique functions (Konovalova & Pachur, 2021). Such fallacies not only hinder students' data interpretation abilities but also limit their comprehension of more complex statistical topics. Mevarech and Kramarski (2003) contended that these errors arise not solely from computational inaccuracies but also from more profound cognitive difficulties associated with statistical reasoning. High school students frequently neglect the importance of variety in data, resulting in their dependence on intuitive yet erroneous heuristics for data set comparisons. DelMas and Liu (2005) noted that misconceptions regarding the standard deviation calculation, particularly the rationale for squaring deviations from the mean, hinder students' ability to effectively evaluate and comprehend variability.

Empirical research underscored the worldwide nature of these issues in statistics instruction. Watson et al. (2003) identified substantial weaknesses in Australian students' comprehension and use of statistical variance. While students frequently succeeded in calculating averages, their understanding of intricate measurements like standard deviation and range was insufficient. The gap encompassed not just procedural knowledge but also a deficiency in conceptual comprehension, as students did not acknowledge the importance of these measurements in real-world scenarios (Watson, Kelly, Callingham, & Shaughnessy, 2003). Chan and Ismail (2013) identified a similar issue in Malaysia, where high school pupils faced challenges in statistical reasoning about variability. In their poll, 41.5% of students erroneously responded to questions regarding standard deviation, highlighting the widespread misconceptions about data dispersion. This study emphasizes the global aspect of the problem and indicates a necessity for more efficient educational solutions.

Research on younger kids further substantiates the necessity for early and successful interaction with statistical ideas. English and Watson (2015) investigated the significance of variation as a fundamental element of statistical reasoning in young children. Their findings indicated that early engagement in activities related to data variety can improve students' comprehension of these ideas in senior high school environments when misconceptions about variation continue to exist. By cultivating a fundamental comprehension of data variability, educators may establish a robust basis for advanced statistical learning. This result has significant ramifications, indicating that intervention techniques targeting younger kids may be crucial in mitigating the comprehension deficits observed in high school pupils.

In Malaysia, Chan and Ismail (2013) employed a statistical reasoning evaluation to investigate the comprehension of variability among 412 high school students, revealing two previously unrecognized misconceptions in the literature. Their findings demonstrate the intricacy and profundity of pupils' misconceptions, which surpass fundamental faults. Watson et al. (2003) utilized hierarchical coding and Rasch analysis to investigate the conceptual comprehension of variance among Tasmanian children in grades 3 through 9. While certain students exhibited practical knowledge, their capacity to grasp variance, particularly in the comparison of multiple distributions, was constrained. Vermette and Savard (2018) examined high school mathematics educators' assessments of student misconceptions regarding variability, indicating that professional development for instructors is crucial for effectively addressing these concerns. In a related study, English and Watson (2015) revealed that involving fourth-grade students in practical exercises with authentic data enhanced their comprehension of measurement variance, underscoring the efficacy of early intervention in cultivating statistical thinking.

Initial findings from Winneba SHS indicated that students face challenges analogous to those reported in international studies. Educators indicate that numerous students find it challenging to differentiate between various measures of variation, such as range and standard deviation, and frequently do not grasp the significance of more intricate metrics like variance. The enduring misconceptions among Senior High School students suggest that the issue transcends academic performance, highlighting essential instructional and cognitive difficulties in comprehending statistical concepts. Sorto (2012) discovered that numerous educators possess an inadequate understanding of statistical variance, which impedes their capacity to communicate these topics effectively. This problem is especially pronounced at Winneba SHS, where conventional teaching techniques prioritize procedural accuracy rather than conceptual comprehension. This study aimed to solve a significant gap in knowledge by investigating the unique misconceptions and errors faced by senior high school students at Winneba SHS in computing measures of variance.

1.3 Purpose of the Study

The purpose of this study was to investigate the specific errors and misconceptions encountered by senior high school students at Winneba SHS in understanding and calculating statistical measures of variation, such as range, variance, and standard deviation. While statistics is a vital component of mathematics education, evidence indicates that students globally, including those at Winneba SHS, struggle significantly with concepts of variability, leading to pervasive inaccuracies and misinterpretations.

1.4 Objective of the Study

Specifically, the study were to:

1. Assess Student Performance on Measures of Variation in the Winneba Senior High School.
2. Identify Common Errors and Misconceptions in Measures of Variation among Students in the Winneba Senior High School.
3. Explore the Causes of Errors in Understanding Measures of Variation among Students in the Winneba Senior High School.

1.5 Research Questions

The research sought to answer the following key questions based on its outlined objectives:

1. What is the performance level of senior high school students in understanding and applying statistical measures of variation?
2. What are the common errors and misconceptions that senior high school students demonstrate when working with measures of variation in statistics?
3. What are the underlying causes of errors among senior high school students in understanding measures of variation in statistics?

1.6 Significance of the Study

The analysis of high school students' failures in comprehending statistical measures of variance yielded substantial insights spanning knowledge, policy, theory, and practice. The research enhanced comprehension of students' cognitive difficulties in statistics, particularly regarding prevalent mistakes within the context of the Modified Newman Error Hierarchy. Findings indicated that comprehension and transformation errors are common, highlighting students' challenges in understanding statistical concepts such as range, mean, and standard deviation (Garfield & Ben-Zvi, 2008). Students had a significant frequency of understanding errors for complicated statistical concepts, consistent with prior studies suggesting that students encounter difficulties with the abstract components of statistics (Tishkovskaya & Lancaster, 2012). By identifying these locations, the study enhanced the understanding of where pupils encounter cognitive disruptions, thereby establishing a foundation for the creation of customized educational interventions.

The results influenced educational policy, particularly in curriculum development and teaching methodologies in mathematics education. The study emphasized the prevalence of comprehension and process skill errors, indicating the necessity for curricula that promote enhanced conceptual understanding and problem-solving skills in statistics (Engelbrecht, Harding, & Potgieter, 2019). The findings indicate that policy should emphasize instructional innovations that diminish dependence on rote memorization, associated with elevated errors in transformation and process skills in statistical tasks (Pfannkuch & Wild, 2004). Policymakers might endorse initiatives that prioritize conceptual learning, including teacher training centred on active learning methodologies that enhance statistical reasoning and literacy (Sharma, 2017).

The study utilized the Modified Newman mistake Hierarchy to validate and enhance the theoretical comprehension of mistake kinds in statistical education. The investigation demonstrated that faults at various stages—namely reading, comprehension, and encoding impede students' statistical reasoning, hence validating Newman's model as a proficient diagnostic instrument (Lovett & Greenhouse, 2000). The study revealed that errors in transformation and comprehension frequently arise from an insufficient grasp of statistical terminology and symbols, corroborating Wild and Pfannkuch's (1999) results regarding the significance of symbol fluency in statistics. The study corroborated Newman's hypothesis on the hierarchical nature of errors by demonstrating the prevalence of encoding errors in students' final replies, indicating that foundational faults, such as understanding, result in subsequent transformation and encoding issues.

The study proposed instructional strategies that effectively mitigate these learning impediments. Utilizing real-world examples and graphical aids may reduce comprehension and encoding errors, as students would get experience in evaluating statistical data (Garfield & Ahlgren, 1988). Furthermore, the findings prompted educators to emphasize the development of procedural fluency using scaffolded exercises that assist students in navigating multi-step statistical procedures (Chance et al., 2007). The study's findings also supported task-based learning, wherein students engage with and resolve practical problems, hence diminishing tendencies towards rote memorization. This corresponds with research indicating that experiential, context-driven training enhances recall and comprehension of intricate statistical ideas (Lovett & Greenhouse, 2000).

This study made substantial contributions by pinpointing specific challenges faced by students in statistics, offering evidence for educational reform, confirming theoretical error frameworks, and proposing ways for enhanced teaching practices. These contributions highlight the importance of addressing both conceptual and procedural elements in statistics education to improve students' comprehension and mitigate recurring errors in mathematical problem-solving.

1.7 Delimitation of the Study

This study focused exclusively on students at Winneba Senior High School in the Effutu Municipality, excluding private institutions and other Senior High Schools in the area. It purposively sampled third-year students due to their availability at the time of the study. A mixed-methods approach, incorporating research triangulation, was used to collect data for both qualitative and quantitative analyses. This design enabled a comprehensive exploration of students' performance and the types of errors encountered in statistical measures of variation, providing a well-rounded understanding of the educational context specific to this school.

1.8 Limitation of the Study

Although the situation of students' errors and misconceptions in measures of variation seemed to be nationwide, it was not possible to undertake the study nationwide due to the cost, energy and volume of work involved. It was restricted to feasibly systematic area. The depth of the study covered Senior High School students' errors and misconceptions in measures of variation. The findings of the study can also not be generalized because purposive sampling technique was employed during the sampling.

1.9 Organization of the Study

This thesis comprised five chapters, each contributing to a thorough comprehension of the research. Chapter One laid the groundwork and rationale for the research. It examined the correlation between mathematics and statistics, offered the context, and delineated the research problem and objective. This chapter delineated the study's aims, research questions, significance, and breadth, while also addressing its scope. Chapter Two included a comprehensive overview of pertinent literature, concentrating on student errors and misconceptions regarding measures of variance. This chapter contextualized the investigation within the current body of knowledge, facilitating the formulation of a theoretical framework. Chapter Three delineated the research technique, outlining the study's population, instruments, and processes for data collecting and analysis. This chapter also examined the study's credibility and has a declaration regarding researcher subjectivity. Chapter Four delineated the demographic attributes of the sample, along with the results and conclusions obtained from data analysis. This chapter emphasized significant findings and establishes correlations between these results and those of pertinent studies. Chapter Five, the concluding chapter, presented a summary of the study, and a synthesis of key findings, conclusions, and suggestions. It also delineated topics for further investigation based on the study's findings.

CHAPTER TWO

LITERATURE REVIEW

2.0. Overview

This chapter discusses the literature on theoretical framework, conceptual framework and related studies.

2.1. Conceptual Framework

The conceptual framework consists of conceptual principles, constructs, concepts and tenants of the theory (Grant & Osanloo, 2014).

2.1.1 Overview of Measures of Variation

Measures of variation, often referred to as measures of dispersion, play a crucial role in statistics by quantifying how much data points in a data set deviate from the central value. These measures offer valuable insights into the distribution and variability of data, essential for precise data interpretation and informed decision-making. The main indicators of variation consist of the range, variance, standard deviation, and interquartile range (IQR). The range serves as the most straightforward measure of dispersion, determined by the difference between the highest and lowest values in a dataset. Although it provides a rapid indication of variability, the range is particularly susceptible to outliers and fails to convey details about the distribution of values within the extremes (Weiss, 2012). The variance quantifies the average of the squared differences from the mean, offering insight into the dispersion of data points relative to the central value. A higher variance signifies increased dispersion. Nonetheless, since variance is represented in squared units, its interpretation may not be as straightforward (Moore, McCabe, & Craig, 2017). The standard deviation represents the square root of the variance, effectively returning the measure to the original units of the data, thereby improving interpretability. This method is commonly employed in statistical analyses to evaluate the distribution of data points about the mean. A smaller standard deviation indicates that data points are closely clustered around the mean, while a larger standard deviation suggests that the data is more dispersed (Triola, 2018). The interquartile range (IQR) indicates the span that encompasses the central 50% of data points, determined by subtracting the first quartile (Q1) from the third quartile (Q3). The interquartile range is especially valuable for comprehending the dispersion of the central segment of the data and is minimally influenced by outliers, rendering it a reliable indicator of variability (Mann, 2010).

Grasping these measures is crucial for accurately interpreting data distributions. For example, in data that follows a normal distribution, roughly 68% of the data points are found within one standard deviation from the mean, while around 95% are located within two standard deviations, as outlined by the empirical rule (Bluman, 2014). This understanding assists in evaluating the likelihood of events within a dataset. In educational settings, especially at the high school stage, understanding these concepts is crucial for fostering statistical literacy. However, studies show that students frequently face challenges in grasping measures of variation. Ng and Chew (2023) examined the errors made by high school students in solving problems related to measures of dispersion and discovered that a significant number of students function at the action level, suggesting a restricted grasp of the underlying concepts. In a similar vein, Cooper and Shore (2008) examined students' misunderstandings regarding the interpretation of measures of centre and variability, emphasizing the difficulties in differentiating between various measures and their suitable applications. The challenges highlighted here point to the necessity for teaching strategies that prioritize conceptual understanding rather than merely focusing on procedural knowledge. Involving students in activities centred around data collection, analysis, and interpretation can significantly improve their understanding of variability. Shaughnessy et al. (1999) explored students' conceptions of variation using hands-on sampling problems, illustrating that experiential learning enhances understanding. In summary, understanding measures of variation is essential for analyzing data distributions and guiding informed decision-making through statistical analysis. Educators need to prioritize the enhancement of students' conceptual understanding of these measures to cultivate statistical literacy. Clarifying prevalent misunderstandings and utilizing engaging instructional techniques can improve learners' understanding of variability, equipping them for higher-level statistical education and practical use.

2.1.2 Theoretical Framework

The theoretical basis for comprehending measures of variation in statistics is rooted in cognitive development theories, learning theories, and the essence of statistical thinking. These frameworks illustrate how students understand variability and the reasons behind their potential difficulties with it. This section provides a thorough examination of the pertinent theories and their implications for understanding the errors and misconceptions of senior high school students.

Constructivist Learning Theory

The constructivist learning theory highlights the active role of learners in building their understanding, drawing from their previous knowledge and experiences (Piaget, 1952). In the realm of statistics, this indicates that students' capacity to grasp measures of variation is shaped by their foundational knowledge of interconnected concepts, including mean and range. When students possess incomplete or flawed prior knowledge, misconceptions regarding variability emerge. For example, research conducted by Garfield and Ben-Zvi (2008) showed that numerous students struggle to differentiate between measures of central tendency and measures of dispersion due to insufficiently developed mental models.

APOS Theory

The APOS (Action, Process, Object, Schema) theory is the description of mental activities and mental constructions that tends to make when formulating their understanding of mathematical concept (Triguros and Oktac, 2019). APOS theory outlines that learning takes place in four distinct stages:

Action: Executing procedures without a thorough grasp of the underlying concepts.

Process: Assimilating these actions to develop cognitive frameworks.

Objective: Understanding the concept as a unified whole.

Schema: Incorporating the concept into extensive knowledge structures.

Ng and Chew (2023) utilized the APOS theory to investigate students' comprehension of measures of variation. It was observed that numerous students tend to stay at the action stage, depending on rote memorization of formulas instead of understanding the fundamental significance of statistical measures such as standard deviation or variance.

Dual Process Theory

Kahneman and Frederick's (2002) dual-process theory, which differentiates between intuitive (System 1) and analytical (System 2) thinking, provides valuable insights into the challenges students face regarding variability. Intuitive thinking can cause students to oversimplify statistical concepts, whereas analytical thinking, which involves careful reasoning, is crucial for grasping variability. For instance, students may assume that a narrow range indicates a lack of variation, overlooking the importance of more thorough metrics such as standard deviation or interquartile range.

Understanding Statistics and Analyzing Data

Statistical literacy encompasses the capability to interpret and critically assess statistical information, whereas statistical reasoning involves comprehending and drawing inferences from data (Gal, 2002). Both are essential for understanding measures of variation. Watson and Moritz (2000) pointed out that numerous students struggle with the reasoning skills required to grasp variability, as they frequently do not link measures of variation to practical situations. The absence of sound reasoning can lead to persistent errors and misunderstandings, such as the incorrect assumption that low variance indicates all data points are the same.

Zone of Proximal Development

Vygotsky's (1978) Zone of Proximal Development (ZPD) theory highlights the importance of scaffolding in the learning process. Students tend to grasp intricate statistical concepts, such as measures of variation, more effectively when they receive structured guidance. Lehrer and Schauble (2004) showed that focused teaching strategies, like offering visual representations of variability, assist students in moving from a fundamental to a more sophisticated grasp of statistical concepts.

Implications for Comprehending Mistakes and Misunderstandings

1. Theoretical frameworks emphasize various factors that contribute to errors and misconceptions in the comprehension of measures of variation:
2. Cognitive overload frequently occurs among students when they engage with abstract concepts such as variance or standard deviation, attributed to their inherent complexity (Sweller, 1988).
3. Inadequate Instruction: Educational methods that emphasize procedural knowledge at the expense of conceptual understanding may result in students being poorly prepared to analyse variability (Garfield & Ben-Zvi, 2008).

4. Cultural and contextual factors, such as language barriers and variations in curriculum, can significantly hinder students' understanding of statistical terminology and concepts (Shaughnessy et al., 1999).

2.2 Justification of Theories about the Study

The constructivist hypothesis, advocated by Piaget (1952), underscores the active participation of learners in the construction of knowledge informed by previous experiences and concepts. The research on Senior High School students' errors and misconceptions concerning measures of variation demonstrates that these misconceptions arise from deficient or insufficient basic knowledge. Garfield and Ben-Zvi (2008) demonstrated that errors, such as mistaking central tendency with measures of dispersion, frequently stem from inadequately established mental models. This is evident in students' failure to differentiate between range and standard deviation within the study's context. The thesis findings corroborate this hypothesis by emphasizing the necessity of addressing prior knowledge to rectify entrenched errors and facilitate conceptual understanding, especially for abstract statistical measures.

The APOS (Action, Process, Object, Schema) hypothesis proposed by Dubinsky (1991) outlines the phases of cognitive comprehension, ranging from rote memorizing to cohesive knowledge integration. The study's emphasis on students stagnating at the "Action" stage simply applying formulas without comprehending their importance illustrates the relevance of this idea. Ng and Chew (2023) noted same trends in which children rely predominantly on rote recollection, overlooking more profound conceptual understanding. This theory supports the study's focus on advancing pupils to higher levels, specifically in cultivating schemas that enable them to relate measurements of variation, such as variance and standard deviation, to wider statistical frameworks.

Kahneman and Frederick's (2002) Dual Process Theory differentiates between intuitive (System 1) and analytical (System 2) cognition. The study emphasizes inaccuracies resulting from dependence on intuitive thinking, such as presuming minimal variability in datasets with proximate means. Analytical thinking, according to the notion, is crucial for comprehending measurements such as standard deviation. The research illustrates this hypothesis by examining how students' dependence on intuitive heuristics results in oversimplification and recurring mistakes. It emphasizes the necessity for pedagogical approaches that promote advanced analytical reasoning, allowing students to assess variability thoroughly.

Vygotsky's (1978) Zone of Proximal Development underscores the significance of scaffolding in advancing learners from their existing comprehension to elevated cognitive capacities. The study's results, especially the effectiveness of guided instructional interventions, align with this notion. Lehrer and Schauble (2004) established that visual aids and focused assistance can markedly improve comprehension of intricate topics such as variability. The study noted that structured assistance could facilitate students' shift from procedural to conceptual understanding, therefore minimizing errors associated with measures of variance.

The study finds various elements that contribute to errors, which theoretical frameworks assist in contextualizing. Sweller's (1988) cognitive load theory emphasizes the intrinsic complexity of metrics such as standard deviation as a source of errors. Students perplexed by the abstract concept of variance may resort to mechanical calculation without grasping the underlying principles. Garfield and Ben-Zvi's (2008) critique of procedural-centric pedagogy corresponds with the study's finding that a focus on formulaic calculations promotes superficial understanding. Shaughnessy et al. (1999) highlight the influence of linguistic and curricular discrepancies on the comprehension of statistics terminology and concepts. The study's findings indicate that language hurdles and curriculum design impede comprehension at Winneba SHS.

Gal (2002) and Watson and Moritz (2000) contend that statistical literacy, which includes the interpretation and critical assessment of data, is crucial for comprehending measures of variation. The study substantiates this by highlighting deficiencies in students' reasoning and their incapacity to apply statistical principles to real-world data. Ongoing misconceptions, like conflating low variation with uniformity, highlight the necessity for pedagogical approaches that incorporate practical applications to enhance statistical reasoning and literacy.

The utilization of cognitive development and learning theories in this study offers a comprehensive framework for elucidating the errors and misconceptions identified among students in the present study. Each theory emphasizes the importance of addressing basic information, facilitating learning, and fostering analytical reasoning to improve students' understanding of measures of variation. The study's results corroborate these theoretical frameworks and provide practical recommendations for enhancing teaching methods, thus closing the divide between procedural knowledge and conceptual comprehension.

2.3 Empirical Review

The empirical framework of this study was constructed through an integration of historical and contemporary insights into mathematics education within the Ghanaian and broader African context. Beginning with an exploration of the evolution of school mathematics in Ghana, the framework traces the pedagogical shifts from rote-based arithmetic during colonial education to the adoption of “modern mathematics” in the post-independence era. These reforms, aimed at aligning school curricula with contemporary needs, laid the foundation for current instructional practices but also introduced new complexities in content delivery (Mereku, 2010). Building on this historical trajectory, the framework incorporates findings from empirical studies that examined student misconceptions and systemic instructional challenges, especially in statistical topics like measures of variation. It draws on Radatz's (1979) classification of mathematical errors, Fischbein and Barash's (1993) model of intuitive versus formal knowledge, and Newman's (1977) error analysis hierarchy. These theories collectively highlight the layered nature of student misunderstandings—ranging from reading and comprehension to transformation and encoding errors. The framework is thus grounded in both curriculum evolution and diagnostic research on learning difficulties. By contextualizing these insights within the realities of Ghanaian classrooms, the study constructs a practical and theoretically informed lens for analyzing student errors in statistical reasoning.

2.4 School mathematics and it teaching in Ghana

Before Ghana's independence from British rule in 1957 arithmetic, was mainly mechanical number facts and tables of measurements were the mathematics studied in Ghanaian elementary schools. As a result, the textbook in use by then,' 'Larcombe's Arithmetic series' was purposely designed to promote and develop good mental skills in all students. The textbook was characterized by the speed test in mental arithmetic. However, at the secondary level, arithmetic, algebra and geometry were taught (Mereku, 2010). As a result of the discovery of new mathematics in the 1960s, the African continent, and as such Ghana, underwent curricular changes in mathematics education. After World II, it became necessary for the school system to be shifted from an 'elitist' to a 'comprehensive' one which led to an expansion of facilities and student populations. As such it became necessary for the school mathematics curriculum to be re-organized to also make it more comprehensive to satisfy the needs of the growing number of students in the schools (Mereku, 2010).

According to Mereku (2010), new systems and policies of education were sought to help develop rapidly the human resources of new nations in Africa. Hence, after many conferences and workshops, many projects were launched which introduced new textbooks to the nations. Some of these include the Entebbe Modern Mathematics, the 'New Mathematics for Primary School (NMPS)', which aimed to make the learning of mathematics more interesting and more meaningful to Ghanaian children, the Joint School Project (JSP), which aimed to produce new mathematics course for West Africa Secondary School Certificate level. The movement for change in the content of school mathematics led to the new name 'modern mathematics' which was to permit an approach to mathematics which will facilitate the learning of basic language and structure of mathematics quickly. It was the main aim of the 'new mathematics' to ensure a connection between school mathematics and university mathematics. As a result, there were major changes in pre-university mathematics education in Ghana in the late 1960's and the early 1970's (Mereku, 1999).

2.5. A general discussion of errors and misconceptions

The foundation for research on student conceptions comprises three major traditions. Each tradition has its epistemological assumptions (Confrey, 1990). These epistemological positions are: Piagetian studies in the tradition of genetic epistemology, applications of the philosophy of science in the tradition of conceptual change, and research on systematic errors. According to Confrey (1990), research in the first two traditions tends to be on student conceptions in science and mathematics, whereas research in the third area focuses on mathematics and computer programming. These three categories are not exhaustive, nor are they mutually exclusive. Among these traditions, the first and the third are closely related to the research under study. Piagetian work on student conceptions examined the development of student understanding of particular mathematical and scientific concepts over time. Piaget's fundamental assumption was that knowledge is a process, not a state. Hence knowledge needs to be examined about its developmental associations. In line with this thinking, Piaget studied conceptions, not misconceptions. Researchers in the tradition of systematic errors have documented that students hold mini-theories about scientific and mathematical ideas. Numerous studies have shown that students have many naive theories, preconceptions, or misconceptions about mathematics that interfere with their learning (Posamentier, 1998). Because students have actively constructed their misconceptions from their experiences, they are very attached to them. They find it very difficult to give them up. Radatz (1979) proposed that student errors could be categorized by following through problem-solving stages. According to Radatz, various causes of errors in mathematics can be identified by examining the mechanisms used in obtaining, processing, retaining, and reproducing the information in mathematical tasks. He identified four error categories. Those are (1) errors due to processing iconic representations, (2) errors due to deficiencies of mastery prerequisite skills, facts, and concepts, (3) errors due to incorrect associations or rigidity of thinking leading to inadequate flexibility in decoding and encoding new information and the inhibition of processing new information, and (4) errors due to the application of irrelevant rules or strategies. Fischbein and Barash (1993) developed a theory in their seminal analysis of students' mathematical performances. This theory is related to three components of knowledge: algorithmic, formal, and intuitive. According to them, algorithmic knowledge is the ability to use theoretically justified procedures. This is the ability to activate procedures in solving problems and understand why these procedures work. The formal aspect refers to axioms, definitions, theorems, and proofs (Fischbein, 1994). This relates to rigor and consistency in deductive reasoning and it is free from the constraints imposed by concrete and practical situations. The intuitive knowledge is described as immediate, self-evident cognition imparting the feeling that no justification is required.

Sometimes, these three components converge. Usually, in the process of learning, understanding, and problem-solving, conflictual interactions will appear (Fischbein, 1994). Often, intuitive background knowledge manipulates and hinders the formal interpretation or the use of algorithmic procedures (Fischbein & Barash, 1993) sometimes, a solving schema is applied inadequately because of superficial similarities in disregard of formal similarities. Other times, a solving schema deeply rooted in the student's mind is mistakenly applied despite a potentially correct, intuitive understanding. The three components, according to Fischbein and Barash (1993), are inseparable and they play a vital role in students' mathematical performance. Usually, it is the intuitive interpretation based on a primitive, limited, but strongly rooted individual experience that annihilates the formal control or the requirements of the algorithmic solution, and thus distorts or even blocks a correct mathematical reaction (Fischbein, 1994). The solving procedures, acting as overgeneralized models, may sometimes lead to wrong solutions by disregarding the corresponding formal constraints. Matz (1980) extended the research on students' error behaviours in rule-based problems to build a generative theory that accounts for as many common errors as possible that students make in problem-solving. The theory states two extrapolation mechanisms for generating algebra errors. They are the use of a known rule in a new situation where it is inappropriate and incorrectly adapting a known rule so that it can be used to solve a new problem. The examples for these categories again emanated from the over generalization of the distributive law (Matz, 1980; Matz, 1982; Kaput, 1982; Kirshner, 1985). Kirshner (1985) said that over-generalization of rules is common in almost every student up to a certain stage. Even successful students tend to go through a phase of overgeneralizing distributive before achieving fluency in manipulative skills. Errors are logically consistent and rule based rather than random (Ben-Zeev, 1998). Investigating errors, therefore, presents an opportunity for uncovering the mental representations underlying mathematical reasoning (Ben-Zeev, 1998). In preparing taxonomy of errors, Ben-Zeev (1998) discussed the need to have a clearer distinction among various stages of the problem-solving process such as execution errors and encoding errors.

2.6. Mathematical Errors and Misconceptions

Among studies done on learning mathematics in general, there are those which point to the same direction as this study. In further relation to this study, there is also some research which has been conducted on learners' errors and misconceptions displayed while trying to cope with mathematics (Olivier, 1996). Drawing also from Sarwadi and Shahrill (2014) who argue that some existence of errors and misconceptions dates back to early learning and makes it difficult for learners to cope with the subsequent demands of mathematics, hence affect their performance in tests or assessment tasks. This problem persists and prevails throughout the learners' period of schooling if not dealt with and thus end up affecting their general attitude towards the subject (Dowker, 2004). Literature which serves to identify errors and misconceptions from learners' verbal and written work exists, but more needs to be suggested on what then to do. It is not sufficient just to identify the errors and misconceptions (Smith et al., 1994). In light of the need raised by Smith et al. that future research should focus on using the misconceptions and errors to build on learners' conceptions, Sarwadi and Shahrill (2014) make a contribution towards the instruction from a teacher's end. The authors posit that teachers need to be 'made' aware of how these errors and misconceptions come about and accordingly device pedagogical means to incorporate them in their teaching. They should develop diagnostic expertise so as to be able to deal with errors and misconceptions from learners' written and verbal work constructively Prediger (2010). This special skill required of a teacher because constructivism accounts for the fact that errors and misconceptions are pathways for constructing knowledge, and must not be eradicated, but instead be capitalized on and used as 'springboards for inquiry' (Borasi, 1994). This is all because learners are viewed by constructivism as NOT passive recipients of imposed facts and information/opinions, but rather as active participants in the construction of their own knowledge (Hatano, 1996). It is through this process whereby learners engage with new information and process it to what eventually becomes knowledge. Misconceptions tend to emanate from this process as by-products. These lead to learners making errors which are persistent and resistant to change. Teachers also find it difficult to convince learners that what they know which might be incorrectly structured but making sense to them is wrong mathematical conceptions (Brodie & Berger, 2010). Learners' thinking abilities can also be recognized from the mathematical conversations they participate in. It is from this platform whereby teachers can pick up ideas from learners and use them to facilitate the learners' process of constructing knowledge despite it being a discouraging process (Brodie, 2007). Despite the issues acknowledged and raised by authors some of whom I have referred to in this review of the literature, it is expected of a teacher to ensure that he or she is able to create a learning environment which embraces the above-mentioned teaching strategies so that learners' ideas/thinking can be integrated in the learning process to enhance knowledge construction Jacobs, Lamb & Phillips's (2010). Still on learning through acknowledgement of errors and misconceptions, Bray and

Santagata (2013) also maintain a very strong view that for teaching to result with learning of actual mathematical concepts, teachers need to implement an instructional strategy to expose learners' errors and misconceptions, and deal with them openly. Since this is a complex phenomenon which involves both the teacher and the learner, it is also vital to bear in mind what it commands of the teacher up and above what I have mentioned in the preceding sentence. The concept of hearing and listening are also vital in the process of teaching-and-learning as part of the teacher's role. This is because the teaching and learning processes encompass lots of other sub-processes. Coles (2002) established a valuable link between the concepts of hearing and listening, and their implications on the teaching strategies. Her findings emanated from her analysis of classroom interactions where she used the three forms of listening from Davis (1997). Due to the thin line that seems to exist between the two concepts i.e. listening and hearing, it makes it easy for teachers to confuse one with the other. Hearing needs a full and conscious effort to tune into the 'how' and the 'what' of the student's idea, and thereby enabling understanding of the students' meanings and thinking (Coles, 2002). But the extent to which a teacher can understand the students' meanings and thinking is somewhat dependent on the form of listening and the teaching strategies they employ. This implies that a teacher also needs to carefully look into his or her teaching strategies, which Coles (2002) refers to as any activity undertaken by a teacher in relation to organizing his or her teaching and learning space, the teaching resources, the assessment procedures and the nature problems he or she chooses. This does not leave out a teacher's personal view of the mathematics subject and the manner in which he or she chooses to interact with the learners. Coles (2002) brings about the fact that, of the three forms of listening by (Davis, 1997), transformative listening was found to carry some special feature of enabling the slowing down and opening up of discussions, affording students with opportunities to ask questions and work with their own questions, thereby allowing a teacher to engage with learners' thinking. This form of listening enables a teacher to get insights if learners ideas, which may in turn enable him/her to pick up conceptions and misconceptions through a learning and teaching process. In addition to the mere identification of errors and misconceptions, Makonye and Luneta (2013) explain their possible roots with their focus specifically on the function concept. My study is aiming to go slightly beyond Makonye and Luneta (2013)'s work. That is, establishing if capitalizing on the identified errors and misconceptions on teaching functions and using them to shape up the teaching can enhance the teaching and learning process and enable a better understanding and mastery of the concept. The study also takes into consideration the constructivist theory of learning and how closely related it is to learners' errors and misconceptions as they (Makonye & Luneta, 2013) write of other scholars who argue of a strong link between constructivism and learners' mathematical misconceptions (Smith et al., 1994; & Nesher, 1987).

2.7. Conceptual and procedural knowledge

Conceptual knowledge is a product of devising new tactics or modify existing strategies to meet new challenges (Rittle-Johnson et al., 2016) In this web of knowledge, the links that connect the various nodes are just as important as the connections that exist between them (Groth and Bergner, 2006; Miller and Hudson, 2007). The connecting process in mathematics learning is formed when students can identify specific rules or procedures from more abstract concepts (Hiebert and Carpenter, 1992). Procedural knowledge, on the other hand, is the ability to answer a mathematical issue by going through a set of rule-based processes (Canobi, 2009; Sáenz, 2009). Skemp (1987) refers procedural knowledge as instrumental understanding, or rule knowledge. This can involve familiarity and grasp of the symbols used to construct algorithms, and the procedural rules needed to solve problems, without necessarily knowing the underlying mathematical concepts. Both the conceptual knowledge and procedural skills are intertwined in mathematics problem solving (Hurrell, 2021; Nesher, 1987).

Table 2.1 summarizes the categories of errors and description establish from the learners' work by Movshovitz-hadar, Zaslavasky and Ibar error analysis, namely misused data; distorted theorem or definition; unverified solution and technical error.

Table 2. 1 Description of errors adapted from Movshovitz-Hadar,Zaslavasky and Ibar (1987)

Types of error	Description of the error
Misused data	-Ignoring the given data that is important to find a solution. -Using a data which is different with the given data. -Adding irrelevant or extraneous data. -Using a numerical value of on variable for another variable.
Conceptual error	-Inability to relate initial concept with newly given one. -Misgeneralisation.
Procedural error	-Incorrectly citing a definition, theorem, rule or formula. -Non-systematic
Unverified solution	-The final result is not the solution to the problem: error in examining the final result.
Technical error	-Error in calculation due to carelessness. -Error in manipulating algebraic symbol or operation. -Error in applying an algorithm

Another error analysis model or framework that has been commonly used in the literature is the Newman's (1977) error analysis model which has five distinct levels: Reading, Comprehension, Transformation, Process Skills, and Encoding. Each error is classified into one of these categories to pinpoint specific difficulties students encountered. Table 2.2 shows how the skills in solving mathematical tasks are categorized under the modified Newman's error analysis model.

Table 2. 2: Newman's error analysis model (Newman, M.A. (1977)

Skills Criteria	Newman's Error Level	Description of Errors
Identifying relevant information from the data	Reading	Students fail to recognize or interpret key symbols, terms, or data in the problem statement.
Understanding the problem's context	Comprehension	Students misunderstand what is being asked, resulting in incorrect interpretations of the problem's intent.
Transforming the problem into a mathematical model	Transformation	Students make errors in converting the problem scenario into an appropriate mathematical representation.
Performing correct calculations or operations	Process Skills	Errors occur when students misapply arithmetic operations or fail to continue the necessary calculations.
Presenting correct answers with proper notation and labels	Encoding	Students are unable to express their results clearly and accurately, often due to poor notation or carelessness.

Table 2.2 presents an overview of Newman's Error Model, which categorizes errors into five hierarchical levels: Reading, Comprehension, Transformation, Process Skills, and Encoding. This structured approach allows for a detailed diagnosis of the various stages at which students encounter difficulties when solving problems related to measures of variation in statistics.

At the reading stage, students are expected to extract and identify relevant information from the problem. Errors at this level indicate a failure to recognize key terms or symbols, which often leads to a breakdown in subsequent problem-solving steps. For instance, students may misinterpret statistical symbols or neglect important details in the question, preventing them from effectively engaging with the problem.

Errors in the comprehension phase reflect the students' inability to understand what is being asked. Even if students can read the problem, they may struggle with comprehending the core requirements of the task. Such errors are indicative of deeper conceptual misunderstandings, where students misinterpret instructions or fail to grasp the problem's objective.

The transformation stage involves converting the verbal or written problem into a mathematical model. Errors at this level suggest that students face difficulties in translating a real-world scenario into a statistical equation or model. For example, students may fail to correctly apply a formula, such as the one for variance, due to confusion over which mathematical operations are necessary. This phase is particularly critical because failure here disrupts the entire problem-solving process, resulting in significant errors downstream, as highlighted by the high percentage of transformation errors in the quantitative data.

At the process skills level, errors occur when students make mistakes in executing the necessary mathematical operations. This could involve miscalculations, incorrect application of procedures, or an inability to proceed to the next steps in the problem-solving process. These errors often stem from previous misunderstandings in the transformation phase. For example, a student may correctly identify the need to use a formula but incorrectly execute the steps required to arrive at a solution.

Finally, encoding errors occur when students fail to clearly and accurately present their solutions. This might involve incorrect use of notation, failure to label graphs, or poor organization of the final answer.

2.8. The Need for Statistical Literacy, Reasoning and Thinking

Nowadays ability to analyze, interpret and communicate information from data has become skills necessary for daily living and helps an individual to be an effective citizen. Concepts in statistics are occupying an increasingly important role in mathematics curricula. Decision-making in society and learning about the world are increasing being based on evidence derived from a set of data. Statistical methods and ways of thinking are taking over a varied range of human activities such as in psychology, government policy, engineering, health science and sustainable environments (Pfannkuch, 2008), and also in the media through numerous kinds of argument, advertisement, or suggestion (Ben-Zvi & Garfield, 2004).All these activities are using data to give meaning and understanding to real setting and situations, and statistics education has a crucial to play in this regard. Despite the attention given to statistical in school curricula, people continue to show sign of poor statistical literacy and reasoning after formally studying statistics as a subject in school (Ben-Zvi 2004; delMas & Liu, 2005; Matthew & Clark, 1997, Armah & Aseidu-Addo, 2014). Rubin (2002) noted that, adults and most students at the college-level do not have in-depth understanding of data, beyond the simple graphical representations like the bar and pie chart that are often misleadingly presented in the media. It was widely acknowledged by mathematics educators that the foundations of statistical reasoning must be built in the early years of schooling rather than reserving it for higher levels like high school (NTCM, 2000). According to Rumsey (2002), many statistics instructors agree to the fact that it is the objective of any introductory statistics course is to raise the awareness of students' data in our everyday life experiences and prepare them for a future career in the present information age. To achieve this, statistics educators must work towards achieving two principal goals for our introductory statistics course. First, students must be trained to be good statistical citizens, who understand statistics very well to be able to consume the information that they are flooded with daily, thinking about bit critically, and using it to make good decisions. This, Rumsey, (2002) stated, some researchers call "statistical literacy."

The second, and to her, the often understand goal for our introductory statistics course, is to develop scientific research skills in our students. This involves the ability to identify questions and problems, collect data, discover ad apply tools to interpret it, communicate and exchange results. Though not all our student may conduct scientific studies of their own, it is almost impossible to imagine a student in today's society, not encouraging data or statistical results in their course of career. In every aspect of this scientific method, statistics involved (Rumsey, 2002). For an individual to have a deep knowledge of a concept, he is required to be able to be abused in many ways, despite its complexities (Bennett & Briggs, 2002) it is imperative for students who are the future generation to be knowledgeable in statistics so as to analyze data critically and evaluate the information they receive. There have been strong calls within the past decades for statistics education to emphasize more on statistical literacy, reasoning, and thinking (Aliaga et. 2005; Cobb, 1992). One such call was from the International Collaboration for Research on Statistical Reasoning, Thinking and Literacy (SRTL). On the home page, this collaboration points out their goal which is to foster current and innovative research studies that examine the nature the nature and development of statistical literacy, reasoning, and thinking, and explore the challenge posed to educators at all levels- to develop these desired learning goals student. Aliaga et al., (2005) also reiterated the fact that the desired goal of all introductory statistics course is to produce statistically educated students, which means student must develop statistical literacy and the capacity to think statistically. There is the need therefore to distinguish between these terms that are used to statistics education: statistical literacy, statistical reasoning, and statistical thinking since their definitions and usage among the literature sometime seem to be coinciding.

2.9 Statistical Vocabulary Knowledge

Students' fluency of mathematical language is important to the development of one's conceptual understanding of content knowledge and skills. Previous study showed that focused instruction on mathematical vocabulary may help the low performing students in learning mathematics. Rubenstein and Thompson,(2020) claimed that there are at least 11 categories of difficulties associated with learning the language of mathematics. As for statistics, statistical language consists of a blend of general English, mathematical English, and statistics-specific English, often known as statistical English. Rangecroft and Rothery, (2002) identified six categories of words used in statistics based on the meaning of the words in terms of general English, statistical English and mathematical English. Dunn et al.,(2006) further described the difficulties students encounter when learning statistics due to the terminology used. Some "lexically ambiguous words" that have a more specific meaning in statistics than in general English are said to generate difficulty and confusion in the learning of statistics by students. Without proper vocabulary instruction, students were confused with the application and definition of the statistical vocabulary especially when these terms are abstract.

2.9.1 Rote learning

Mayer categories learning into three scenarios i.e., no learning, rote learning, and meaningful learning. Students who have attended to the materials but cannot understand the relevant information is characterized as rote learning. In the meaningful learning, a student not only possesses the relevant knowledge, but is able to transfer that knowledge to solve problems and understand new concepts. Rote learning in mathematics is the mastering of a rule or procedure through the process of repeated learning without understanding the reasons that make it work. Students' misconception and low achievement in solving and reasoning mathematics problems could be due to inefficient rote learning. When mathematics curricula is rigid and emphases academic achievement rather than the process of learning, students will tend to memorizing procedures, instead of seeking solution; memorizing formulas instead of observing patterns; and doing exercise instead of formulating conjectures.

2.9.2 Symbol sense

Symbols are the component of the mathematics language that enables the communication, manipulation and reflection upon abstract mathematical concepts. Arcavi, (1994) defined symbol sense as the skill to appreciate the power of symbols, the right application and manipulation of symbols in a range of context. Past studies revealed that students often struggle and confuse over the symbolic representations in mathematics mainly due to the conciseness and abstraction. Students often perceived their personal meaning to symbols, and failure to manipulate and understand mathematics symbols has attributed to students' difficulties in mathematics learning. Rubenstein and Thompson, (2020) categorized the challenges related to learning mathematical symbols into three areas, namely:

i) Verbalization challenges (i.e. the translation of symbols into spoken language);

ii) Reading challenges (i.e. conceptual understanding of the symbols); and

iii) Writing difficulties (i.e. producing symbols). These challenges are complex and often occur simultaneously.

2.9.3 Statistical reasoning

Garfield and Chance, (2000) defined statistical reasoning as a way of reasoning with statistical ideas and understanding statistical information. Other researchers defined statistical reasoning as making sense of the statistical information, interpreting statistical results, summarizing statistical data, and draw conclusion from data. In addition, previous researchers characterized students' reasoning across four levels: idiosyncratic, transitional, quantitative and analytical. At the idiosyncratic level, students' reasoning is narrowly and consistently bound to idiosyncratic or subjective reasoning. Students provide irrelevant information and often focused on personal experiences or subjective beliefs. At the transitional level, students began to reasoning quantitatively, but are inconsistent in their use of such reasoning. At the quantitative level, students' reasoning is consistently quantitative and they can identify the problem but do not necessarily make sense and apply the relevant mathematical ideas in solving the problem. At the analytical level, students are able to represent the multiple aspects of a problem into a meaningful structure such as creating multiple data displays, or making a reasonable prediction.

2.9.4 Statistical thinking

Statistical thinking is the cognitive actions that students engage in during the data handling processes of describing, organizing and reducing, representing, and analyzing and interpreting data. This definition is different from the definitions emerged from the statisticians which focuses more on the practical experiences and reflections. Wild and Pfannkuch, (1999) categorized statistical thinking into general types of thinking which seeking explanations and applying techniques; and fundamental statistical thinking which involves recognition of need for data, consideration of variation, and reasoning with statistical models. In Garfield middle school student statistical thinking framework, he characterized middle school students' thinking in statistical situation into four processes: i) Describing data; ii) Organizing and reducing data; iii) Representing data; and iv) Analyzing and interpreting data. The four statistical processes are closely interrelated and determining students' ability to solve statistical problem.

2.10 Studies in Students' Conception of Variation

Statisticians and statistics educators acknowledged variation to be the core of statistics (Orta & Sanchez, 2011). Garfield and Ben-Zvi, (2005) identified variability to be at heart of statistics, and a major component of statistical thinking (Pfannkuch, 1997). The variability of a data set can be examined through its distribution which Wild, (2006) stated, functions as a lens. In Pfannkuch's, (2008) study, Ray, a statistician identified statistics as basically the science of variation and statistical thinking to be mainly about evaluating variation. According to Snee, (1999), "if there was no variation, there would be no need for statistics and statistician". Variation occurs everywhere in our daily experience different result are obtain when different people measure the same quantity with the same instrument (Moore, 1990) or even when an individual measures the same quantity with the same instrument at different times. Cobb and Moore, (1997) claims that what gives statistics a particular content and set it apart from mathematics is the emphasis on variation. Hence, understanding the concept of variability or spread of data is major factor in understanding distribution, and this is necessary for statistical inferences (Garfield & Ben-Zvi, 2008). In Cobb's 1992 report which served as the basis for the GAISE college report, it was recommended that to emphasize statistical thinking, teachers were to help students recognize the ubiquitous nature of variability which is the essence of statistics as a discipline. Every introductory statistics course is aimed at producing statistically educated students. This means students must develop statistical literacy and the ability to think statistically. To achieve this, a number of goals were enumerated in the GAISE College report among which was that students must be taught to be certain of the fact and also understand why variability, the essence of statistics, is natural or ever present, quantifiable and explainable. Though the statistical techniques to help achieve these goals were important, the knowledge students' gain after going through these techniques was more important (Aliaga et al., 2005). Even at the lower level of education, it is recommended that teacher assist students to understand the nature and sources of variability since statistical problem solving and decision making depend on individual ability to understand, explain, and qualify the variability in data. This is because it is focus on variability in data that sets apart statistics from mathematics (Frankling et al., 2005).   In study in Turkey 8th grade students' statistical literacy of average and variation, Yolcu and Haser, (2013) found that although the majority of participants were able to explain the concept of variation in various contexts, their responses in other contexts indicated they considered there was more variation where the data set consisted of values of the same numbers. These responses showed a sign of possible misconception about the concept of variation of the students. Shaughnessy, (1997) observed a lesser attention given to research on variability. Giving reasons to this Shaughnessy, (1997) indicated statisticians have traditionally been very enamored with standard deviation as the measure of spread or variability, and teachers and curriculum developers often avoided dealing with spread because they felt they could not do so without introducing standard deviation. The standard deviation, he noted, is computationally complex and difficult to motive, particularly with beginning students. Another reason for this lack attention he cited was that people are comfortable using measures of centers or average for predicting into the future or for comparing groups, even though they are not always used correctly. When spread or variation is used in these predicting or comparing processes it only creates confusion in people's ability to make clean predictions or comparisons. Lastly, he indicated that the concept of variability may neither be within the comfort zone of many people nor their zone of believe. Since descriptive statistics are mostly used in our everyday activities, it is necessary to understand basic statistical concepts in descriptive statistics such as the standard deviation. The standard deviation as a measure of variability is another fundamental concept taught in every introductory statistic class. It is the commonly used among the measures of variation and commonly reported in statistical reports. Orta and Sanchez, (2011) describe it as the most suitable measure of variability, but students find it difficult to comprehend. Al-Saleh and Yousif, (2009) identified it as a vague concept. Unlike other summary statistics, the standard deviation is one concept not fully understood by students. Most students can compute standard deviation for data set; though do not comprehend its value and importance. (Al-Saleh and Yousif, 2009; Garfield & Ben-Zvi, 2008). Nevertheless, delMas and Liu (2005) indicated that an incomplete understanding of the standard deviation will affect students' understanding of other advanced and complex concepts like sampling distribution, inference, and p-values. Turegun, (2011) recounted his situation when he started teaching an introductory statistics course. He confessed he had not developed any conceptual understanding of the statistical topics himself. He had little or no difficulty performing calculations using his procedural understanding of the formulae and recipes, but could not give an explanation nor describe the idea behind those formulae and procedures. Turegun, (2011) found similar inconsistencies between conceptual and procedural understanding of introductory statistics topics among his students. One of such concepts was the standard deviation. Using the algorithm, he, as well as his students, could calculate the standard deviation with their calculators once they knew which keys to use. However, explaining the concept of the standard deviation in the context of a particular data set was problematic. Reading & Shaughnessy, (2004) attributed part of students' difficult in working with the standard deviation to the lack of accessible models and metaphors for students' conceptions of the concept. delMas and Liu, (2005) also observed that most teachers tend to stress on teaching of formula of the standard deviation and practicing it with performing calculations. The authors also stated that teachers link the standard deviation to empirical rule of normal distribution, thereby, depriving student students of its actual conceptual understanding. According to Delmas and Lui, (2005), a conceptual model of the standard deviation is required to develop an instructional procedure that helps students to understand the concept. The claimed that a model of that nature should involve the coordination of numerous fundamental statistical concepts out of which the concept of standard deviation is build. In the study of Matthew and Clark (2007) which was to examine the conceptions of mean, standard deviation and Central Limit Theorem most successful students had immediately after an introductory statistics course, their result showed that student did not have a grounded conception of standard deviation. Out of the students who were interviewed, it was found that some of them had an action conception of standard deviation, the lowest level of understanding according to APOS Theory. At this level, the students only saw the standard deviation just as a rule or formula to be followed, and they were not able to describe the algorithm for finding the standard deviation.

Contrary to the case of the range, none of the students' demonstrated an appropriate process conception of standard deviation. Matthew and Clark, (2007) therefore attributed this to the fact that most of the student had not been exposed to the concept of standard deviation before elementary probability and statistics course. Moreover, the algorithm used for the computation of the standard deviation was evidently more complex, and their use was groundless in the minds of the students. The lack of a correct process conception of the standard deviation exhibited by the students indicated that the standard pedagogical treatment of this topic for these students was ineffective (Matthew and Clark, 2007). Even though Clark, Kraut, Matthews, and Wimbish (2007) replicated the study of Matthew and Clark (2007) with some modifications, the results were similar. Their results also indicated that a third of the students used in this study only demonstrated to have a partial action conception of standard deviation. This is because in some cases, some were unable to calculate or describe the standard deviation. Of those who had even progressed past the action conception of standard deviation, they seemed to also have a limited process conception of the procedure for computing the standard deviation; others demonstrated they have only an instrumental understanding of this process. Those who understood the process relational, found the successive distance of the measurement from the range, while those who understood it instrumentally were involved in merely subtracting and squaring the numbers with no cognitive link to the concept of distance. In any case, even though the student will in terrorize the procedure for calculating the standard deviation into the process, it will be done in an abstract sense by imputing data, performing calculations and yielding an output.

2.11 Summary

The review points to the fact that most students commit errors which lead to misconceptions of the concept of measure of variation. Though students are able to do computations on these statistical concepts, they are not able to explain their result in connection to the set data from which the answers were obtained. Statisticians and statistics educators have attributed this to the tradition method and instructional strategies used by teachers in the introduction statics courses. The traditional methods of learning whereby students passively listen to teachers and work in isolation have the causes for rote memorization and learning of fragment facts (Meletiou-Mavrotheris & Lee, 2002) leading to instrumental learning. In such a teaching environment, research shown that students do not learn what their teachers expect of them (Chance, delMas, & Gaefield, 2004; delMas & Liu, 2009) and these challenges encounted by student consequently lead to misconceptions in statistical reasoning (Chan & Ismail, 2013).

CHAPTER THREE

METHODOLOGY

3.0 Overview

This chapter delineated the research methods utilized to examine the errors and misconceptions of senior high school pupils about statistical measures of variation. This study's methodological framework is based on the pragmatism paradigm, facilitating a mixed-methods approach to thoroughly investigate both quantitative and qualitative dimensions of students' learning obstacles. An exploratory sequential methodology was employed to systematically amalgamate data from several sources, enabling a more comprehensive investigation of the trends in students' statistical reasoning and errors. The chapter delineates the study's population, sampling techniques, data collection tools, and analysis methodologies. The study addresses trustworthiness and researcher subjectivity, establishing a solid basis for understanding the results within the educational setting of Winneba Senior High School.

3.1 Research Paradigm and Philosophy

This study is based on the pragmatism paradigm, emphasizing the practical application of research methods to address real-world issues and to find solutions that connect theoretical understanding with practice. Pragmatism is well-suited for mixed-methods research, as it facilitates the integration of quantitative and qualitative approaches, thereby enhancing the explanatory power of the data. Morgan (2007) posits that pragmatism endorses a pluralistic approach to knowledge, urging researchers to prioritize methods that effectively address the research question instead of strictly conforming to conventional philosophical dichotomies such as positivism and constructivism. Aligning with this paradigm allows the study to prioritize practical outcomes and actionable insights regarding students' statistical misconceptions, thereby enhancing the research's relevance to educational practice and policy. The focus of pragmatism on practical outcomes (Patton, 2015) aligns effectively with the mixed-methods approach utilized in this study, which aimed to measure and interpret students' statistical errors comprehensively. Through its philosophical perspective, pragmatism integrates both deductive reasoning, prevalent in quantitative analysis, and inductive reasoning, crucial in qualitative analysis. This alignment allows the study to interpret statistical errors through both numeric performance and the meanings students ascribe to statistical concepts, thereby capturing their depth of understanding or misconceptions (Tashakkori & Teddlie, 2010). This study employed mixed method research approach and for that matter both quantitative and qualitative data were collected for analysis which makes the pragmatism the best research paradigm to study. The pragmatic paradigm was chosen due to its methodological flexibility and its ability to emphasize both theoretical frameworks and practical solutions.

3.2 Research Approach

A mixed-methods approach was utilized in the study, integrating both quantitative and qualitative methods to thoroughly analyze and understand high school students' errors and misconceptions regarding statistical measures of variation. The mixed-methods approach is especially appropriate for this study, as it facilitates a comprehensive exploration of students' understanding through both statistical analysis and detailed contextual insights (Creswell & Plano Clark, 2018). Mixed-methods research adeptly combines numerical data to evaluate error patterns with qualitative insights that delve into the cognitive and interpretive factors underlying these patterns. This approach is crucial for examining intricate educational challenges, such as conceptual misunderstandings in statistics. Greene, Caracelli, and Graham (1989) further justify this approach by highlighting that mixed-methods research improves research validity through triangulation, where various data types reinforce one another, resulting in more robust findings. This study employed triangulation to merge quantitative performance scores with qualitative interview data, thereby ensuring that the findings effectively reflect both the breadth and intricacy of students' statistical misconceptions. For instance, quantitative test data can uncover the frequency of particular errors, whereas qualitative interviews can elucidate the reasons behind these errors, presenting a holistic perspective on student comprehension that a single-method approach would overlook (Tashakkori & Teddlie, 2010). Furthermore, employing mixed methods is particularly advantageous in the realm of educational inquiry as they effectively connect research findings with practical implementations in real-world settings. According to Johnson, Onwuegbuzie, and Turner (2007), mixed methods are closely aligned with pragmatism, emphasizing outcomes that are both actionable and relevant in the context of educational practice. This study employed a combined approach that offers both statistical and narrative insights, which can guide practical instructional improvements, making the findings more applicable for educators, curriculum developers, and policymakers. The selection of an exploratory sequential design enhances the study's aims, as it enables the initial collection of quantitative data to identify trends in student errors, subsequently followed by qualitative data to delve deeper into the reasoning behind these errors (Creswell & Creswell, 2017). This design proved beneficial for elucidating quantitative findings through qualitative data, thereby facilitating a deeper exploration of students' misconceptions (Ivankova, Creswell, & Stick, 2006).

3.3 Research Design

The study utilised an explanatory sequential design, which is a mixed-methods approach that entails gathering and analyzing quantitative data first, followed by qualitative data to enhance and clarify the initial results. This design proved to be highly effective in the realm of education, as quantitative trends can uncover the frequency and nature of student errors, while qualitative insights provide a deeper understanding by examining students' reasoning and misconceptions. Creswell and Plano Clark (2018) indicate that an exploratory sequential design is particularly effective for investigations focused on elucidating the fundamental reasons behind observed trends, as it facilitates the direct integration of qualitative data with quantitative findings. The rationale for employing an exploratory sequential design was rooted in its ability to yield an in-depth understanding of intricate phenomena, like cognitive errors in statistical reasoning, by initially identifying the types of errors present and subsequently investigating the underlying reasons for their occurrence. Morse (1991) contends that sequential designs are crucial when initial quantitative data establishes a foundational baseline to guide a subsequent interpretive phase that is qualitative. This study begun with a quantitative phase that identifies error patterns among a sample of senior high school students. Following this, the qualitative phase gathers student perspectives, providing a deeper understanding of statistical misconceptions. By following a sequential approach, the design guaranteed that the qualitative data effectively responds to particular gaps or patterns highlighted in the quantitative findings, resulting in a more focused and enlightening analysis (Ivankova et al., 2006). One more benefit of the exploratory sequential design is its capacity for providing explanations. After quantitative data identifies the prevalence and types of errors, qualitative data can offer narratives and examples that clarify how students understand statistical concepts.

According to Teddlie and Tashakkori (2009), employing mixed-method designs with sequential analysis enhances the explanatory validity of a study by directly aligning interpretive phases with quantitative findings, thereby aiding in the refinement of theoretical and practical implications within educational contexts. The qualitative follow-up phase enhances the understanding of quantitative trends, contributing to the development of more effective teaching interventions that address students' conceptual misunderstandings. This design choice ultimately showcased a practical approach, highlighting findings that are not only statistically reliable but also contextually significant. Morgan (2007) highlights that the sequential approach provides the flexibility needed to adapt qualitative methods in response to specific quantitative findings, leading to insights that are both actionable and relevant to educational practice.

3.4 Study Population

The study population included all final-year students at Winneba Senior High School in the Effutu Municipality, concentrating on five particular classes: General Science 4, General Agriculture 1, Business 3, Home Economics 1, and General Arts 3. The selection of these classes was informed by their previous engagement with statistical concepts, especially measures of variation, which serves as the central theme of this investigation. By including only students who have engaged with this topic, the researcher ensured that participants have the foundational knowledge necessary to explore their errors and misconceptions, which in turn strengthens the reliability of the findings (Teddlie & Yu, 2007). Choosing final-year students as the focus for this study was especially suitable for investigating intricate cognitive dimensions in statistical comprehension, given that these individuals have encountered a broader range of mathematical and statistical subjects during their academic journey. This decision was supported by findings that show advanced students frequently exhibit more complex and varied misconceptions, positioning them as an excellent group for a thorough investigation of cognitive errors (Watson & Callingham, 2003). Moreover, students in their final year find themselves at a pivotal point in their education, where grasping statistical concepts is essential for achieving academic success and applying knowledge in real-world scenarios beyond the classroom. The deliberate choice of classes in the Senior High School environment is warranted, as it facilitates a thorough analysis of error patterns within a uniform educational framework, thereby strengthening the study's internal validity (Patton, 2015). This study narrowed its focus to students within a single school and municipality, effectively controlling for variations in curriculum and instructional methods that could differ across different educational settings. This approach allowed for a more precise understanding of the specific misconceptions that are common among this particular group of students.

3.5 Sampling and Sample Size

This research employed purposive sampling to identify five classes from the final-year cohort at Winneba Senior High School: General Science 4, General Agriculture 1, Business 3, Home Economics 1, and General Arts 3. The selection of these classes was deliberate, as they had successfully navigated the curriculum addressing statistical measures of variation, thereby guaranteeing participants' comprehension of the concepts being examined. These classes were purposively selected because they have gone through the topic under study that is measures of variation in statistics , hence the use of the purposive techniques. A subset of 140 students was gathered from a total of 300 using convenience sampling from the selected classes. To guarantee that this sample demonstrated a fundamental comprehension of the subject matter, only those students who achieved a score of 40% or higher on an initial assessment were incorporated, thereby refining the study's emphasis on pertinent errors and misunderstandings among sufficiently prepared individuals. Convenience sampling is frequently utilized in educational research as a result of logistical limitations, proving effective in acquiring a practical and readily available sample that can provide insights promptly (Creswell & Creswell, 2017). Furthermore, a selection of ten students were intentionally chosen from the pool of 200 participants to engage in qualitative interviews, thereby offering a more profound insight into the cognitive factors contributing to statistical errors. Onwuegbuzie and Leech (2007) assert that purposive sampling in qualitative research facilitates the selection of participants who possess rich information, thereby providing detailed insights that augment the interpretive depth of the study. The integration of purposive and convenience sampling harmonizes practicality with specificity, facilitating a thorough investigation of statistical misconceptions within the specified population. This dual approach guarantees that the sample remains both accessible and pertinent to the study’s objectives, thereby augmenting the reliability and contextual applicability of the findings (Patton, 2015).

3.6 Data for the Study

This research predominantly employed primary data gathered directly from student participants via assessments and interviews. Quantitative data were acquired through a structured assessment designed to evaluate students' comprehension and application of statistical measures of variation. Direct assessments hold significant value in pinpointing particular error types and misconceptions, by research that underscores the necessity of contextualized testing for precise cognitive evaluation (Shaughnessy, 2007). Furthermore, qualitative data were gathered via semi-structured interviews, facilitating a more profound investigation into the reasoning processes of students regarding their responses. This mixed-data approach offers an extensive perspective on students' knowledge deficiencies and cognitive obstacles, a methodology endorsed by Creswell and Plano Clark (2018), who champion the integration of quantitative accuracy with qualitative richness in educational research. The collection of primary data guarantees that the results accurately represent the learning environment of the population, thereby augmenting the validity and significance of the study about instructional methodologies (Patton, 2015).

3.7 Instrumentation

This study used two main instruments for data collection: a test and an interview guide. These tools were chosen to gather both quantitative and qualitative data, providing a thorough exploration of students' errors and misconceptions regarding statistical measures of variation.

3.7.1 Description of the Test

The test was designed to quantitatively assess students' understanding of statistical concepts, specifically focusing on measures of variation such as mean, range, standard deviation, and percentiles. It included questions that aligned with curriculum standards and aimed to identify specific errors in calculation, comprehension, and problem-solving. To ensure its validity, the test items were aligned with the learning outcomes relevant to the senior high school statistics curriculum. This approach is supported by studies indicating that content validity is essential for diagnostic assessments (Shaughnessy, 2007). The test was administered to 200 students, resulting in a response rate of 87.5%. Responses were analyzed to identify patterns of statistical errors (see Table 4.1 for a statistical breakdown).

3.7.2 Interviews

Semi-structured interviews were conducted with a purposively selected sample of ten students. These interviews provided qualitative data that complemented the quantitative findings and allowed for a deeper exploration of students' cognitive processes and the reasoning behind their answers. The interview guide included open-ended questions that encouraged students to discuss their thought processes when responding to test items, offering insights into the specific misconceptions and cognitive barriers they encountered. According to Creswell (2014), in-depth qualitative data is essential in mixed-methods research for understanding the nuanced thought patterns that underlie quantitative outcomes, particularly in educational settings where complex cognitive skills are assessed.

3.8 Validity and Reliability

The focus on ensuring validity and reliability was fundamental to the methodological rigour of this study. The test instrument's content validity was established by ensuring that the questions corresponded with the statistical concepts outlined in the senior high school curriculum, with a particular emphasis on measures of variation. The alignment with curriculum standards is crucial as it guarantees that the test accurately mirrors the expected learning outcomes for students, facilitating a valid assessment of their comprehension and any misconceptions (Shaughnessy, 2007). The test items underwent expert review to enhance clarity, relevance, and appropriate challenge for the target demographic. This approach aligns with the recommendations of Cohen, Manion, and Morrison (2018) aimed at reinforcing content validity. The researcher conducted a pilot test with a comparable cohort to address reliability, enabling the identification and resolution of any inconsistencies in the test items. The reliability of the test was evaluated through Cronbach's alpha, resulting in a coefficient of 0.82, which signifies a strong internal consistency (Taber, 2018). To improve the reliability of qualitative data obtained from interviews, a semi-structured format was employed. This approach ensured consistency in the delivery of questions while also permitting deeper exploration of students' individual experiences. Patton (2015) advocates for this method to enhance reliability in qualitative research by promoting consistency throughout interviews while also ensuring depth in each response. The integration of validity and reliability in this approach highlights the strength of the study's instruments, confirming that the findings genuinely represent students' performance and the conceptual difficulties they face in grasping statistical measures of variation.

3.9 Pilot Testing

A pilot test was conducted before the main study to refine both the test and the interview guide, ensuring their validity and reliability. This pilot study involved a similar group of final-year students from another high school in the region. Feedback from the pilot revealed areas of ambiguity in certain test items, prompting revisions for improved clarity and relevance. Additionally, the pilot provided the researcher with the opportunity to assess the time required for test completion, ensuring it was manageable within the allocated session. The pilot test also enabled an evaluation of the reliability of the test items using Cronbach's alpha, which indicated a high level of internal consistency ($\alpha = 0.82$), consistent with the recommended standards in educational research (Taber, 2018). Furthermore, pilot testing the interview guide enhanced the reliability of qualitative data collection, as adjustments were made to improve the flow and depth of the probing questions (Patton, 2015). These refinements laid a strong foundation for data collection, ensuring that the instruments would produce meaningful and reliable insights in the main study.

3.10 Data Collection Procedure

The data collection for this study was conducted using a structured and sequential approach to effectively capture both quantitative and qualitative data. The process began by administering a carefully designed test to 200 final-year students from Winneba Senior High School. This test assessed their understanding of statistical measures of variation, including concepts such as mean, range, standard deviation, and percentiles. The test items were aligned with the senior high school curriculum, enabling a diagnostic analysis of specific errors and misconceptions (Shaughnessy, 2007). Students completed the test under supervised conditions to ensure consistency and reduce potential variability in their responses. The quantitative data collected were analyzed descriptively to identify common patterns and errors in student performance. Following the test, semi-structured interviews were conducted with a purposively selected subset of 10 students. These interviews aimed to explore the reasoning processes behind the students' answers, investigating the cognitive and interpretative challenges they faced. The semi-structured format allowed for both structured and open-ended questions, providing a more nuanced understanding of the test results (Creswell & Plano Clark, 2018). Each interview was audio-recorded and transcribed for accuracy, with the responses analyzed thematically to complement the quantitative findings and offer comprehensive insights into student errors. To ensure rigour in the data collection, all instruments and procedures were pre-tested.

3.10.1 Test Administration

The test administration was meticulously organized to uphold consistency, reduce bias, and create a standardized environment that facilitates precise data collection. The test took place in a classroom environment at Winneba Senior High School, involving all

200 final-year students who participated in a single session overseen by the researcher and trained proctors. Before the test, students received a detailed explanation regarding the study's objectives and were guaranteed that their responses would remain confidential, by the ethical standards outlined for educational research (Cohen, Manion, & Morrison, 2018). This briefing was designed to reduce test-related anxiety and promote candid, unbiased responses. The assessment was organized into a 45-minute format, consisting of various sections that concentrated on measures of variation, such as the mean, range, standard deviation, percentiles, and associated statistical concepts. Students were directed to address all questions independently, guaranteeing that their answers demonstrated their comprehension and educational achievements. The controlled setting ensured the preservation of test integrity and reduced chances for collaboration, which could potentially skew the results (McMillan & Schumacher, 2014). The presence of proctors ensured adherence to instructions among students and facilitated uniform responses to any procedural questions across the cohort. The test scripts were promptly collected upon completion to eliminate the possibility of any alterations by students following their submission. The gathered scripts underwent anonymization and coding for analytical purposes, ensuring the protection of students' identities and the preservation of data confidentiality by ethical research standards (American Educational Research Association, 2011). The results from the test administration were scored and examined, yielding quantitative insights into the frequency and categories of errors related to the different statistical concepts evaluated. The results guided the subsequent phase of the study, during which selected students engaged in interviews aimed at delving deeper into their cognitive processes and the rationale behind their test responses.

3.10.2 Student Interviews

After the test was administered, semi-structured interviews were carried out with a purposive sample of 10 students chosen according to their test performance to gather insights into their understanding and misconceptions. The interviews sought to explore students' cognitive frameworks, specifically examining their understanding of statistical measures of variation, along with prevalent mistakes and reasoning patterns observed during the testing phase. Interviews were carried out one-on-one in a serene and secluded environment, allowing students to express their reasoning freely, uninfluenced by their peers. The discussions were structured around open-ended questions, which enabled students to provide detailed explanations of their strategies for particular test questions, articulate any misunderstandings, and clarify the challenges they faced. This structure enabled comprehensive responses while allowing for the exploration of distinct insights provided by each participant (Creswell & Creswell, 2017). To guarantee the precision of the data, every interview was recorded with the student's consent and subsequently transcribed for thorough examination. A thematic analysis of the transcripts was performed to uncover recurring cognitive patterns, challenges, and misconceptions, thereby enhancing the understanding of the quantitative data derived from the tests. This qualitative data enhanced the study by providing insights into the fundamental factors that affect student performance in statistics.

3.11 Data Analysis

A diagnostic test covering topics in measures of variation was administered to 198 students. However, only 140 students’ responses were considered for analysis since after collecting the test papers it was found that 58 of them left over 90% of test items unanswered or blank. This yielded a response rate of 70.7%. The data analysis for this study was conducted using a structured approach, with distinct procedures for both the quantitative and qualitative data collected. The goal was to identify patterns in students' understanding and misconceptions about statistical measures of variation. This analysis aimed to provide a comprehensive, evidence-based interpretation of the students' errors.

3.11.1 Quantitative Data Analysis Procedure

The quantitative data collected from students' test responses were analysed using descriptive statistics to identify patterns in error types and performance levels. Each test item was scored individually, and aggregated by the measures of variation topics tested which are range, histogram and table of values, mean, quartiles & percentiles, variance and standard deviation. Table 3.1 shows the weighing of test items in each topic.

Table 3.1 Weighing or marks awarded to test items in each topic

Items in measures of dispersion test	Weighting[1] (or number) of test items
Overall test	75 (100)
Range	7 (9)
Histogram & Table of values	7 (9)
Mean	17 (23)
Quartiles & Percentiles	11 (15)
Variance	9 (12)
Std. Dev.	24 (32)

[1]Marks awarded to items in topic; [2]Percent in parenthesis

It clear from Table 3.1 that, the topics tested were not given equal scores. While ‘range’ ‘histogram and table of values’ had weights less that 10% since items on these topics are largely factual and conceptual knowledge questions. Consequently, the mean and standard deviation had the largest weighing (i.e., 23% and 32% respectively) because they require procedural knowledge and metacognitive knowledge to solve questions on those topics.

To further examine how errors were distributed across different statistical tasks, the data were categorized into specific error types based on the Modified Newman Error Hierarchy: reading, comprehension, transformation, process skills, and encoding errors (Clements, 1982). This categorization allowed for the identification of recurring error patterns, enabling a detailed diagnostic analysis of students' cognitive difficulties at each stage of problem-solving.

3.11.2 Qualitative Data Analysis Procedure

To analyse the qualitative data from student interviews, thematic analysis was used to identify recurring themes and patterns in their explanations and reasoning. The transcripts were thoroughly reviewed and coded based on specific cognitive themes, such as misunderstandings of statistical terms, procedural errors, and difficulties in interpreting data representations. This coding process enabled an organised examination of students' misconceptions, providing deeper insights into the nature of their errors (Braun & Clarke, 2006). Through iterative refinement, the codes were grouped into broader themes that aligned with the error categories observed in the quantitative data. This approach provided a richer, explanatory context for the statistical findings. By combining both quantitative and qualitative analyses, we gained a robust, triangulated understanding of student misconceptions in statistical variation, which facilitated meaningful conclusions and recommendations for improving instruction.

3.12 Trustworthiness

Qualitative research acknowledges that it is difficult to avoid the problem of trustworthiness (Guba & Lincoln, 2005). Trustworthiness is a way for qualitative researchers to monitor potential sources of bias in the design, execution, review, and interpretation of the study that corresponds to the principles of internal scientific studies (Lincoln & Guba, 1986). Some of the techniques that foster trustworthiness in a sample are triangulation of multiple data sources, and many others (Merriam, 1991). Thus, this issue of trustworthiness, quality, rigor or the soundness of any qualitative research work cannot be overlooked. Whereas quantitative researchers look at reliability, objectivity and validity to ensure the trustworthiness of their findings, qualitative researchers look at credibility, dependability, transferability and conformability, as criteria for trustworthiness (Guba, 1981; Schwandt, Lincoln, & Guba, 2007). These four criteria: credibility, dependability, transferability and conformability, are discussed in the design and implementation of this current study.

3.12.1 Credibility

Credibility is the extent to which research is believable and appropriate with special reference to the level of agreement between participants and the researcher. Credibility is established through member checking, peer debriefing and mamboing.

Member checking is a participant feedback strategy which involves giving the participants the opportunity to verify the findings. After analysing and summarising data, the researcher gave the participants the summarised data for them to check if it addressed what the researcher obtained from their responses (Carlson, 2010). In the qualitative framework, the individual perspectives are accommodated. Participants were given a chance to cross-check if the results presented were their viewpoints to eliminate the researcher’s bias (Creswell, 2009). This member checking aided in ensuring that the researcher had accurately interpreted the participants’ responses.

Peer debriefing is a strategy whereby an experienced colleague in the field of study is given a chance to examine and discuss the findings with the researcher. The person who is given the mandate to examine the findings should be neutral. Peer debriefing strategy helped the researcher to find out new things that were omitted when the researcher analysed the data. On one hand, it helped the researcher to give authentic results (Guba & Lincoln, 2005). Peer debriefing enabled the researcher to find out if the methodology intended for use was employed. After a careful examination of the results, the peers provided the researcher with feedback hence ensuring trustworthiness of the research findings (Guba & Lincoln, 2005).

Memoing involves reading data several times to get meaning from it. It is a process in which, the feelings and thoughts of participants are recorded according to how they perceived the situation. It reflects what the participants have revealed when answering the research questions (Charmaz, 2006). Memoing was employed by the researcher who read the transcribed data several times and related it to the recorded one. This helped the researcher to understand participants' views by studying the data intensively to get the meaning from the context (McMillan & Schumacher, 2010). The concept of memoing helped the researcher to retain ideas that might be lost and facilitated the development of study design. Moreover, the researcher's journal was used to monitor subjectivity, perspectives and bias.

3.12.2 Transferability

The term transferability describes the extent to which results of qualitative study can be transferred to other setting with different respondents (Bitsch, 2005). It represents how the findings of study may be applicable to other situations. Researchers are particularly concerned with the extent to which the results of the work at hand can be applied to wider population (Shenton, 2004). Guba & Lincoln (1982), have describe it to be parallel to external validity or generalizability in quantitative studies.

To facilitate transferability, Bitsch (2005) opines that the researcher must provide a detailed description of the study and also use purposive sampling in selecting respondents. By detailed description Anney (2014) explains, it involves the researcher clarifying all the research processes from data collection, context of the study to production of the final report. According to Shenton (2004), providing adequate detailed of the context of the study helps a reader to decide whether the existing environment is comparable to another situation he or she is familiar with. It also helps other researcher to replicate the study with similar conditions in different settings (Anney. 2014; Guba, 1981).

Teddlie and Yu (2007) defined purposive sampling as “selecting units (e.g., individuals, groups of individuals, or institutions) based on specific purposes associated with answering a research study‘s questions”. It requires that one considered the characteristics of members of sample as those characteristics are related directly to the research questions (Devault, 2017). When used, purposive sampling helps the researcher to concentrate on key respondents particularly knowledgeable on the issues under study (Schutt, 2006). According to Cohen, Manion, and Morrison (2011), in-depth findings are obtained when purposive sampling is used than in probability sampling methods. It also helps the researcher in his decision as to why a specific group of respondents must be in a study (Bernard, 2000).

In the current study both strategies have been employed to ensure transferability. The study employed the used of purposive sampling technique to ensure specific and varied information is emphasized than a generalized and aggregate information, which would have been the case in the quantitative research. Also a detailed description of the processes used in the study, as well as the results have been spelled out to aid replication if need be.

3.12.3 Dependability

In parallel with the concept of reliability in the quantitative studies, dependability discusses how stable the results of a study are over time. It answers the question: will the results of a study be the same when replicated with same or similar respondents in a similar context (Bitsch, 2005)? McAninch posits that the question of dependability refers to the situation in which a different researcher repeats the same work, in the same setting, with the same methods and respondents and obtains similar results (McAninch, 2015). As such, in ensuring dependability of research findings, a researcher must provide evidence that if the study were to be replicated with the same or similar respondents in the same or a similar context, findings would be repeated. To do this, Bitsch (2005) proposes that a detailed and comprehensive documentation of the research process must be provided as well as every methodological decision. This, according to Shenton (2004) will help future researcher to be able to replicate the work. For this study, the researcher made sure all research procedures were described in detailed. Also the views of supervisors, advisors and experts in the field of qualitative research were sought to ensure the right research procedures were followed to confirm dependability.

3.12.4 Confirmability

The term confirmability is described as the neutrality and objectivity of the study. This means that the results of the study present perceptions of participants (Merriam, 2009). It denotes the extent to which results of a study might be confirmed by different researchers (Baxter & Eyles, 1997). Confirmability describes the degree of impartiality or the extent to which the findings of a study are shaped by the participants and not the biasedness, motivation, or interest of the researcher. Bitsch (2005), posits that data, its interpretations, and findings must be based on individuals and situations apart from the researcher. Thus, according to Gasson (2004), Confirmability concern itself with the main issues that “findings should represent, as far as is possible, the situation that is being research instead of the beliefs, pet theories, or biases of the researcher”. It is also concerned with establishing that data and interpretations of the findings are not fabrications of the inquirer’s imagination, but are clearly derived from the data” (Tobin & Begley, 2004)

Data triangulation was used as one of the strategies to ensure confirmability and to reduce the effect of biasedness of the researcher. Again, audit trial, which describes the detailed methodological or step by step description of how data was gathered and processed in this current study, is given to ensure confirmability. Also, consultation with advisors during the collecting and processing of data helped to control biasedness on the part of the researcher

3.13 Ethical Considerations

The study meticulously addressed ethical considerations to uphold the integrity of the research process and safeguard participants' rights. All participants, along with their guardians when applicable, provided informed consent by the ethical guidelines established for educational research (Cohen, Manion, & Morrison, 2018). The study’s objectives, procedures, and participants' right to withdraw at any time without repercussions were communicated, thereby ensuring that participation was entirely voluntary. The research placed significant emphasis on maintaining anonymity and confidentiality. To safeguard the identities of participants, the data underwent a process of anonymization, wherein unique codes were assigned in place of personal identifiers. The storage and access of responses were conducted with a focus on security, ensuring that only the researcher had access, in compliance with the data protection standards outlined by the American Educational Research Association (2011). This guaranteed the protection of sensitive information during the research process. Additionally, the study followed the principle of non-maleficence, guaranteeing that participants experienced no harm whether physical, psychological, or emotional. The tests and interviews were carried out in a setting that promoted student comfort, thereby reducing stress or anxiety throughout the data collection process. Delicate subjects were handled meticulously to prevent unnecessary unease. The research received ethical approval from the University of Education, Winneba, confirming adherence to institutional ethical standards. The approval highlighted the researcher's dedication to responsibly executing the study, and adhering to the ethical standards of educational research. The implementation of these measures systematically guaranteed that the study adhered to the highest ethical standards, safeguarding participants' rights and preserving the integrity of the research process.

CHAPTER FOUR

RESULTS AND FINDINGS

4.0 Overview

The chapter examined the performance of senior high school students and the types of errors they made when tackling statistical measures of variation. This analysis employed a mixed-methods approach, combining quantitative data from tests with qualitative insights gathered from interviews. Each analysis is systematically aligned with the research questions the study set out to interrogate, ensuring that the findings directly address the objectives of the investigation. Errors were categorized using Newman error analysis framework, which identified key challenges in reading, comprehension, transformation, process skills, and encoding. The qualitative insights added depth to the findings, revealing a common reliance on rote learning, frequent symbolic misinterpretations, and procedural deficiencies.

4.1 Demographic Background of the Respondents

Table 4.1 shows the distribution of males and females, age categories and programmes offered by the students sampled for the study.

Table 4.1 Demographic Characteristics of the Respondents

	Teachers		**Demographics**	**Teachers**	
Demographics	Number	Percent		Number	Percent
Gender			*SHS Programme of Study*		
Male	241	83	Gen. Science	35	25
Female	59	17	General Arts	24	17
Total	300	100	Business	14	10
Age			Home Econs	40	29
16 years or younger	16	11	Agric	27	19
17 years old	57	41	Total	140	100
18 years or older	67	48			
Total	140	100			

Table 4.1 shows that 58 (41%) males and 82 (59%) females were sampled for the study. The table shows a majority of female respondents compared to the males in the sample. In Ghana currently, most students enter senior high school at the age of 16. The majority (48%) of the students involved in the study were 18 years old with only 11% of them being younger than this age (see Table 4.1)

4.2 Senior High School Students' Performance on Measures of Variation in Statistics (Research Question 1)

The first research question aimed to assess senior high school students’ performance in statistical measures of variation in the Effutu Municipality, specifically. Winneba Senior High. In order to confirm whether or not the senior high school students in the study have good conceptual knowledge and procedural skills for dealing with topics under measures of dispersion, the researcher began to present the results for the first research question by examining the students’ overall performance on the test.

The students' test scripts were scored out of 75, and the descriptive statistics of the raw scores out of 75 and percent scores are presented in Table 2 and the box plot (Figure 1) present the of overall performance as well as performance on the individual topics in measures of dispersion.

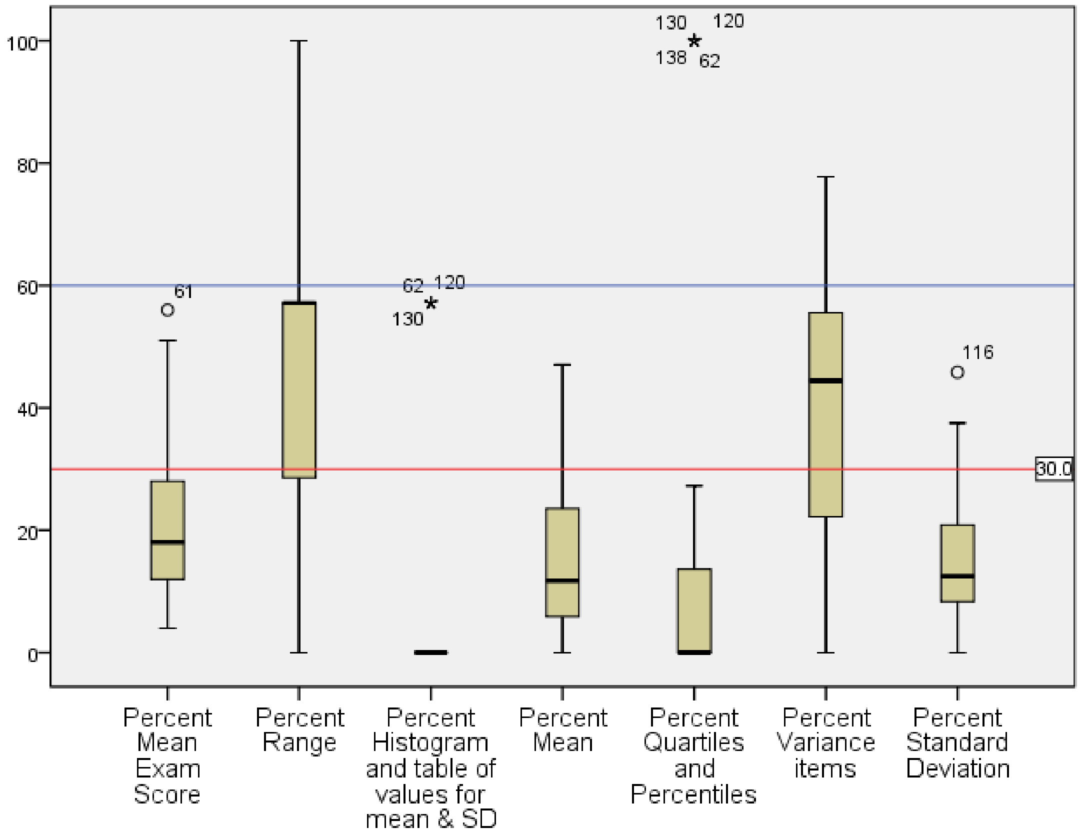
Table 4.2 Results of the students' performance on the measures of dispersion achievement test (N = 140)

Items	Weighting[1] of topic in test	Number attemptting items	Zero score	Min.	Mode	Mean score	Std. Dev.	Max
Overall Exam Score	75 (100)	140 (100)	0 **(0)**	3 (4)	19 **(25)**	16 (22)	9 (11)	**42 (56)**
Range	7 (9)	140 (100)	2 (9)	0 (0)	4 (57)	4 **(52)**	2 (22)	7 (100)
Histogram & Table of values	7 (9)	36 **(26)**	31 **(86)**	0 (0)	0 (0)	1 (8)	1 (20)	4 (57)
Mean	17 (**23**)	140 (100)	14 (10)	0 (0)	1 (6)	4 (21)	3 (16)	12 (71)
Quartiles & Percentiles	11 (15)	140 (100)	20 **(56)**	0 (0)	0 (0)	2 (18)	4 (34)	11 (100)
Variance	9 (12)	36 **(26)**	11 (8)	0 (0)	4 (44)	4 (**45**)	2 (23)	8 (89)
Standard Deviation	24 (**32**)	140 (100)	7 (5)	0 (0)	1 (4)	4 (18)	3 (13)	15 (63)

[1] Marks awarded to items in the topic [2] Percent in parenthesis

The results in the top row of Table 4.2 (shaded blue) indicate the overall mean score for the measures of dispersion test was 22% which is very low, and thus confirms the researcher's observation which provided the spur for the study. Though the highest mark scored on the test was 56% and lowest score was 4%, the most popular score was 25% (i.e., the modal score) which are indications that generally all the students have a poor grasp of the concepts and procedures required to carry out measures of dispersion tasks in statistics.

Figure 4.1 **Results of the students' performance on the measures of dispersion achievement test (N = 140)**



Also, it is clearer from the box plot in Figure 4.1 that the upper quartile of the distribution is below the 30% score line (or red line) suggesting over 75% of the students failed to make a total score of 30%.

The students' performances on the individual topics under measures of dispersion were also examined. The topics are - range, histogram and table of values, mean, quartiles & percentiles, variance and standard deviation. The performances on the items under *range* and *variance* were found to record means which were close to 50% (i.e., 52% and 45% respectively) making them to look like topics that the students demonstrated the highest performances. Even though the students' performance was much better on these topics, the information in the box plot in Figure 4.1 shows that the upper quartiles of the topics range and variance were below the below the 60% score line (or blue line) suggesting over 75% of the students failed to make a total score of 60% on the topics.

The topics the students found most difficult were histogram and table of values and quartiles and percentiles. These were also the topics that were skipped most and majority (86%) of the few (26%) who attempted it obtained zero scores. The mean score for all the test items under 'histogram and table of values' and 'quartiles and percentiles' were found to be less than 1% (i.e., 0.2% an 0.5% respectively). It is also interesting to note that for range, five of the students attempting the items obtained half of the full score, and for quartiles and percentiles, five of the students who attempted the items obtained the full mark.

Finally, two important topics under measures of dispersion whose mastery is very necessary in solving and making meaning of statistical problems are the mean and standard deviation, and the latter is an extension of the former when it comes to its definition and calculation. In this regard, the weighting or marks awarded to the test items under mean and standard deviation were 23% and 32% respectively, making the two areas to constitute a little over half of the scores for the entire measures of dispersion test. In spite of the emphasis on these content areas, the mean scores recorded for these two important topics were 21% and 18% indicating that the students cannot cope with several tasks involved in determining, computing and interpreting means and standard deviations.

Table 4. 3 **Descriptive statistics of performance of students in the sample on topics in measures of variation**

Measures of variation topic	Students who took the test	Attempted test items on topic	Skipped or didn't attempt	Weighting of test items	Mean	Std Dev	Min	Max	Skewness	Kurtosis
Range	140	140	0	9	52	22.3	0	100	-.17	-.72
Mean	140	140	0	23	21	16.0	0	71	.58	-.47
Standard Deviation	140	140	0	32	18	13.1	0	63	.78	.02
Quartiles and Percentiles	140	36	104	15	18	34.1	0	100	2.01	2.46
Histogram/Table of values	140	36	104	9	8	20.0	0	57	2.18	2.91
Variance	140	140	0	12	45	23.3	0	89	-.39	-.70
Mean Exam Score	140	140	0	75	22	11.3	4	56	.48	-.18

Each of the measures of variation topics (i.e., mean, standard deviation, minimum, maximum, skewness and kurtosis) was further analyzed by examining the skewness and kurtosis examined. Figure 2 is the histogram of the scores obtained for range in the test.

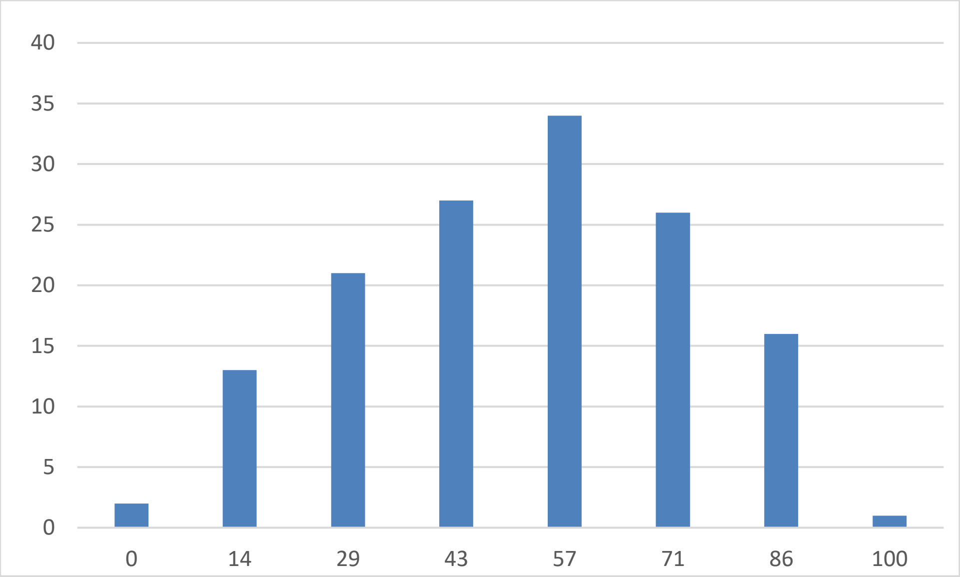


Figure 4. 2 Histogram of Range

The mean score of 52% for the Range metric indicates that students' performance this topic is average, suggesting that, students' performance was neither high or low. The standard deviation of 22.3% reflects moderate variability, meaning that while most students scored around the mean, some variation in performance is present. The skewness value of -0.17 reveals a slight leftward skew, implying that a small subset of students scored significantly lower than the majority. Furthermore, the kurtosis value of -0.72 indicates a flatter distribution, with fewer extreme scores, as corroborated by the accompanying histogram in Figure 4.2. The further analysis of the other topics have been moved to Appendices since, the information presented in Table 4.3 provide the picture of the skewness and kurtosis of the students' performance distribution.

This indicates that a subset of students is achieving much better results than their classmates. Most of the distributions exhibit negative kurtosis, which suggests that they have flatter shapes and lighter tails compared to a normal distribution. This suggests that the occurrence of extreme values (scores that are very high or very low) is less common compared to a normal distribution. However, certain metrics such as the application of standard deviation (q17) exhibit greater peaks, indicating a clustering of scores around specific values. Moreover, the examination of percentiles and quartiles reveals that a substantial proportion of students are clustered within specific score ranges. This could suggest either clustering resulting from comparable levels of understanding or the possible impact of instructional excellence or curriculum on student achievement. Ultimately, the overall scores exhibit a notable rightward skewness, with an average of 13.57 out of a maximum of 52. This indicates that although a significant number of students are facing difficulties in achieving high scores overall, there are a small number of students who are scoring far better than the average, hence increasing the mean score.

4.3 Research Question 2: What is the error prevalent among senior high school students in measures of variation in statistics

This research question aimed to identify and analyze the errors commonly observed among senior high school students when dealing with measures of variation in statistics. To achieve this, an integrated approach, combining both quantitative data from student assessments and qualitative data from interviews, was employed. This mixed-methods analysis provides crucial insights into the specific types of errors students make in their statistical reasoning. The analysis is supported by marked test scripts, which reveal patterns of misunderstanding, and in-depth student interviews, which further elucidate the reasons behind these mistakes. In line with feedback received, particular attention is given to addressing student misconceptions, numerical clarity, and the incorporation of qualitative insights into the discussion.

In the statistics test, participants were required to demonstrate proficiency in several key areas, including:

- Identifying relevant information from the provided data,
- Understanding the context and requirements of the problem,
- Expressing the data in an appropriate and coherent format,
- Applying relevant statistical techniques, and
- Presenting solutions clearly with correct notation and labels.

The students' test scripts were assessed using a structured marking scheme that categorized their errors based on Newman's five distinct levels or error hierarchy. The framework, as detailed in Chapter 2 was used to classify errors that students were likely to make into one of these categories to pinpoint specific difficulties students encountered. Based on the above skills, using the marking scheme the script was marked and participants' scores, difficulties and errors were identified. Table 4.2 shows how the skills in solving the statistic tasks were categorized under the Newman's error analysis model.

Newman's categorization of errors in Table 2.2 provides a structured lens through which the challenges in students' statistical reasoning can be analyzed. The application of Newman's error model reveals not only the types of mistakes students make but also their root causes, such as a lack of conceptual understanding, over-reliance on memorized formulas, or confusion with statistical symbols. Table 4.2 provides a comprehensive breakdown of student error types across nine statistical questions (Q12–Q20) using **Newman's Error Model**.

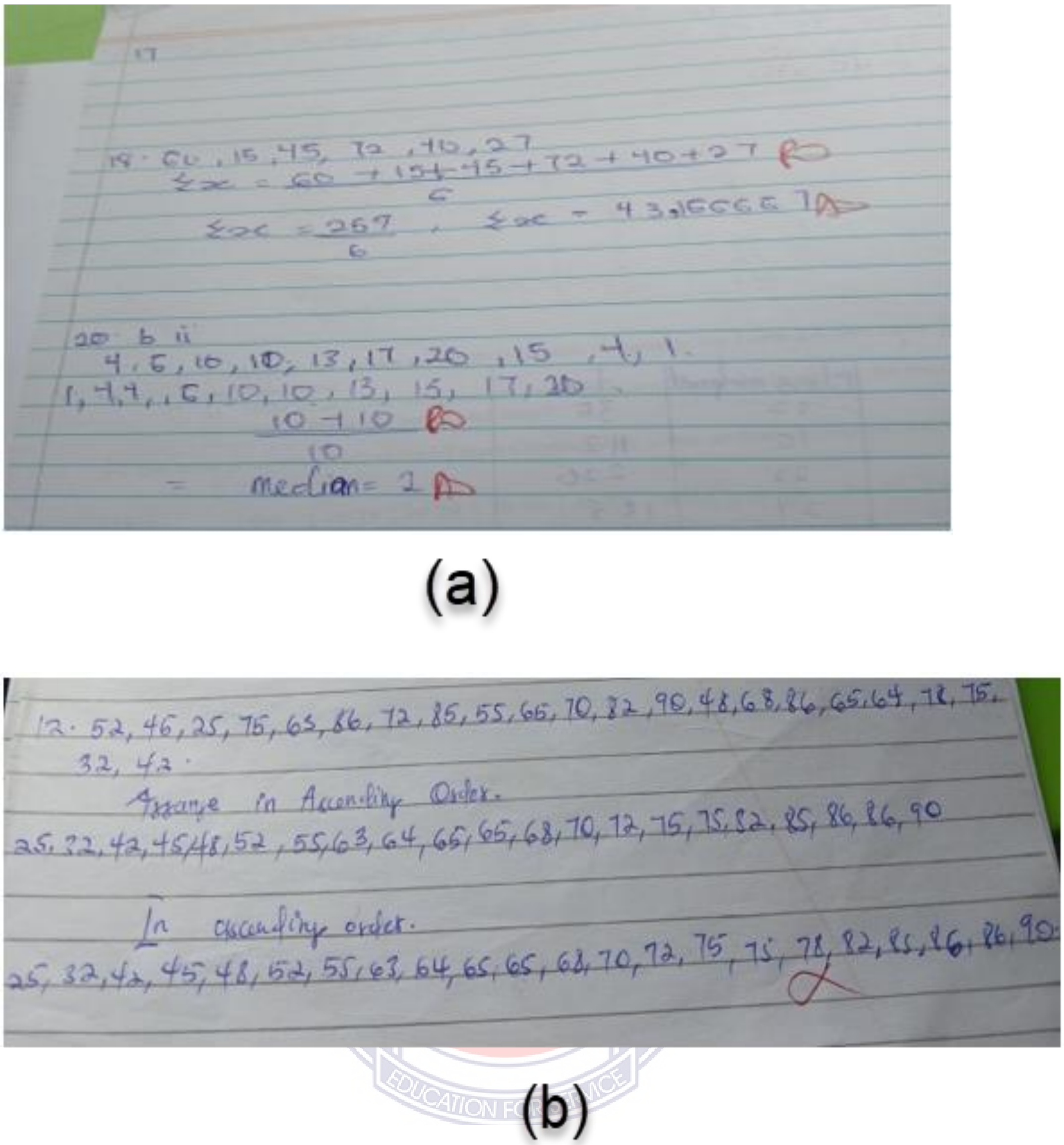
Table 4. 4: Error Types in Student Responses

Error Type	Q12 (%)	Q13 (%)	Q14 (%)	Q15 (%)	Q16 (%)	Q17 (%)	Q18 (%)	Q19 (%)	Q20 (%)
Reading	1 (0.5)	0 (0)	0 (0)	0 (0)	0 (0)	7 (3.5)	2 (1)	0 (0)	2 (1)
Comprehension	0 (0)	0 (0)	30 (14)	3 (1.5)	28 (14)	3 (1.5)	9 (4.5)	0 (0)	0 (0)
Transformation	0 (0)	18 (9)	15 (7.5)	0 (0)	2 (1)	0 (0)	1 (0.5)	2 (1)	12 (7.5)
Process Skills	0 (0)	31 (3.5)	27 (13.5)	1 (0.5)	4 (2)	0 (0)	0 (0)	3 (1.5)	15 (7.5)
Encoding	1 (0.5)	62 (31)	43 (21.5)	23 (11.5)	62 (31)	17 (8.5)	21 (10.5)	26 (13)	26 (13)

The data in Table 4.2 reveals distinct patterns of error prevalence, with a noticeable concentration of errors in specific phases, particularly **Encoding** and **Comprehension**. These findings highlight key areas where students encounter difficulties in solving statistical problems related to measures of variation.

4.3.1 Reading Error

Reading errors occur when students fail to correctly interpret or recognize important symbols, terms, or information provided in a question. Although the overall frequency of reading errors in this study is relatively low, they can significantly impede a student's ability to solve the problem correctly. As shown in **Table 4.4**, reading errors were recorded in Q12 (0.5%), Q17 (3.5%), Q18 (1%), and Q20 (1%). These figures, while not as high as other error types like comprehension or encoding, reflect a critical weakness in students' foundational understanding of the statistical content.



17

18 · 60 , 15 , 45, 72 , 40 , 27

$\Sigma x = \frac{60 + 15 + 45 + 72 + 40 + 27}{6}$

$\Sigma x = \frac{257}{6}$, $\Sigma x = 43.16666 7$

20 · b ii

4 , 5 , 10 , 10 , 13 , 17 , 20 , 15 , 4 , 1.

1, 4, 4, , 5 , 10 , 10 , 13, 15, 17, 20

$\frac{10 + 10}{10}$

= median = 2

(a)

12 · 52, 46, 25, 75, 63, 86, 72, 85, 55, 66, 70, 82, 90, 48, 68, 86, 65, 64, 78, 75.
32, 42.

Arrange in Accending Order.

25, 32, 42, 45, 48, 52 , 55, 63, 64, 65, 65, 68, 70, 72, 75, 75, 82, 85, 86, 86, 90

In ascending order.

25, 32, 42, 45, 48, 52, 55, 63, 64, 65, 65, 68, 70, 72, 75, 75, 78, 82, 85, 86, 86, 90.

(b)

Figure 4. 3: Examples of Reading Error (a) Question No.12, (b) No. 17, 18, and 20

In Figure 4.3, the student was required to calculate the “range” from a given dataset. However, rather than finding the range (i.e., the difference between the largest and smallest values in the dataset), the student instead arranged the data in ascending order, stopping there without completing the task. This error reflects a fundamental misinterpretation of the question—specifically, the student failed to understand the meaning of "range," which is a basic statistical concept. This error falls under the **Reading** category because the student did not correctly process the key term "range" in the question, which inhibited their ability to move forward in solving the problem. The fact that this error only appeared in 0.5% of students suggests that while most students understood the task, a small subset may struggle with basic statistical vocabulary, a problem that can hinder their progress in more complex statistical problems.

Figure 4.3(b) provides further examples of reading errors across multiple questions. In Question 17, students were required to apply the concept of **standard deviation**, but the student left this question blank, indicating an inability to engage with the task. This is a clear reading error, as the student likely did not understand the statistical terms or symbols necessary to proceed. In Question 18, the task was to compute a “new mean and standard deviation” from a given dataset. The student failed to interpret the question correctly, evidenced by the lack of appropriate work shown and the empty response space, reflecting confusion about what was required in terms of symbol interpretation and statistical operations. Similarly, in Question 20, students were tasked with drawing a cumulative frequency curve and interpreting the graph to estimate key statistical measures (e.g., the 80th percentile, interquartile range, and mode). The student in **Figure 4.3(b)** displayed a reading error by misunderstanding the terms and symbols associated with these tasks, which led to incomplete or incorrect responses.

The errors depicted in **Figures 4.3(a) and (b)** support the quantitative data from **Table 4.4**, which shows that reading errors, though not the most frequent type, still present a significant barrier to effective problem-solving. For instance, in Q17, where the reading error rate was 3.5%, we see from the image that some students did not even attempt to engage with the statistical task, likely due to an inability to interpret the key terms such as "standard deviation."

The visual evidence from **Figures 4.3(a) and (b)** also underscores the specific nature of reading errors—these are not necessarily failures in calculation or logic, but in the basic recognition and interpretation of statistical language. In the case of Question 12, the student's misinterpretation of "range" as requiring data ordering instead of subtraction demonstrates how crucial it is to ensure students have a firm grasp of statistical terminology. The empty spaces in **Figure 4.3(b)** further illustrate that when students encounter unfamiliar terms or symbols, they are more likely to leave questions blank or fail to proceed beyond the initial reading phase.

The reading error rates of 0.5% in **Q12**, 3.5% in **Q17**, and 1% in **Q18** and **Q20** may seem low in isolation, but they represent a critical early-stage barrier that affects students' ability to engage with the problem-solving process. These reading errors are particularly concerning in complex questions such as Q17 and Q20, which require students to apply statistical concepts like **standard deviation** and interpret graphical data. The data from **Table 4.4**, combined with the examples from **Figures 4.3(a) and (b)**, suggest that students who encounter reading errors are likely to struggle throughout the rest of the problem-solving process, as they are unable to proceed past the initial interpretation of the question.

4.3.2 Comprehension Error

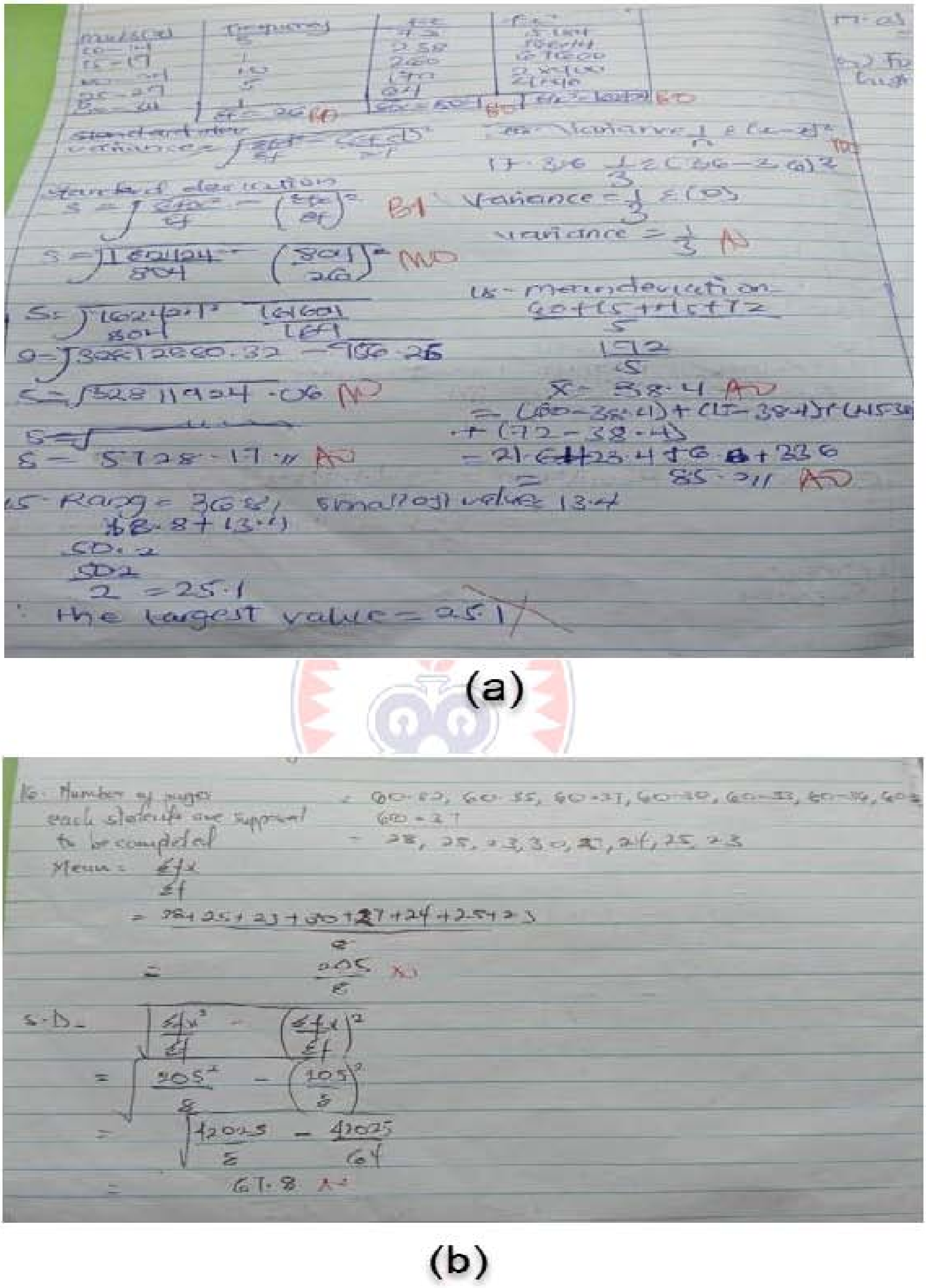


Figure 4. 4: Error in comprehension for Questions (a) No.14,15, 17 and 18, (b) No. 16

Comprehension errors occur when students can read the problem but fail to fully understand what is being asked, leading to incorrect or incomplete responses. These errors reflect a deeper misunderstanding of the question's requirements, often tied to gaps in students' conceptual knowledge of statistical terms and operations.

As seen in **Table 4.4**, comprehension errors were most significant in **Q14 (14%)** and **Q16 (14%)**, with smaller but still notable error rates in **Q15 (1.5%)**, **Q17 (1.5%)**, and **Q18 (4.5%)**. The relatively high frequency of comprehension errors compared to reading errors indicates that, although students can recognize and read statistical terms, they struggle to fully understand and interpret them correctly, hindering their ability to apply the correct methods and solve the problem accurately.

In **Figure 4.4(a)**, the student demonstrates multiple comprehension errors. In Question 14, the student was asked to estimate the **variance and standard deviation** of a given dataset. However, instead of calculating these statistical measures, the student simply wrote down some information from the question, showing an inability to comprehend what the task required. This is a clear example of a comprehension error, where the student fails to interpret the key terms "variance" and "standard deviation" correctly, reflecting a lack of understanding of how these concepts are applied. Similarly, in Question 15, the student failed to identify and calculate the **largest value** from the dataset provided. Rather than solving the problem, the student only listed some basic information without addressing the actual question. This further demonstrates a comprehension gap, where the student recognizes some elements of the problem but does not fully grasp the task at hand.

Figure 4.4(b) shows a similar pattern of comprehension errors in Question 16. The student was required to compute key statistical measures such as the **range** and

interquartile range from a dataset. However, the response reflects a misunderstanding of what was being asked, with the student providing an incorrect or incomplete explanation. This highlights a failure in grasping the problem’s context, leading to an inability to progress to the transformation and calculation phases. The student’s difficulty in interpreting terms like "interquartile range" underscores a lack of conceptual clarity, which is critical for solving such statistical problems.

The examples in **Figures 4.4(a) and (b)** vividly illustrate how comprehension errors manifest in students' responses. These errors are not simply due to carelessness or misreading, but rather reflect fundamental gaps in understanding statistical concepts and operations. In **Figure 4.4(a)**, the student's inability to differentiate between simply copying information and solving the problem shows a lack of comprehension regarding the tasks of calculating variance and standard deviation. Similarly, the incomplete or incorrect responses in **Figure 4.4(b)** highlight how students struggle to understand the requirements of complex statistical measures, such as the interquartile range.

The data in **Table 4.4** show that comprehension errors are a major source of difficulty for students. The relatively high error rates for **Q14 (14%)** and **Q16 (14%)** suggest that students consistently struggle with understanding the requirements of questions that involve key statistical concepts. In these questions, students are asked to compute measures like variance, standard deviation, and interquartile range—concepts that require not just rote memorization of formulas but a deeper understanding of their meaning and application.

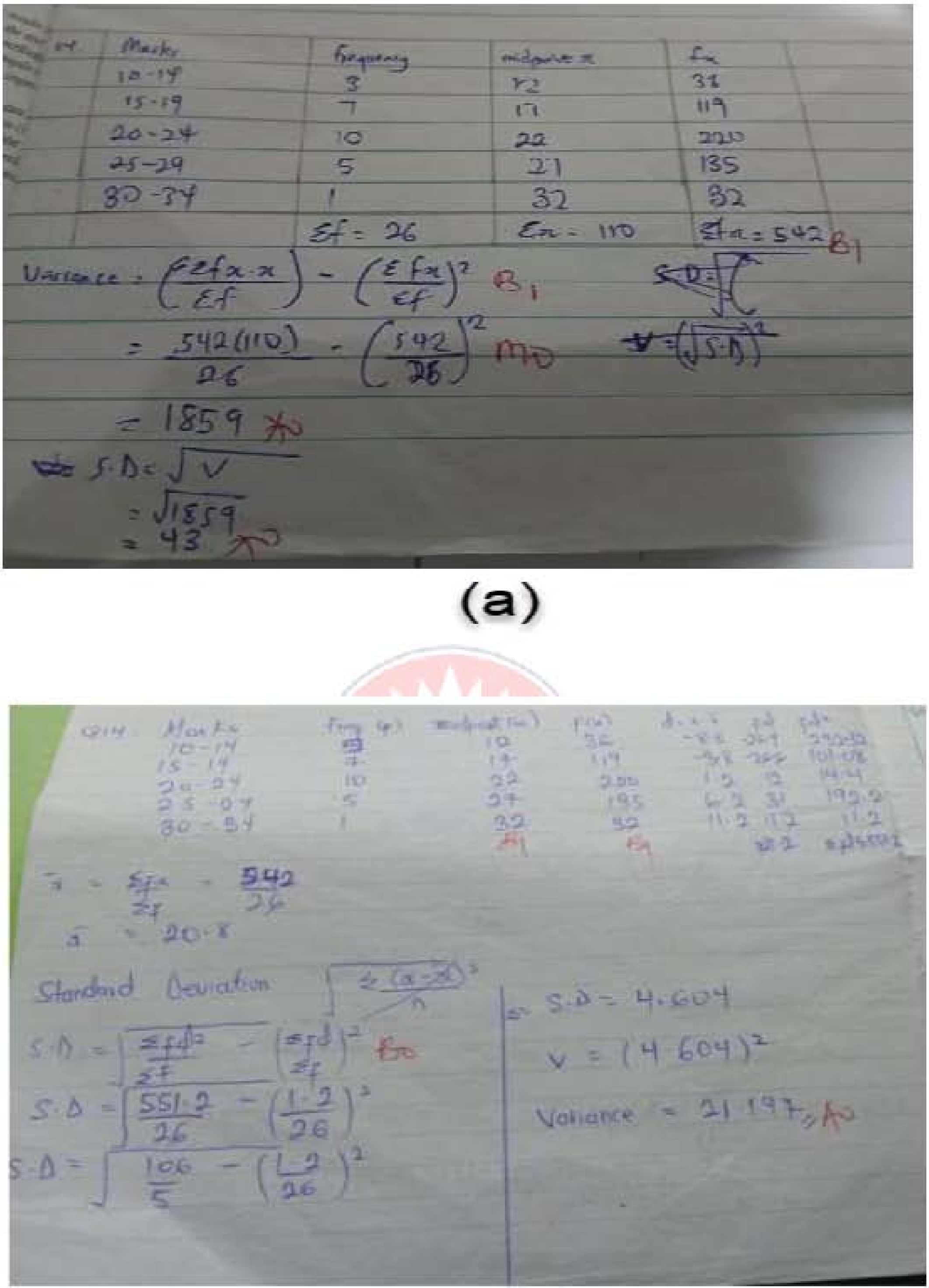
In particular, the 14% comprehension error rate in **Q14** highlights a widespread issue with students' ability to interpret tasks involving variance and standard deviation. This is further supported by the evidence from **Figure 4.4(a)**, where the student provided irrelevant or incomplete information, failing to address the question's actual requirements. Similarly, the 4.5% comprehension error rate in **Q18** underscores the challenges students face in tasks requiring the calculation of a **new mean and standard deviation** from a modified dataset. These error patterns indicate that students may be familiar with the terms involved but lack the conceptual understanding necessary to solve the problems accurately.

The evidence suggests that comprehension errors are particularly prevalent in questions requiring more complex reasoning and multiple steps. For instance, the **14% comprehension error rate** in **Q16** reflects difficulties with interpreting statistical terms like "interquartile range," as demonstrated by the incomplete responses in **Figure 4.4(b)**. These errors often arise from students' failure to grasp what is being asked in the question, preventing them from applying the appropriate statistical techniques.

4.3.3 Transformation Error

Transformation errors occur when students struggle to convert a problem’s verbal or written information into the correct mathematical or statistical model. In this phase, students may fail to use the appropriate formulas or methods required to solve the problem, even if they correctly understand what is being asked. Transformation errors often arise from a lack of deeper conceptual understanding or confusion over when and how to apply specific statistical techniques.

As highlighted in **Table 4.4**, transformation errors were most prevalent in **Q13 (9%)**, **Q14 (7.5%)**, and **Q20 (7.5%)**, with smaller error rates in **Q16 (1%)** and **Q18 (0.5%)**. These errors are particularly concerning because they reflect a fundamental breakdown in students' ability to translate a problem into a solvable mathematical form, which is a critical step in the problem-solving process.



14	Marks	Frequency	midpoint x	fx
	10-14	3	12	38
	15-19	7	17	119
	20-24	10	22	220
	25-29	5	27	135
	30-34	1	32	32
		Σf = 26	Σx = 110	Σfx = 542 B1

$$\text{Variance} = \left(\frac{\Sigma fx \cdot x}{\Sigma f}\right) - \left(\frac{\Sigma fx}{\Sigma f}\right)^2 \quad B_1$$

$$= \frac{542(110)}{26} - \left(\frac{542}{26}\right)^2 \quad M0$$

$$= 1859 \quad A0$$

$$S.D = \sqrt{V} = \sqrt{1859} = 43 \quad A0$$

$$V = \left(\sqrt{S.D}\right)^2$$

(a)

$$\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{542}{26}$$

$$\bar{x} = 20.8$$

$$\text{Standard Deviation} \quad \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}}$$

$$S.D = \sqrt{\frac{\Sigma fd^2}{\Sigma f} - \left(\frac{\Sigma fd}{\Sigma f}\right)^2} \quad B0$$

$$S.D = \sqrt{\frac{551.2}{26} - \left(\frac{1.2}{26}\right)^2}$$

$$S.D = \sqrt{\frac{106}{5} - \left(\frac{1.2}{26}\right)^2}$$

$$\therefore S.D = 4.604$$

$$V = (4.604)^2$$

$$\text{Variance} = 21.197 \quad A0$$

(b)

Figure 4. 5: Example of Transformation Error for Question (a) No.14, (b) No.14

In **Figure 4.5(a)**, the student was required to calculate the **variance** of a dataset. However, instead of applying the correct formula, the student wrote down an incorrect formula and performed incorrect calculations. This error demonstrates a failure to transform the problem into the appropriate mathematical model. Rather than using the correct variance formula, the student substituted incorrect elements, leading to a wrong answer. This reflects the student's lack of understanding of how to apply the concept of variance to the dataset, resulting in a transformation error.

Similarly, **Figure 4.5(b)** shows another transformation error in Question 14. In this case, the student failed to correctly structure the mathematical model needed to calculate **standard deviation**. The formula that should have been used, **SD** = $\sqrt{\left(\frac{\sum fx^2}{\sum f}\right) - \left(\frac{\sum fx}{\sum f}\right)^{2}}$, was either omitted or misapplied, leading to an incorrect answer. This transformation error indicates that the student knew the task required a formula but was unable to correctly identify or apply the right one, a common issue when students rely too heavily on memorized formulas without fully understanding their conceptual basis.

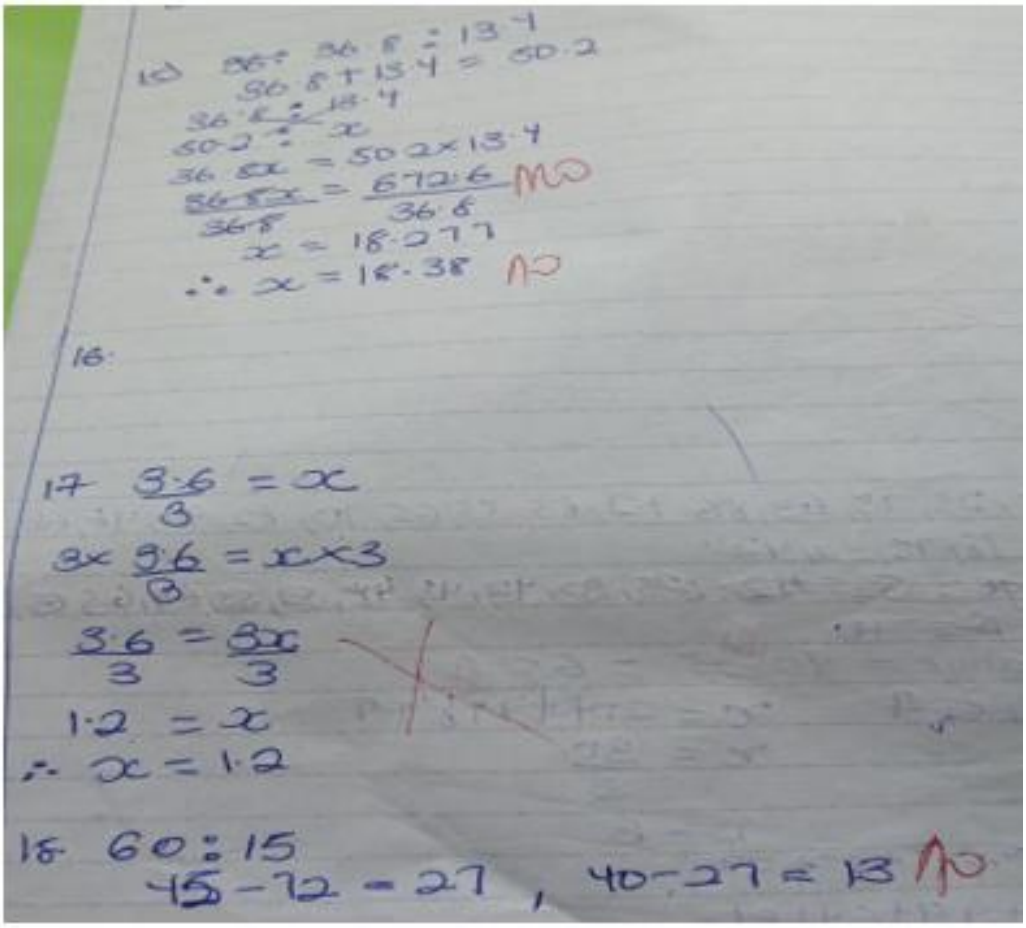
The examples in **Figures 4.5(a) and (b)** illustrate how transformation errors manifest as incorrect applications of statistical formulas. In both cases, students either misapplied the formula for variance or standard deviation, or used incorrect elements in their calculations. These errors suggest a lack of understanding of the conceptual framework behind the formulas, highlighting a reliance on rote memorization rather than a true comprehension of when and how to apply specific statistical models. Without the correct mathematical model, students are unable to execute the necessary steps to arrive at the correct answer. As seen in **Figure 4.5(a)**, the student's misunderstanding of the variance formula resulted in completely erroneous calculations, and **Figure 4.5(b)**, the failure to properly apply the standard deviation formula reflects confusion over how the statistical measures relate to the dataset.

The data from **Table 4.4** show that transformation errors were significant across several questions, with the highest error rates in **Q13 (9%)**, **Q14 (7.5%)**, and **Q20 (7.5%)**. These questions required students to apply specific statistical formulas, such as those for **variance** and **standard deviation**, but many students were unable to successfully convert the word problem into the correct mathematical representation. The **9% transformation error rate in Q13** is particularly concerning, as this question required students to calculate the **mean** and **standard deviation** from grouped data. The high error rate suggests that students struggle with understanding how to represent the grouped data mathematically, particularly when dealing with statistical measures that require more than basic arithmetic. Similarly, in **Q14**, which had a **7.5% transformation error rate**, students were asked to calculate variance and standard deviation, and many failed to apply the correct formulas or misinterpreted the dataset. The evidence from **Figures 4.5(a) and (b)** aligns with this data, showing students' failure to correctly transform the problem into a workable mathematical model.

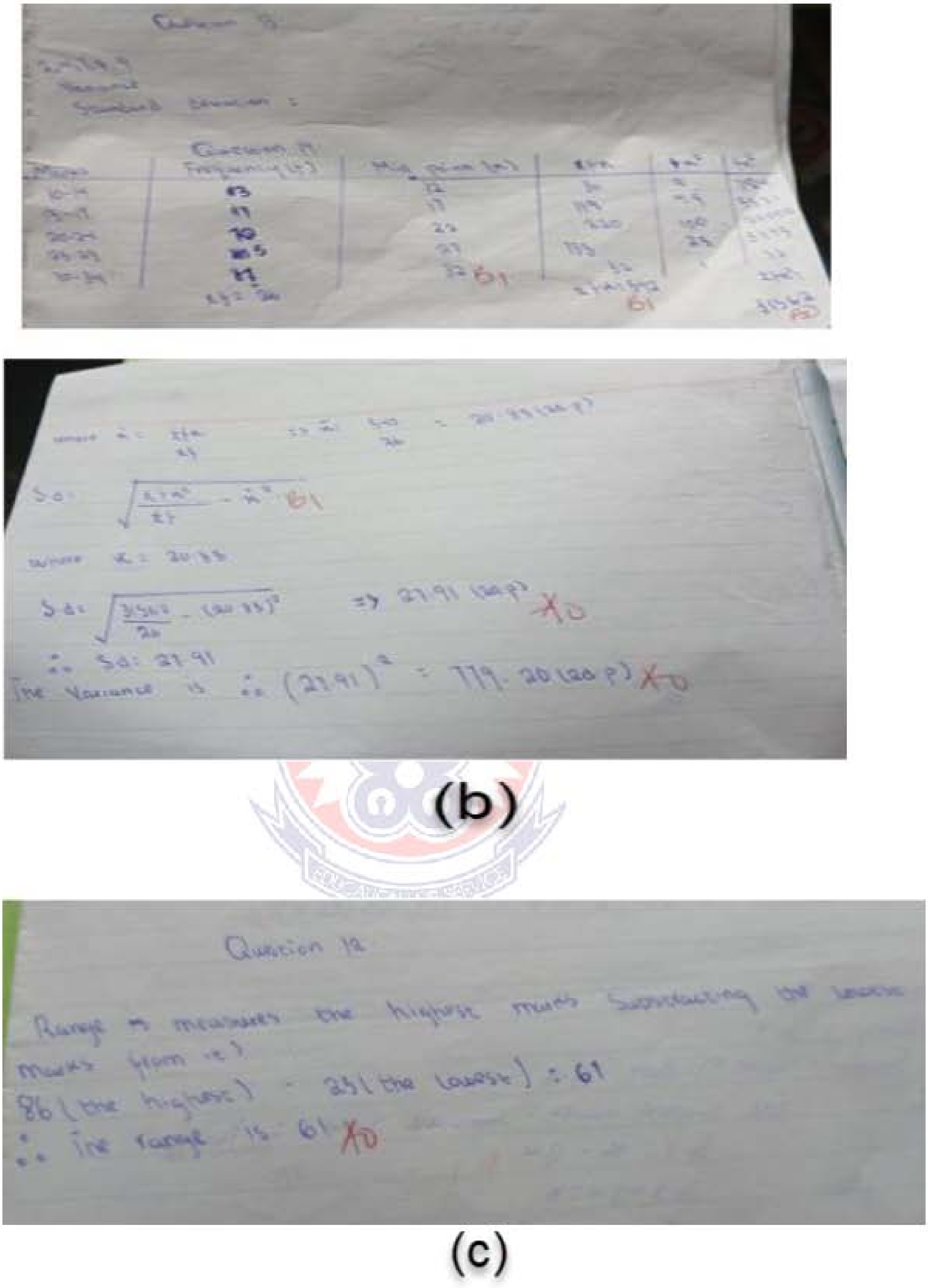
The **7.5% transformation error rate in Q20** is also notable. This question involved drawing a **cumulative frequency curve** and calculating statistical measures such as the **interquartile range**. The relatively high error rate suggests that students struggled to understand how to convert the graphical information into a mathematical framework for further calculation, a common difficulty when interpreting visual data. In contrast, the lower transformation error rates in **Q16 (1%)** and **Q18 (0.5%)** suggest that fewer students encountered difficulties in transforming these particular problems. However, even in these cases, transformation errors prevented students from accurately applying the necessary formulas to solve the problem.

4.3.4 Process Skill

Process skill errors arise when students are unable to correctly execute the steps of a mathematical or statistical procedure, even if they have successfully identified the correct model or formula to use. These errors typically occur due to mistakes in calculations, misapplication of arithmetic operations, or failure to carry out all necessary steps in the problem-solving process.



(a)



S.d: $\sqrt{\frac{\Sigma fx^2}{\Sigma f} - \bar{x}^2}$ B1

S.d: $\sqrt{\frac{35563}{36} - (20.83)^2}$ ⇒ 27.91 (2d.p) X0

∴ S.d: 27.91

The Variance is ∴ $(27.91)^2$ = 779.20 (2d.p) X0

Question 12

Range ⇒ measures the highest marks subtracting the lowest marks from it)

86 (the highest) − 25 (the lowest) = 61

∴ The range is 61 X0

Figure 4. 6: Example of Error in Process Skill for Question (a) No.15, (b) No.14

As indicated in **Table 4.4**, process skill errors were particularly prominent in **Q13 (15.5%)** and **Q14 (13.5%)**, with lower but still significant occurrences in **Q15 (0.5%)**, **Q19 (1.5%)**, and **Q20 (7.5%)**. These errors reflect a general difficulty among students in executing the procedures required to arrive at a correct answer, often due to lapses in basic arithmetic skills or misunderstanding the sequential steps involved in statistical calculations. In **Figure 4.6(a)**, the student made an error in calculating the **largest value** from a dataset. The correct calculation should have been **36.8** (largest value) + **13.4**, yielding **50.2**. However, the student incorrectly calculated the result as **18.38**, which indicates a fundamental mistake in arithmetic operations. This process skill error demonstrates that while the student may have understood the task (to find the largest value), they lacked the procedural accuracy needed to correctly perform the calculation. This kind of error often arises when students fail to apply basic arithmetic rules properly or misinterpret the operations required to solve the problem. **Figure 4.6(b)** provides another example of a process skill error in Question 14, where the student was tasked with calculating **variance**. The correct calculation for the term $\sum fx^2$ should have been **11,964**, but the student incorrectly calculated it as **31,562**. This significant error in arithmetic reflects a failure in the calculation process, which led to an incorrect final answer for the variance. Process skill errors like this are often the result of students mismanaging steps in the calculation or using incorrect intermediate values in their procedure, leading to compounded errors.

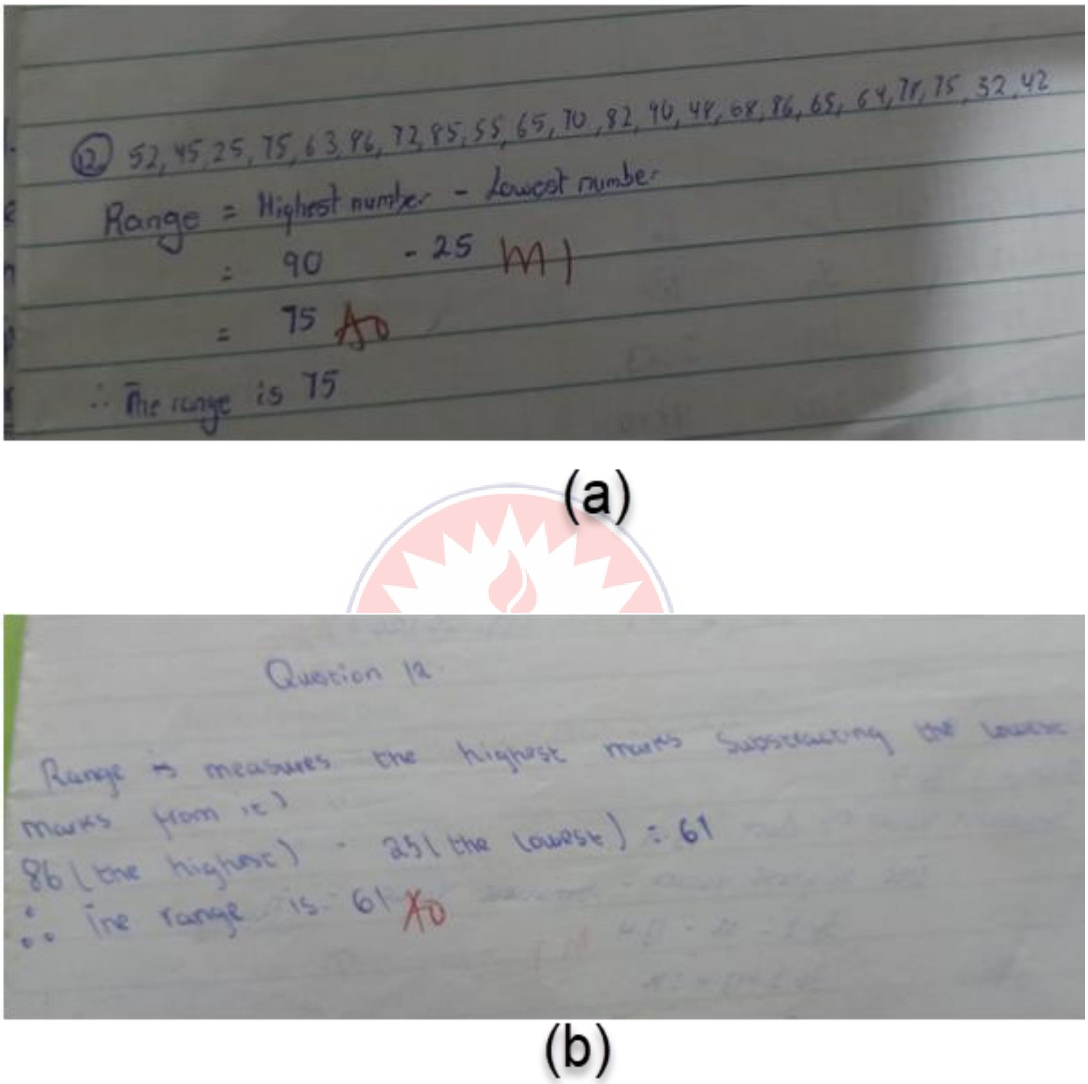
In **Figure 4.6(c)**, the student continued to struggle with calculating variance. The error shown here highlights an incorrect handling of the formula for variance, which resulted in a completely wrong calculation of the final answer. This process skill error likely stems from earlier transformation mistakes, where the student failed to establish the correct formula, leading to missteps in performing the required calculations.

The data from **Table 4.4** show that process skill errors are a significant issue, particularly in **Q13 (15.5%)** and **Q14 (13.5%)**. These questions required students to perform detailed calculations involving grouped data, variance, and standard deviation, all of which involve multiple steps. The high error rates in these questions suggest that many students struggle to correctly perform the necessary procedures, even when they have identified the correct statistical model or formula.

For instance, the **15.5% error rate in Q13** reflects widespread difficulty in performing the arithmetic operations needed to calculate the **mean** and **standard deviation** for grouped data. The process skill errors in this case may stem from earlier transformation issues, as students who fail to set up the problem correctly are more likely to make mistakes in carrying out the calculations. Similarly, in **Q14**, the **13.5% process skill error rate** shows that even when students recognize the need to use a formula for variance or standard deviation, they often fail to perform the steps correctly, as evidenced by the incorrect values for $\sum fx^2$ shown in **Figure 4.6(b)**. In contrast, the lower error rates in **Q15 (0.5%)** and **Q19 (1.5%)** suggest that these questions involved simpler calculations or fewer steps, making it easier for students to avoid process skill errors. However, the **7.5% error rate in Q20** highlights the challenges students face when dealing with visual data representations, such as **cumulative frequency curves**. In these cases, students not only need to interpret the graph correctly but also perform arithmetic calculations based on the data, which can lead to errors if the procedural steps are not followed accurately.

4.3.5 Encoding Error

Encoding errors occur at the final stage of problem-solving, where students fail to express their solutions clearly and accurately, often due to incorrect notation, mislabeling, or careless recording of their results. These errors, while not as frequent as other types, are critical because they reflect students' difficulties in translating their calculated answers into an acceptable, well-organized final response.



⑫ 52, 45, 25, 75, 63, 86, 72, 85, 55, 65, 70, 82, 90, 48, 68, 86, 65, 64, 78, 75, 32, 42

Range = Highest number - lowest number

= 90 - 25 M1

= 75 A0

∴ The range is 75

(a)

Question 12

Range = measures the highest marks Substracting the lowest marks from it)

86 (the highest) - 25 (the lowest) = 61

∴ The range is 61 A0

(b)

Figure 4. 7: Example of encoding 6(a), (b)

According to **Table 4.4**, encoding errors are the most frequent type of error observed across several questions. The highest rates were recorded in **Q13 (31%)**, **Q16 (31%)**, and **Q20 (13%)**, with notable occurrences in **Q14 (21.5%)**, **Q15 (11.5%)**, **Q17 (8.5%)**, and **Q18 (10.5%)**. The significant prevalence of encoding errors suggests that many students, even when they manage to correctly solve parts of the problem, struggle to properly express their answers in a coherent and correct format.

In **Figure 4.7(a)**, the student made an error in presenting the calculated result. Although the student may have performed some of the calculations correctly, the final response is incomplete and unclear. The encoding error is evident in the student's failure to use the appropriate notation and labels for the **variance**. Instead of recording the solution properly, the student presented an unfinished and incorrectly labelled answer. This reflects an encoding failure, where the student was unable to translate the computational work into an organized final answer that accurately represented the result.

In **Figure 4.7(b)**, the encoding errors are more pronounced. For **Question 17**, which required calculating the **standard deviation**, the student left the space blank or provided an answer without the correct labels and notations, indicating a complete breakdown in encoding. In **Question 18**, which involved finding the **new mean and standard deviation**, the student's response lacked clarity and appropriate statistical symbols. This failure to accurately record the statistical values and formulas reveals a major encoding error. The student may have understood the procedures but was unable to express the final solution in a structured and meaningful way. The data in **Table 4.4** reveal that encoding errors were the most frequent type of error in this study, with notable peaks in **Q13 (31%)**, **Q16 (31%)**, and **Q20 (13%)**. The prevalence of encoding errors suggests that many students struggle to express their answers even after they have performed the necessary calculations or steps. The **31% error rate in Q13** shows that nearly one-third of students were unable to properly record their answers for a problem involving the calculation of **mean** and **standard deviation** from grouped data. This is a significant finding, as it indicates that encoding issues are not limited to more advanced questions, but also affect fundamental tasks in statistical reasoning.

Similarly, the **31% encoding error rate in Q16** highlights that students have difficulty presenting results for tasks involving the **interquartile range** and other statistical measures. These errors suggest that even when students comprehend the problem and perform the necessary calculations, they frequently fail to express their solutions using the correct statistical notations and labels. This failure in communication undermines their overall performance, as even correct calculations can be rendered incorrect if they are not properly presented.

The **13% encoding error rate in Q20**, a question that involved drawing and interpreting a **cumulative frequency curve**, suggests that students also struggle with presenting answers that require both graphical and numerical representation. Many students may have understood how to calculate statistical measures from the graph, but were unable to correctly label or express their findings, leading to a high rate of encoding errors. In contrast, encoding errors in **Q14 (21.5%)**, **Q15 (11.5%)**, **Q17 (8.5%)**, and **Q18 (10.5%)** suggest that while students may be capable of solving the earlier stages of the problem, they falter when it comes to recording their final answers. This aligns with the qualitative evidence from **Figures 4.7(a) and (b)**, where students left questions blank or used incorrect notation, demonstrating that encoding failures often stem from a lack of familiarity with statistical conventions.

The analysis of errors in understanding statistical measures of variation among senior high school students, presented in both quantitative data and supported by qualitative interviews, reveals crucial insights into the nature and causes of these mistakes. The triangulation of data provides a comprehensive view of the student’s difficulties, highlighting the most prevalent error types and their underlying causes. This discussion integrates these findings with relevant literature, focusing on the critical issues of reading, comprehension, transformation, and process errors, as well as the absence of encoding errors.

The quantitative data from Table 4.2 in the report highlights that the majority of students struggle with questions related to complex statistical concepts such as variation. The average error rate of 88.78% indicates pervasive challenges across the student population, with students either answering incorrectly or failing to attempt the questions at all. The data further reveals that the error rate was particularly high for questions **Q16** to **Q20**, where students demonstrated widespread confusion and disengagement, as evidenced by the high percentage of non-responses (e.g., 66.5% for Q16 and 89% for Q18). Reading errors were the most frequent error type identified, particularly in complex questions like **Q16 (31%)** and **Q14 (21.5%).** Interviews with students provide further insights, revealing that many students struggled to extract relevant information from the questions. For instance, student **(S44)** misunderstood the term "range" in **Q14**, interpreting it incorrectly as the sequence of numbers from 1 to 20. This suggests a fundamental reading problem that inhibited the student's ability to process the question correctly. Comprehension errors, where students can read the question but fail to grasp its meaning, were another common issue, with rates as high as **14%** for **Q13.** Interviews support these findings, as a student **(S53)** was unable to define the "interquartile range," providing only vague and incomplete explanations. This reflects a lack of deep conceptual understanding, which is a barrier to effective problem-solving. Transformation errors, which involve difficulties in identifying and applying the correct mathematical operations, were particularly prevalent in **Q14,** where **(13.5%)** of students made such errors. Interviews revealed that students often relied on memorized formulas without fully understanding when or why to apply them. For instance, student **(S61)** admitted to memorizing formulas for quartile deviation and standard deviation but struggled to apply them correctly during problem-solving. Errors related to process skills, which involve mistakes in executing the correct mathematical procedures, were less common but still notable. For example, **(3.5%)** of students in Q13 made process errors despite identifying the correct operations. In interviews, student S53 expressed confusion about why their final answer was incorrect, despite correctly identifying the steps needed to solve the problem. Interestingly, encoding errors, which involve the failure to correctly express a solution in written form, were absent across all questions in the analysis. This suggests that once students were able to navigate the earlier stages of problem-solving (i.e., reading, comprehension, transformation, and process), they were generally able to express their solutions accurately. However, the absence of encoding errors could also be attributed to the fact that many students never reached this stage due to errors in earlier steps, as evidenced by the high rates of non-responses and incorrect answers. However, it highlights the importance of addressing the earlier stages of problem-solving to ensure that students can reach the point where they can accurately encode their solutions.

4.4 Research Objective Three: Causes of Errors in Measures of Variation in Statistics

Research question three sought to investigate the underlying causes of errors in statistical measures of variation among senior high school students. The analysis, based on students' marked test scripts and qualitative insights from interviews, reveals several key factors contributing to these errors.

4.4.1 Inadequate Grasp of Statistical Vocabulary

One of the major contributing factors to students' errors was their insufficient understanding of key statistical terms. Interviews revealed that many students struggled to define basic statistical vocabulary, which directly impacted their ability to solve related problems. For example, several students demonstrated confusion over the term **"standard deviation."** In one interview, a student remarked,

For instance, when asked about **standard deviation**, **S45** admitted:

"I don't know what standard deviation is or how to calculate it,"

When probed further for the definition of **standard deviation**, the student simply replied:

"… I don't know …" (Interview Transcript 1).

Another student, **S44**, misunderstood the concept of **range**, explaining:

"Range is these numbers… all these 1 to 20" (Interview Transcript 2).

This lack of understanding was evident in **Q13** and **Q14** of **Table 4.4**, where high comprehension error rates were observed (14% in both questions). Students often misinterpreted or failed to apply the terms "variance" and "standard deviation" correctly. The confusion over terminology, as illustrated in **Figure 4.7(a)** of Section **4.2.2**, was a recurring theme across multiple questions. Without a firm grasp of these foundational concepts, students were unable to progress to more advanced problem-solving stages, leading to significant comprehension and transformation errors.

4.4.2 Reliance on Memorization Rather than Conceptual Understanding

Another significant issue identified was the students' heavy reliance on rote memorization of formulas, rather than developing a conceptual understanding of statistical principles. Many students admitted that they had memorized formulas but did not understand when or how to apply them appropriately. One student, when asked about their approach to solving a question, stated,

For example, **S52** explained their process for solving a question by stating:

"I don't know. I just follow the format" (Interview Transcript 5).

Similarly, when asked about the formula for **quartile deviation**, **S61** recited the formula but could not explain its purpose:

"I memorized the formula" (Interview Transcript 6).

"I memorized the formula for standard deviation, but I wasn't sure how to use it." (Interview Transcript 7).

This over-reliance on memorization led to frequent transformation errors, as seen in **Q14** of **Table 4.4**, where **7.5%** of students failed to correctly apply the formula for variance. These students often wrote down incorrect or partially correct formulas, indicating that they did not fully understand the relationship between the statistical measures and the dataset they were analyzing. The inability to apply memorized formulas to different contexts, as shown in **Figure 4.7.(a)** and **Figure 4.7(b)** of Section **4.2.3**, suggests that students lacked the deeper understanding necessary to adapt their knowledge to solve novel problems.

4.4.3 Misunderstanding of Statistical Symbols

Interviews also highlighted a widespread misunderstanding of statistical symbols, which caused further errors in problem-solving. Many students confused key symbols, such as **n** (the number of data points) and $\bar{x}$ (the mean). This confusion was evident in **Q14** of **Table 4.4**, where a significant number of students (13.5%) made transformation errors by incorrectly interpreting the symbols used in the formula for standard deviation.

One student, for example, misread the symbol for variance, leading to an incorrect calculation.

In one instance, **S52** incorrectly used **x** instead of **n** when calculating the total number of data points:

"Yes. What is 8? What symbol do you use to represent the total number of data? We use 'n' not 'x' alright?" (Interview Transcript 10).

These misunderstandings of statistical notation are a significant barrier to students' success in solving problems, as they often lead to errors in the transformation and calculation phases. **Figure 4.7(a)** and **Figure 4.7(b)** of Section **4.2.3** illustrate these transformation errors, which stem from misinterpreting or misapplying statistical symbols. Without a clear understanding of how to read and interpret statistical symbols, students struggled to move beyond the initial stages of problem-solving.

4.4.4 Lack of Procedural Fluency in Statistical Calculations

A recurring issue identified in both the test results and interviews was the students' lack of procedural fluency when performing statistical calculations. Several students expressed uncertainty over how to correctly follow the steps required to calculate measures like variance or standard deviation. This was particularly evident in **Q13** and **Q14** of **Table 4.4**, where process skill errors were recorded at rates of **15.5%** and **13.5%**, respectively.

For example, **S61** made an arithmetic error when calculating the largest value in **Q15**, acknowledging:

"I didn't rearrange the data" (Interview Transcript 20).

Similarly, **S53**, despite using the correct formula for mean, admitted when asked about standard deviation:

"I don't know why it is like that. I just memorized" (Interview Transcript 8).

This lack of confidence in performing multi-step procedures often resulted in incomplete or incorrect calculations. Students who struggled with procedural fluency were unable to accurately execute the calculations needed to find the correct answer, even when they had identified the appropriate formula. The examples of process skill errors shown in **Figure 4.6(a)**, **Figure 4.6(b)**, and **Figure 4.6(c)** of Section **4.2.4** illustrate how miscalculations and incorrect operations affect the outcomes of the student's work.

4.4.5 Difficulty Interpreting Graphical Data

Finally, the analysis revealed that students had significant difficulty interpreting graphical data, particularly when required to extract or calculate measures from graphs. In **Q20** of **Table 4.4**, where students were asked to draw and interpret a **cumulative frequency curve**, a large number of encoding errors were observed (13%). Many students failed to label their graphs correctly or could not use the graph to estimate statistical measures such as the **interquartile range** or **80th percentile**.

For instance, **S72** misunderstood the quartile positions and explained when asked to calculate the interquartile range:

"75 minus 25?" (Interview Transcript 17).

In another case, **S79** incorrectly calculated the total frequency as **99** instead of **100**, leading to a series of errors in their calculations:

"I see. Why did you take 99?" (Interview Transcript 18).

These responses highlight a fundamental challenge that students face when working with visual data representations. Without the ability to accurately interpret graphs, students were unable to perform the calculations required for the task, leading to incomplete or incorrect answers. The encoding errors related to graphical data interpretation are further illustrated in **Figure 4.7(a)** and **Figure 4.7(b)** of Section **4.2.5**, where students failed to properly label or present their answers, leading to confusion and errors in communication.

The findings from this analysis suggest that the primary causes of errors in students' understanding of statistical measures of variation stem from a combination of conceptual misunderstandings, reliance on memorization, difficulties with statistical symbols, and a lack of procedural fluency. Additionally, the inability to interpret graphical data further compounded these challenges. Addressing these issues will require targeted interventions to strengthen students' understanding of statistical vocabulary, enhance their conceptual knowledge, and improve their ability to perform and express complex calculations accurately.

This study utilised qualitative data collected from semi-structured interviews with selected students, which were analysed using thematic analysis to explore cognitive processes and misconceptions related to statistical measures of variation. This method, as described by Braun and Clarke (2006), allows for the systematic identification and analysis of patterns or themes within qualitative data. The approach was particularly suitable because it enabled the researcher to identify both the types of errors students made and the underlying reasons behind those errors, providing a more comprehensive explanatory context for the quantitative findings.

The analysis began with a familiarisation process, during which all interview transcripts were reviewed multiple times to develop a thorough understanding of students' expressions, explanations, and reasoning. At this stage, core concepts and consistent themes were identified, forming the basis for systematic coding.

A combination of descriptive, in vivo, and process coding methods was used to capture the nuances in student responses. Descriptive codes identified explicit content in the data, including terms such as “incorrect definition of variance” and “wrong application of standard deviation formula.” In vivo codes, using the precise language of participants, effectively preserved the authentic voice of students. For example, expressions such as “range means the average of everything” and “standard deviation is just the big number” were coded to highlight specific linguistic misconceptions. Process coding was employed to identify students' cognitive strategies, including "guessing answers," "repeating memorised formulas," and "confusing steps in calculation." The coding strategies enabled systematic documentation of both surface-level responses and underlying reasoning processes.

Following the initial coding, the data were organised into broad thematic categories that covered groups of related codes. Several important themes emerged during this process. One identified theme was Terminological Confusion, which highlighted students' difficulties in understanding and correctly using statistical terminology. Another key theme, Procedural Dependency, showed that students relied heavily on memorised formulas without fully grasping the underlying concepts they represent. A third theme, Conceptual Misalignment, pointed to students' tendency to confuse statistical measures; for example, equating a higher mean with greater variability or using range and standard deviation interchangeably. The final theme, Graphical Misinterpretation, drew attention to students' struggles in connecting graphical data representations, such as histograms and boxplots, with fundamental statistical concepts.

Themes were reviewed and refined to accurately reflect students' thinking and ensure a solid grounding in the data. Selected representative quotes illustrate each theme, thereby deepening the interpretive analysis. Each theme was clearly defined in terms of its importance to statistical learning, and connections were established between the qualitative themes and the quantitative error categories identified through the Modified

Newman Error Analysis Model. Implementing cross-validation enhanced the internal consistency of the study and supported triangulation of findings from various data sources.

4.5 Discussions

4.5.1 Senior High School Students Performance in Measures of Variation

The study's findings indicate that Senior High School pupils face significant difficulties in comprehending fundamental statistical concepts such as mean, range, and standard deviation. The student's overall performance demonstrated moderate to low competency, with average scores sometimes falling below anticipated levels. For example, students attained a mean score of 3.59, somewhat above the criterion of 3.5, although they encountered considerable difficulties with more intricate metrics, such as standard deviation, where the mean score was merely 5.03 out of 18. These findings highlight a deficiency in students' statistical understanding, especially in advanced topics, reflecting earlier research indicating a widespread challenge in comprehending statistical variability. Garfield and Ben-Zvi (2008) identify comparable difficulties, observing that pupils frequently struggle to comprehend concepts necessitating abstract statistical reasoning. This corresponds with the present study's findings, wherein students demonstrated superior performance on fundamental concepts such as mean, while encountering difficulties with variability measurements like standard deviation, signifying a deficiency in their statistical literacy.

The performance diversity, indicated by elevated standard deviation scores among students, corroborates Tishkovskaya and Lancaster's (2012) claim that teaching practices centred on rote memorization may lead to these discrepancies. When children are primarily exposed to formulaic methods devoid of contextual comprehension, they may attain just superficial proficiency. Engelbrecht et al. (2019) assert that prioritizing procedural knowledge over conceptual understanding may impede the cultivation of significant comprehension. The uneven distributions in students' results on multiple measures indicate a larger problem in statistical literacy, as the statistics imply that numerous students faced difficulties with applied concepts like histograms and percentiles. This challenge with application aligns with Sharma's (2017) findings that students frequently lack sufficient contextual information, impeding their capacity to relate statistical concepts to real-world situations.

Qualitative interviews performed for the study indicate that students' difficulties stem not only from rote learning but also from profound conceptual misconceptions. Many students misconstrued fundamental language, which Lovett and Greenhouse (2000) contend can impose an extra cognitive burden on students and impede their general interest in statistical activities. This was seen in the poor scores on questions necessitating computation and interpretation, such as percentile ranks and ogive analysis. The research indicates that these conceptual obstacles may be mitigated by using experiential, application-oriented teaching methods. Chance et al. (2007) assert that active learning environments incorporating practical experiences can enhance comprehension, as students are prompted to relate abstract concepts to concrete scenarios.

The results suggest that meeting students' statistical learning requirements necessitates specialized instructional approaches that consider differing degrees of understanding. Scaffolded training and interactive exercises may increase comprehension of complex subjects such as standard deviation. Pedagogical changes could enhance statistical literacy by developing students' ability to evaluate and apply statistical measurements, thus improving their academic and practical comprehension of statistics.

4.5.2 Types of Error Among Senior High School Students in Measures of Variation in Statistics

The investigation identifies several recurring error types classified by the Modified Newman Error Hierarchy, including reading, comprehension, transformation, process skills, and encoding. The results reveal that comprehension and encoding errors are the most prevalent, underscoring considerable difficulties for students in understanding statistical concepts and articulating their findings effectively. The findings indicate that although students frequently recognize statistical terminology, many encounter difficulties in comprehending and applying these terms contextually a challenge highlighted by Sharma (2017), who noted that secondary students typically struggle with the practical application of statistical concepts when removed from real-life scenarios. The deficiency in contextual knowledge was particularly evident in students' answers to inquiries about advanced subjects such as variance and interquartile range, with comprehension errors occurring in up to 14% of cases.

Interviews done for this study confirm that comprehension difficulties often stem from an excessive dependence on memorization instead of a genuine understanding of statistical processes. Pfannkuch and Wild (2004) assert that rote learning cultivates shallow understanding, which inadequately transfers to new situations, hence constraining pupils' adaptability to unknown information. This was apparent in transformation mistakes, as students encountered difficulties in translating verbal concerns into mathematical models, especially in tasks related to calculating variance and standard deviation. An error rate of 7.5% in these domains indicates that students frequently misapply formulas or neglect essential steps, a situation corroborated by Garfield and Ben-Zvi (2008), who observes that students typically face obstacles in utilizing statistical operations on real-world data due to insufficiently applied learning experiences. These transformation errors not only expose deficiencies in procedural fluency but also highlight the necessity for educational strategies that promote adaptable problem-solving abilities and stress the fundamental principles of statistical methodology. Errors in process skills were notably evident, especially in jobs necessitating multi-step calculations, including those related to grouped data or variance metrics. More than 15% of errors in these domains indicate students' difficulties in preserving precision in arithmetic and adhering to sequential operations challenges that can considerably impede their capacity to resolve statistical problems. Lovett and Greenhouse (2000) noted that weaknesses in procedural abilities frequently arise from insufficient practice with the sequential application of statistical formulas, indicating that students gain from progressively complicated activities that enhance computational fluency. The findings of this study suggest that scaffolding procedural skill development within a controlled learning environment may enhance students' confidence and precision in statistical problem-solving.

Encoding errors, especially in activities necessitating explicit labelling and statistical notation, were identified as the most prevalent error type. About one-third of students experienced challenges with assignments related to cumulative frequency curves and the precise presentation of results, suggesting that although they could execute computations, they found it difficult to articulate their solutions clearly. Tishkovskaya and Lancaster (2012) contend that encoding errors frequently arise when students lack familiarity with standard statistical notations and conventions, potentially obscuring their true comprehension. This study corroborates that assumption, demonstrating that students' final replies often exhibited a lack of structure and clarity, especially in tasks using graphical representations such as histograms and cumulative frequency curves. This corresponds with the findings of Pfannkuch and Wild (2004), who underscored the significance of systematic instruction in statistical notation to improve students' data presentation abilities.

These findings highlight the need for specific instructional interventions emphasizing contextual comprehension, procedural proficiency, and organized expression in statistics education. To rectify the identified flaws, the use of cumulative learning models that enhance vocabulary, formula application, and data presentation skills at many stages could significantly elevate students' statistical proficiency. Chance et al. (2007) suggest that real-world examples and problem-based learning methods may improve students' capacity to relate statistical ideas to practical applications, hence decreasing the incidence of comprehension and transformation errors. Incorporating tasks that emphasize statistical notation and labelling enables students to cultivate the precision required for clear data reporting, enhancing their general capacity to communicate statistical information clearly and effectively.

4.5.3 Causes of Errors Among Senior High Students in Measures of Variation

The examination of fundamental causes for errors among Senior High School students in statistical evaluation tasks uncovers interrelated variables that hinder students' comprehension and application of statistical ideas. Principal concerns encompass restricted language comprehension, overdependence on rote memory, misinterpretation of statistical symbols, inadequate procedural fluency, and challenges in comprehending graphical data. Each of these factors substantially influences students' capacity to effectively perform statistical procedures and convey their findings, ultimately resulting in persistent inaccuracies in their responses. A significant source of inaccuracy is the limited understanding of statistical language since students frequently misinterpret essential concepts such as "mean" and "standard deviation." Garfield and Ahlgren

(1988) discovered that vocabulary deficiencies can hinder comprehension and problem-solving, especially since students find it challenging to associate language with relevant statistical procedures. The interviews conducted in this study corroborate these findings, indicating that students often misinterpret fundamental terminology, which might hinder the entire problem-solving process. For instance, one student misconstrued "range" as simply organizing numbers, instead of determining the difference between the maximum and minimum values, a misunderstanding that corresponds with Sharma's (2017) findings. Mitigating these lexical issues through context-rich learning may augment students' capacity to appropriately grasp questions and enhance their overall performance. Rote memory significantly contributes to the frequency of errors, as numerous pupils depend on memorized formulas without comprehending their conceptual underpinnings. This method restricts pupils' adaptability, hindering their ability to apply formulas correctly in many settings. Garfield and Ben-Zvi (2008) emphasize the shortcomings of memorization-centric learning in statistical education since it hinders students from comprehending the foundational principles of statistical methodology. This was obvious in students' difficulties in applying the formula for quartile deviation to novel datasets, illustrating that inflexible memorization obstructs adaptive reasoning. These findings corroborate the assertions of Pfannkuch and Wild (2004), who contended that prioritizing conceptual comprehension over rote memorization is crucial for substantive engagement with statistics.

A recurring challenge for students is their misinterpretation of statistical symbols, resulting in frequent errors in formula application. Students frequently conflate essential symbols, undermining their capacity to accurately convert verbal problems into mathematical equations. Lovett and Greenhouse (2000) indicate that insufficient expertise in statistical notation might significantly hinder the problem-solving process. This study's interviews revealed situations where students substituted valid symbols with wrong ones, leading to erroneous calculations. Tishkovskaya and Lancaster (2012) urge for enhanced instructional focus on symbols and notation to bridge this gap, contending that proficiency with symbols is essential to reduce transformation errors and promote precise calculation. A significant difficulty is the deficiency in procedural fluency among students, especially in multi-step statistical computations. Procedural fluency is crucial for tasks involving sequential operations, such as calculating variance or standard deviation; nonetheless, numerous students demonstrated recurrent errors in these domains. Garfield and Ben-Zvi (2008) underscore the significance of procedural proficiency in statistical problem-solving, observing that shortcomings in this domain frequently result in computational errors. Students in this study often committed fundamental mathematical errors or omitted critical procedures, so compromising the validity of their conclusions. The results indicate that scaffolded practice, coupled with feedback on multi-step computations, may enhance students' procedural accuracy and confidence in statistical tasks.

Ultimately, students exhibited challenges in interpreting graphical data, a crucial ability in statistics for tasks such as reading cumulative frequency curves and assessing histograms. A considerable number of pupils made mistakes in labelling and interpreting graphs, suggesting insufficient experience with graph-related tasks. Wild and Pfannkuch (1999) noted that students frequently have difficulties in interpreting graphical data owing to a lack of focus on visual data analysis in conventional statistics instruction. Tishkovskaya and Lancaster (2012) advocate for an increased emphasis on graph-based learning tools to enhance students' visual analytical abilities, a position our study corroborates. Integrating tasks that foster proficiency in graphical analysis may improve students' ability to analyze visual data, thereby minimizing encoding errors. The research emphasizes the necessity for holistic educational strategies that promote lexical comprehension, conceptual understanding, symbol identification, procedural proficiency, and graphical analysis. Implementing scaffolded vocabulary activities, as advocated by Garfield and Ahlgren (1988), may enhance students' foundational comprehension, allowing them to interact more effectively with statistical problems. Furthermore, transcending rote memory to cultivate conceptual comprehension can enable students to utilize statistical knowledge more flexibly, hence resolving challenges related to transformation and procedural competencies. Integrating consistent practice with statistical notation and graphical activities enables instructors to improve students' capacity to convey statistical findings accurately, hence promoting clarity and precision in their statistical reasoning.

CHAPTER FIVE

SUMMARY, CONCLUSION AND RECOMMENDATIONS

5.0 Overview

Chapter Five concluded the study on the errors and misconceptions that Senior High School students encounter when understanding measures of variation in statistics. It provided a summary of the research objectives, questions, and methodology, along with a clear synthesis of the findings. The chapter began with an overview of the study, followed by a concise summary of the results. It concluded with targeted recommendations for educators and policymakers, as well as proposed areas for future research to address the identified gaps.

5.1 Summary of the Study

This investigation examined the continuing errors and misunderstandings that Senior High School students exhibit in comprehending and utilizing statistical measures of variation, including range, variance, and standard deviation. Understanding the importance of statistical reasoning in various contexts, the study focused on the pressing challenge of students struggling to comprehend these concepts, which negatively impacts their overall statistical proficiency and critical thinking abilities. The issue is exacerbated by a narrow range of teaching methods that frequently emphasize procedural knowledge at the expense of conceptual understanding, resulting in students being inadequately prepared to tackle intricate statistical reasoning challenges. The investigation sought to pinpoint the particular mistakes and misunderstandings exhibited by students when engaging with measures of variation, comprehend their underlying causes, and suggest practical solutions to tackle these issues. The primary goals of the investigation involved evaluating students' performance regarding statistical measures of variation, pinpointing prevalent errors and misconceptions, and examining the fundamental factors that contribute to these challenges. A mixed-methods research design was implemented to accomplish these objectives. Structured assessments were used to gather quantitative data, evaluate students' proficiency and identify common errors. In addition, qualitative data were obtained through interviews and document analyses to offer a more nuanced understanding of the cognitive and pedagogical factors affecting their comprehension. This methodological triangulation provided a thorough analysis of the research questions, utilizing both numerical trends and interpretive depth. To ensure the validity of the findings, the study utilized strategies including content validation of the assessment tools by experts in the field and piloting the instruments in a comparable educational setting to enhance their reliability. An inter-coder agreement was established for qualitative data to enhance the trustworthiness of the thematic analysis. The study incorporated member checking, enabling participants to assess interpretations of their contributions and employed peer debriefing to ensure consistency in data analysis and interpretation. The reliability of the study was enhanced through the use of standardized procedures for data collection and analysis, which included the consistent administration of assessments and coding protocols. The integration of various data sources and methodologies strengthened the findings by employing triangulation, reducing biases and providing a comprehensive understanding of the research issue. This study conducted a thorough analysis of students' difficulties with statistics, adding valuable insights to the ongoing conversation about mathematics education and presenting data-driven suggestions to enhance the teaching and understanding of statistical principles.

5.2 Summary of Findings

The study uncovered several important insights into senior high school students' comprehension and use of statistical measures of variation. The analysis revealed widespread errors and misunderstandings that obstruct students' capacity to correctly compute and understand concepts like range, variance, and standard deviation. The identified errors were associated with deficiencies in cognitive processes as well as instructional methods.

1. First, students often mixed up measures of variation with measures of central tendency, frequently misunderstanding variability as equivalent to the mean. This merging resulted in erroneous conclusions regarding data dispersion. Furthermore, the improper application of fundamental statistical formulas, like the calculation of standard deviation, indicated a dependence on memorization rather than a deep understanding of the concepts involved.
2. Secondly, there were widespread misunderstandings regarding the unique functions of range and standard deviation, as students frequently interchanged them without discernment. The insufficient grasp of standard deviation as a sophisticated indicator of variability further underscored existing conceptual deficiencies.
3. The results further highlighted the educational challenges that are contributing to these issues. Instructional methods that prioritize procedural fluency do not adequately allow students to interact meaningfully with real-world data and the contextual applications of variability measures. The lack of interactive and exploratory learning environments intensified students' conceptual misunderstandings.

4. Additionally, the investigation revealed cognitive obstacles, including challenges in abstract reasoning, which rendered concepts such as variability and distribution less intuitive for learners. The insufficient scaffolding of statistical concepts within the curriculum and teaching practices exacerbated these barriers.

5.3 Conclusions

Based on the study findings, the following conclusions were drawn:

1. Students exhibited significant misunderstandings of statistical measures of variation, often confusing variability with central tendency and misinterpreting key concepts such as range and standard deviation.
2. There was a heavy reliance on rote memorization of formulas without a deep understanding of their applications, leading to frequent computational errors and misinterpretations.
3. Teaching methods predominantly focused on procedural fluency rather than fostering conceptual reasoning. This lack of emphasis on exploratory and real-world data applications hindered students' deeper engagement with statistical concepts.
4. Students faced challenges in abstract reasoning, making it difficult to internalize non-intuitive concepts like distribution and variability.

5.4 Implication of the study

The findings of this study have significant theoretical implications, particularly regarding how students develop, interpret, and apply statistical knowledge in educational settings. Based on Constructivist Theory, the APOS framework, Dual Process Theory, and Vygotsky's Zone of Proximal Development (ZPD), this research offers valuable insights into the cognitive and pedagogical processes involved in students’ understanding of statistical variation.

Firstly, the study supports the Constructivist view that learners actively develop knowledge through personal experiences and existing mental frameworks. The identified misconceptions—such as confusing the range with the mean or misusing the standard deviation formula—show that students’ prior knowledge, if flawed or superficial, can hinder meaningful learning. These findings reinforce Piaget’s assertion that conceptual change requires cognitive conflict, indicating that targeted teaching methods must directly challenge and reshape students' incorrect mental models rather than simply presenting correct procedures.

Secondly, the results strongly support and expand on the APOS Theory (Action, Process, Object, Schema) by highlighting the stages at which many students are developmentally stalled. A large portion of participants seemed to operate at the "Action" level—performing calculations mechanically but lacking an understanding of the underlying concepts. The lack of progression to the "Process" or "Object" stages—where learners internalise actions and see them as unified cognitive wholes—exposes a pedagogical gap in helping students move from procedural skills to conceptual understanding. This suggests that teaching strategies should focus not only on procedures but also on guiding students to transform these procedures into coherent mathematical schemas.

The findings also confirm the importance of Dual Process Theory (Kahneman & Frederick, 2002) in explaining why intuitive yet incorrect statistical reasoning persists. Many students relied on System 1 thinking—using surface-level heuristics and intuitive judgments, such as assuming that a higher mean automatically indicates greater variability. These intuitive responses often conflicted with formal statistical reasoning

(System 2), which requires deliberate and analytical processing. The study, therefore, highlights the need for instructional strategies that promote analytical rationale, such as through problem-based learning, data interpretation exercises, and reflective questioning.

Furthermore, the study's results support Vygotsky's Zone of Proximal Development, which emphasises the importance of guided instruction in bridging the gap between what learners can do independently and what they can accomplish with support. The observed gains in understanding among students who were scaffolded during the qualitative interviews demonstrate how structured support can enhance cognitive performance. This underlines the vital role of scaffolding in helping students internalise complex statistical concepts, such as variability, distribution, and standard deviation, particularly when introduced gradually and in meaningful, contextual ways.

Together, these findings indicate that models of learning mathematics and statistics should not only focus on knowledge acquisition but also explicitly consider how students' reasoning processes are influenced by instruction, prior knowledge, and stages of cognitive development. Future instructional models must therefore be based on theories that highlight conceptual building, developmental growth, and scaffolding, ensuring that students do not just "do" statistics but genuinely understand the "why" behind it.

5.5 Recommendations

Based on the conclusions drawn from the study, the following recommendations are proposed, both generally and specifically for Winneba SHS:

1. Organize targeted workshops for students that emphasize the differences between measures of central tendency and measures of variability, using examples relevant to their local environment.

2. Introduce project-based learning activities in which students collect and analyze real data sets, applying statistical formulas in meaningful contexts.
3. Equip classrooms with statistical software and tools to facilitate interactive lessons, enhancing students' ability to explore data variability dynamically.
4. Provide additional remedial classes for students who struggle with abstract statistical concepts, ensuring they have ample time for hands-on practice and personalized support.
5. Collaborate with educational stakeholders to pilot a restructured statistics curriculum, aligning teaching methods with best practices derived from research findings.

5.6 Suggestions for Future Studies

Based on the findings and limitations of this study, the following suggestions are proposed for future research:

Future studies could explore the long-term effects of specific instructional interventions, such as project-based learning and the integration of statistical software, on students’ understanding of measures of variation. A longitudinal design would help assess whether conceptual improvements endure over time and result in better applications in higher-level statistical studies or real-world situations.

Additionally, conducting a comparative study on errors and misconceptions regarding measures of variation in different educational settings—such as rural versus urban schools or public versus private institutions—would provide deeper insights into how contextual factors (e.g., resources, teacher training, and curriculum design) influence statistical comprehension. This information could guide the development of targeted, context-specific strategies to enhance statistical education.

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APPENDICES

Appendix A: Test on Measures of Variation

Dear Student

The purpose of this test is to gather data on errors and misconceptions you have in doing tasks in measures of variation.

It is not a test whose results will be used for your continuous assessment, but your **NUMBER** will be required to identify those of you who will require support.
It will take you about 1 hour to complete. Your thoughtful responses will be greatly appreciated; so, please, answer each question to the best of your knowledge.

I assure you that any information shared with me will be protected.

Thank you.

INSTRUCTIONS
Answer all questions on the paper by completing the statements, ticking the appropriate, or showing working and giving reasons.

Part A: Personal Data

a. Student's Code: ________________________________

b. SHS Programme of Study: ________________________________.

c. SHS Form: ______________________

d. Gender: Male [ ] Female [ ] (*Tick one*)

e. Age: ________ years

Part B: Test

1. Which of the following are not methods under measures of dispersion?
   a. Standard deviation
   b. Mean
   c. Range
   d. All of the above
2. Which of the following are characteristics of a good measure of dispersion?
   a. It should be easy to calculate
   b. It should be based on all the observations within a series
   c. It should not be affected by the fluctuations within the sampling
   d. All the above

3. If all the observations within a series are multiplied by five, then……………….
    a. The new standard deviation would be decreased by five
    b. The new standard deviation would be increased by five
    c. The new standard deviation would be half of the previous standard deviation
    d. The new standard deviation would be multiplied by five
4. While calculating the standard deviation, the deviations are only taken from……………..
    a. The mode value of a series
    b. The median value of a series
    c. The quartile value of a series
    d. The mean value of a series
5. The numerical value of a standard deviation can never be …………………
    a. Negative
    b. Zero
    c. Larger than the variance
    d. None of the above
6. The average of squared deviation from the arithmetic mean is known as ………….
    a. Quartile deviation
    b. Standard deviation
    c. Variance
    d. None of the above
    e.
7. Which of the following cannot be calculated for open-ended distributions?
    a. Standard deviation
    b. Mean deviation
    c. Range
    d. None of the above
8. The average daily wage of 100 workers in a shipyard was 200, with a standard deviation of 40. Now if each worker gets an increment of 20% in their wages, how will it affect the mean wage?
    a. The mean wage will remain unchanged

   b. The mean wage will increased by 20%
   c. The mean wage will be 240
   d. Both b and c

9. Which of the following measures of dispersion can attain a negative value?
   a. Mean deviation
   b. Range
   c. Standard deviation
   d. None of the above

10. An example of application of range in a real-world scenario would be………………
    a. Fluctuation in shares price
    b. Weather forecast
    c. Quality control
    d. All of the above

11. Indicate which of the following statements is/are true
    I. The range is a measure of dispersion and its calculation takes into account every unit in the data
    II. The mean is always unique, that is, a set of data has one and only one mean
    III. When we plot class frequencies (vertical axis) against class marks, we obtain a graph known as ogive.
        a. I only
        b. II only
        c. III only
        d. I and II only

12. The statistics test marks of Winneba senior high school are as follows:

    $52, 45, 25, 75, 63, 86, 72, 85, 55, 65, 70, 82, 90, 48,$
    $68, 86, 65, 64, 78, 75, 32, 42.$

    Find the range

13. Find the variance and standard deviation of the data set 2, 4, 7, 8, 9

14. Find an estimate of the variance and standard deviation of the following data for the marks obtained in the test by 26 students.

Marks	$10-14$	$15-19$	$20-24$	$25-29$	$30-34$
Frequency (f)	3	7	10	5	1

15. If the range and the smallest value of a set of data are 36.8 and 13.4 respectively, then find the largest value.
16. A teacher asked the students to complete 60 pages of a record note book. Eight students have completed only 32, 35, 37, 30, 33, 36, 35, and 37 pages. Find the standard deviation of the pages yet to be completed by them.
17. If the standard deviation of a data is 3.6 and each value of the data is divided by 3, then find new variance and new standard deviation
18. For a group of 100 candidates the mean and standard deviation of their marks were found to be 60 and 15 respectively. Later on it was found that the scores 45 and 72 were wrongly entered as 40 and 27. Find the new mean and standard deviation.

19. Students in Form 2A and Form 2B took a test (marked or scored out of 10) in core mathematics at the end of the term. The table below shows the result obtained by the Form 2A students in the test. Use this information to answer questions below

The five number-summary	Form 2A	Form 2B
Minimum	0	1
First quartile	3	4
Median	5	4
The third quartile	6	5
The maximum	8	10
Mean	5	4.5
Standard deviation	1.96	1.46

Looking at the values for the five-number summary, for Form 2A and Form 2B in the table above,

a) How would you compare the performance of the two classes on the test?

b) Which class did better and why?

20. The table gives the distribution of marks obtained by some students in examination.

Marks	0-9	10-19	20-29	30-39	40-49	50-59	60-69	70-79	80-89	90-99
Freq.	4	6	10	10	13	17	20	15	4	1

a) Draw a cumulative frequency curve of the distribution.

b) From your curve, estimate;
   - i) The 80th percentile,
   - ii) The median
   - iii) Semi inter-quartile range
   - iv) The inter-quartile range

c) Draw a histogram

d) Using a histogram, estimate the mode

Appendix B: Marking scheme for the Test

<table>
<tr><th>ANSWER/SOLUTION</th><th>MARK/DESCRIPTION</th><th>VARIABLE LABEL</th></tr>
<tr><td rowspan="11">1. B. Mean
2. D. All the above
3. D. The new standard deviation would be multiplied by five
4. D. The mean value of the series
5. A. Negative
6. C. Variance
7. B. Mean deviation
8. D. Both C and D
9. D. None of the above
10. D. All the above
11. C. III only</td><td>A1 for correct answer</td><td>Q1</td></tr>
<tr><td>A1 for correct answer</td><td>Q2</td></tr>
<tr><td>A1 for correct answer</td><td>Q3</td></tr>
<tr><td>A1 for correct answer</td><td>Q4</td></tr>
<tr><td>A1 for correct answer</td><td>Q5</td></tr>
<tr><td>A1 for correct answer</td><td>Q6</td></tr>
<tr><td>A1 for correct answer</td><td>Q7</td></tr>
<tr><td>A1 for correct answer</td><td>Q8</td></tr>
<tr><td>A1 for correct answer</td><td>Q9</td></tr>
<tr><td>A1 for correct answer</td><td>Q10</td></tr>
<tr><td>A1 for correct answer</td><td>Q11</td></tr>
<tr><td>12. Range= 90-25 = 65</td><td>A1 for correct range</td><td>Q12M1</td></tr>
<tr><td></td><td></td><td>Q12A2</td></tr>
<tr><td>13.</td><td></td><td>Q13M1</td></tr>
<tr><td rowspan="5"><table>
<tr><th>x</th><th>x^2</th></tr>
<tr><td>2</td><td>4</td></tr>
<tr><td>4</td><td>16</td></tr>
<tr><td>7</td><td>49</td></tr>
<tr><td>8</td><td>64</td></tr>
<tr><td>9</td><td>81</td></tr>
<tr><td>$\sum x = 30$</td><td>$\sum x^2 = 214$</td></tr>
</table></td><td>M1 for any correct input of (x) square</td><td>Q13A2</td></tr>
<tr><td>A1 for correct summation of (x)</td><td>Q13M3</td></tr>
<tr><td>A1 for correct summation of square od (x) =214</td><td>Q13M4</td></tr>
<tr><td></td><td>Q13M5</td></tr>
<tr><td>M1 for writing the correct formula for variance</td><td>Q13A6</td></tr>
<tr><td rowspan="2">$Variance = \frac{\sum x^2}{nx} - \left(\frac{\sum x}{x}\right)^2$
$= \frac{214}{5} - \left(\frac{30}{5}\right)^2$</td><td>M1 for correct substitution</td><td>Q13M7</td></tr>
<tr><td>M1 for correct simplification</td><td>Q13M8</td></tr>
</table>

<table>
<tr><td rowspan="3">$= 6.8$
$$std\ deviation = \sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2}$$
$$= \sqrt{\frac{214}{5} - \left(\frac{30}{5}\right)^2}$$
$= \sqrt{42.8 - 36}$
$= \sqrt{6.8}$
$= 2.61$</td><td></td><td></td></tr>
<tr><td>A1 for correct for correct answer =6.8</td><td>Q13M9</td></tr>
<tr><td></td><td>Q13M10</td></tr>
<tr><td rowspan="11">14.
<table>
<tr><th>Marks</th><th>Midpoint (x)</th><th>Number Of students (f)</th><th>fx</th><th>fx^2</th></tr>
<tr><td>10-14</td><td>12</td><td>3</td><td>36</td><td>432</td></tr>
<tr><td>15-19</td><td>17</td><td>7</td><td>119</td><td>2023</td></tr>
<tr><td>20-24</td><td>22</td><td>10</td><td>220</td><td>4840</td></tr>
<tr><td>25-29</td><td>27</td><td>5</td><td>135</td><td>3645</td></tr>
<tr><td>30-34</td><td>32</td><td>1</td><td>32</td><td>1024</td></tr>
<tr><td>Totals</td><td></td><td>$\sum f = 26$</td><td>$\sum fx = 542$</td><td>$\sum fx^2 = 11964$</td></tr>
</table>
ii)
$$variance = \frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2 \Rightarrow \frac{11964}{26} - \left(\frac{542}{26}\right)^2$$
$= 460.15 - 434.56 = 25.59$
$$\text{ii) } SD = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$
$$= \sqrt{\frac{11964}{26} - \left(\frac{542}{26}\right)^2}$$
$= \sqrt{460.15 - 434.56}$
$= \sqrt{25.59}$
$= 5.01 \approx 5$</td><td>A1 for writing correct formula for Std. deviation</td><td>Q14M1</td></tr>
<tr><td></td><td>Q14M2</td></tr>
<tr><td></td><td>Q14M3</td></tr>
<tr><td>A1 for correct substitution</td><td>Q14A4</td></tr>
<tr><td></td><td>Q14A5</td></tr>
<tr><td>M1 for correct simplification</td><td>Q14M6</td></tr>
<tr><td></td><td>Q14M7</td></tr>
<tr><td>A1 correct answer =2.61</td><td>Q14A8</td></tr>
<tr><td></td><td>Q14M9</td></tr>
<tr><td>M1 for any correct input of (x)</td><td>Q14M10</td></tr>
<tr><td>M1 for any correct input of frequency times (x)</td><td>Q14A11</td></tr>
<tr><td>15.
Range =36.4
Smallest value=13.4</td><td>M1 for any correct input of</td><td>Q15M1</td></tr>
</table>

<table>
<tr><td rowspan="2">Range= Largest value-smallest value
36.8= Largest value=36.8+13.4=50.2
∴ The largest value=50.2</td><td>frequency times (x)square</td><td></td></tr>
<tr><td>A1 for correct summation of frequency times (x) =542</td><td>Q15A2</td></tr>
<tr><td rowspan="8">16.
Pages to be completed are

28, 25, 23, 30, 27, 24, 25, 23

$Assumed\ mean = 25$
$n = 8$
<table><tr><th>x</th><th>$d = x - A$</th><th>d^2</th></tr><tr><td>23</td><td>-2</td><td>4</td></tr><tr><td>23</td><td>-2</td><td>4</td></tr><tr><td>24</td><td>-1</td><td>1</td></tr><tr><td>25</td><td>0</td><td>0</td></tr><tr><td>25</td><td>0</td><td>0</td></tr><tr><td>27</td><td>2</td><td>4</td></tr><tr><td>28</td><td>3</td><td>9</td></tr><tr><td>30</td><td>5</td><td>25</td></tr></table></td><td>A1 for correct summation of frequency times (x) square= 11964</td><td>Q16M1</td></tr>
<tr><td>M1 for writing the correct formula</td><td>Q16M2</td></tr>
<tr><td>M1 for correct substitution</td><td>Q16M3</td></tr>
<tr><td>M1 for correct simplification</td><td>Q16A4</td></tr>
<tr><td>A1 for correct answer =25.59</td><td>Q16A5</td></tr>
<tr><td></td><td>Q16M6</td></tr>
<tr><td></td><td>Q16M7</td></tr>
<tr><td></td><td>Q16M8</td></tr>
</table>

<table>
<tr>
<td>
<table>
<tr><td></td><td>$\sum d = 5$</td><td>$\sum d^2$
$= 47$</td></tr>
</table>

$$std\ deviation = \sqrt{\frac{\sum d^2}{n} - \left(\frac{\sum d}{n}\right)^2}$$

$$= \sqrt{\frac{47}{8} - \left(\frac{5}{8}\right)^2}$$

$$= \sqrt{\frac{47}{8} - \frac{25}{64}}$$

$$= \sqrt{\frac{351}{64}}$$

$= 2.3$

$std\ deviation\ of\ the\ page\ to\ be\ completely = 2.3$
</td>
<td>M1 for correct substitution</td>
<td>Q16A9</td>
</tr>
<tr>
<td rowspan="2">
17.

$\sigma = 3.6$

When each data is divided by 3

$$New\ std\ deviation = \frac{3.6}{3} = 1.2$$

$$New\ variance = (1.2)^2 = 1.44$$

$$\therefore\ New\ variance = 1.44$$

$$and\ new\ std.deviation = 1.2$$
</td>
<td></td>
<td>Q17M1</td>
</tr>
<tr>
<td>M1 for correct simplification</td>
<td>Q17A2</td>
</tr>
<tr>
<td>
18

$$n = 100\,,\quad \bar{x} = 60,\quad \sigma = 15$$

$$corect\ values\ 45\ and\ 72\ wrong\ values\ 40\ and\ 27$$

$$\bar{x} = \frac{\sum x}{n} \Rightarrow \sum x = n\bar{x}$$
</td>
<td></td>
<td>Q18A1</td>
</tr>
</table>

$\therefore wrong\ total = 100 \times 60 = 6000$ $correct\ total =$ $wrong\ total - wrong\ values$ $+ correct\ values$ $= 6000 - 40 - 27 + 45 + 72 \Rightarrow 6050$ $new\ mean\ \ \bar{x} = \frac{correct\ total}{n}$ $\bar{x} = \frac{6050}{100} = 60.5$ $std.deviation = \sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2}$ $15 = \sqrt{\frac{\sum x^2}{100} - (60)^2}$ $(15)^2 = \sqrt{\frac{\sum x^2}{100} - (60)^2}$ $\frac{\sum x^2}{100} = 15^2 + (60)^2$ $\frac{\sum x^2}{100} = 3825$ $\sum x^2 = 382500$ $\sum x^2 = 382500 - (40)^2 - (27)^2 + (45)^2$ $+ (72)^2$ $\sum x^2 = 382500 - 2329 + 7209$ $= 387380$ $new\ \sigma = \sqrt{\frac{correct\sum x^2}{n} - \left(\frac{correct\sum x}{n}\right)^2}$	A1 for correct answer =5.01	Q18A2
		Q18A3
		Q18M4
		Q18M5
	M1 for substitution	Q18M6

<table>
<tr>
<td rowspan="2">

$$= \sqrt{\frac{387380}{100} - (60)^2}$$

$$= \sqrt{3873.8 - 3660.25}$$

$$= \sqrt{3873.8 - 3660.25}$$

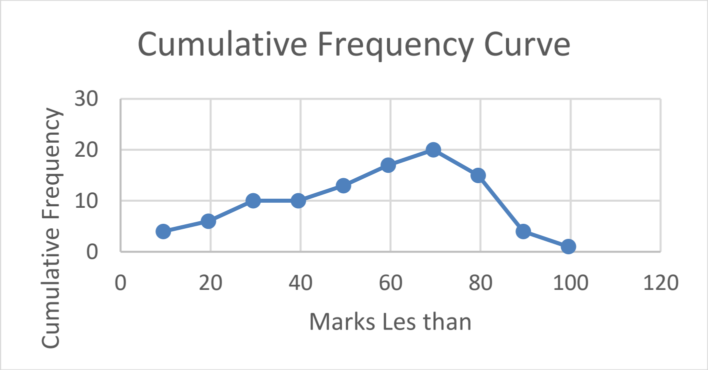
$$= \sqrt{213.55}$$

$$= 14.61$$

$$new\ mean = 60.5\ and\ new\ std.deviation$$

$$= 14.61$$

</td>
<td>A1 for correct answer =50.2</td>
<td>Q18M7</td>
</tr>
<tr>
<td></td>
<td>Q18A8</td>
</tr>
<tr>
<td rowspan="2">19.)
A. From the results in the five-summary, the form 2A had higher values inception of minimum value and the form 2B had a lower value.
B. Therefore, form 2A perform better than the form 2B.</td>
<td></td>
<td>Q19A1</td>
</tr>
<tr>
<td></td>
<td>Q19A2</td>
</tr>
<tr>
<td rowspan="5">20.
<table>
<tr><th>Marks</th><th>f</th><th>$marks\ less\ than$</th><th>CF</th></tr>
<tr><td>$0 - 9$</td><td>4</td><td>9.5</td><td>4</td></tr>
<tr><td>$10 - 19$</td><td>6</td><td>19.5</td><td>10</td></tr>
<tr><td>$20 - 29$</td><td>10</td><td>29.5</td><td>20</td></tr>
<tr><td>$30 - 39$</td><td>10</td><td>39.5</td><td>30</td></tr>
<tr><td>$40 - 49$</td><td>13</td><td>49.5</td><td>43</td></tr>
<tr><td>$50 - 59$</td><td>17</td><td>59.5</td><td>60</td></tr>
<tr><td>$60 - 69$</td><td>20</td><td>69.5</td><td>80</td></tr>
<tr><td>$70 - 79$</td><td>15</td><td>79.5</td><td>95</td></tr>
<tr><td>$80 - 89$</td><td>4</td><td>89.5</td><td>99</td></tr>
<tr><td>$90 - 99$</td><td>1</td><td>99.5</td><td>100</td></tr>
</table>
A) COMMULATIVE FREQUENCY TO BE DRAWN</td>
<td>M1 for any 2 correct deviation from an assumed mean</td>
<td>Q20M1</td>
</tr>
<tr>
<td>M1 for any 2 correct of the square of deviation</td>
<td>Q20M2</td>
</tr>
<tr>
<td>M1 for correct summation of deviation =5</td>
<td>Q20M3</td>
</tr>
<tr>
<td>M1 for correct summation of square of deviation =47</td>
<td>Q20A4</td>
</tr>
<tr>
<td>M1 for writing the formula correctly</td>
<td>Q20M5</td>
</tr>
</table>



i) $80^{th}\ percentile = \frac{80}{100} \times \sum f$

$$= \frac{80}{100} \times 100 = 80$$

$= 80\ on\ the\ commulated\ frequency\ axis. From\ the\ gr$
$, the\ 80^{th} percentile\ is\ 69.5$

ii. $median = \frac{1}{2} \times \sum f$

$$= \frac{1}{2} \times 100 = 50$$

50 on the cumulative frequency axis from the graph the median is 53.5

iii. $upper\ quartile\ Q_3 = \frac{3}{4} \times \sum f$

$$= \frac{3}{4} \times 100 = 75$$

$$from\ the\ graph\ Q_3 = 66.5$$

$$Lower\ quartile\ Q_3 = \frac{1}{4} \times \sum f$$

$$= \frac{1}{4} \times 100 = 25$$

$$from\ the\ graph, \quad Q_1 = 34.5$$

$$Inter - quartile\ range = Q_3 - Q_1$$

$$66.5 - 34.5 = 32$$

iv. $Semi - interquartile$

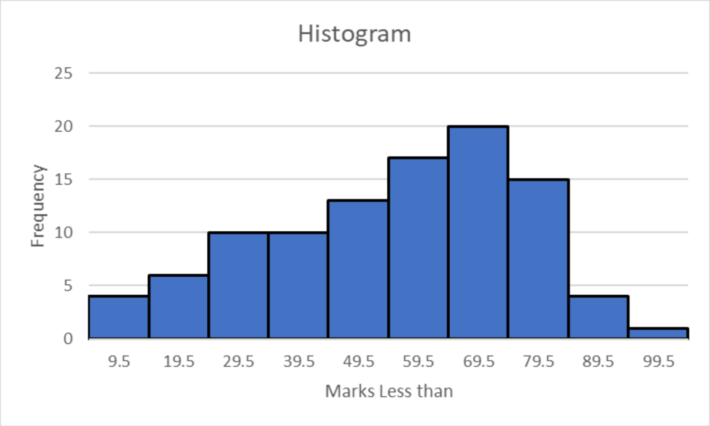
$$= \frac{Q_3 - Q_1}{2}$$

$$= \frac{66.5 - 34.34.5}{2}$$

$$= 16$$

c) HISTOGRAM TO BE DRAWN

M1 for correct substitution	Q20A6
M1 for correct simplification	Q20M7
A1 for correct answer =2.3	Q20A8
M1 for finding correct new std. deviation = 1.2	Q20M9
A1 for correct new variance =1.44	Q20A10
A1 for correct "wrong total" = 6000	Q20M11
A1 for correct "correct total" = 6050	Q20M12
A1 for mean = 60.5	Q20M13
M1 for simplification	Q20M14
A1 for correct summation of (x) = 387380	Q20A15

 d) from the graph the modal mark is 69	M1 for correct substitution	Q20M16
	M1 for correct simplification	Q20M17
	A1 for correct std. deviation = 14	Q20A18
	M1 for correct upper quartile on the cumulative frequency axis =75	
	A1 for finding correct upper percentile using the curve = 66.5	
	M1 for correct lower quartile on cumulative frequency axis =25	
	A1 for finding correct lower quartile using the curve = 66.5	
	M1 simplification of interquartile range	

	1 for correct answer =32	
	M1 for correct substitution of upper and lower quartile into the semi-interquartile range formula	
	A1 for answer = 16	
	M3 for drawing the histogram correctly	
	M1 for using the histogram to determine the mode	
	M1 for correct modal mark = 69	

Appendix C: Introductory Letter

UNIVERSITY OF EDUCATION, WINNEBA
FACULTY OF SCIENCE EDUCATION
DEPARTMENT OF MATHEMATICS EDUCATION
P. O. Box 25, Winneba, Ghana [illegible]
+233 (020) 2041076

Ref: MATHS/HOD/2024.01.10

10th January, 2024

To Whom it may concern

Dear Sir,

LETTER OF INTRODUCTION

I write to introduce Miss Gifty Dzifa Amewu, an MPhil student of the Department of Mathematics Education in the University of Education, Winneba. Miss Amewu is conducting her research work leading to the award of Master of Philosophy in Mathematics Education on the topic **"Senior High School Students' Errors and Misconceptions in determining measures of variation in statistics"**.

She is therefore seeking for your permission to conduct test and interview to collect data in your institution. We kindly plead on your indulgence to assist her in her research.

Yours faithfully,

Dr. Sylvester Ali Frimpong

(Ag. HOD – Mathematics Education)

E-Mail: mali@uew.edu.gh

Phone: 020-831 5368

Appendix D: Results of Further analysis of metrics of topics in variation

The mean score of 4.31 for the Mean metric, which is 33% of the maximum possible score of 13, reflects those students struggled to demonstrate a robust understanding of this concept. When compared to the expected score of 6.5, the below-average performance underscores gaps in students' comprehension of statistical means. Additionally, the high standard deviation of 3.01 points to considerable variability, indicating that while some students demonstrated a reasonable grasp of the material, a significant portion of the cohort exhibited a weaker understanding. This broad spread in performance suggests the need for differentiated instructional interventions to address the varying levels of comprehension. Some students demonstrated a better grasp, while others struggled. This variability underscores the need for differentiated instruction to cater to differing learning needs.

The slight positive skewness of 0.33 indicates a slight clustering of scores at the lower end, indicating more students scored lower. However, kurtosis -0.61 indicates fewer extreme scores exceptionally high or low. A histogram showing the distribution of students' performance on the achievement test on the mean is presented in Figure 4.2.

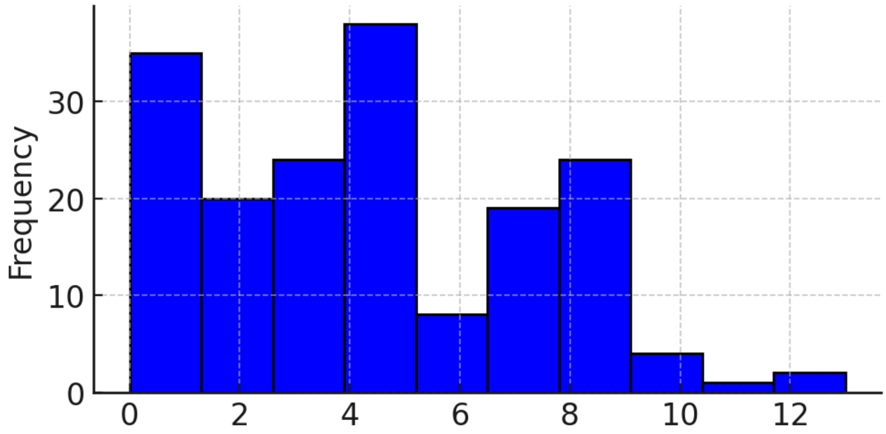


Figure 4. 8: Histogram of Mean

The mean score of 5.03 for the Standard Deviation metric, amounting to only 28% of the maximum possible score of 18, highlights those students faced considerable difficulties with this concept. The expected score for this metric, 9, serves as a reference point, against which the observed performance falls significantly short. The high standard deviation of 4.63 reveals substantial variability in students' responses, confirming the existence of widely differing levels of understanding. The positive skewness of 0.86 further illustrates that a majority of students scored below the average, with a few outliers performing notably better, as reflected in Figure 4.3. The high standard deviation of 4.63 reveals significant variability among students, highlighting diverse levels of understanding. The positive skewness of 0.86 confirms this, showing a skewed distribution with more students scoring lower. Furthermore, kurtosis -0.33 indicates fewer extreme scores, suggesting fewer students excelled or struggled severely. This flat distribution implies that most students clustered around the means, with fewer outstanding.

A histogram showing the distribution of students' performance on the achievement test on the standard deviation is presented in Figure 4.3.

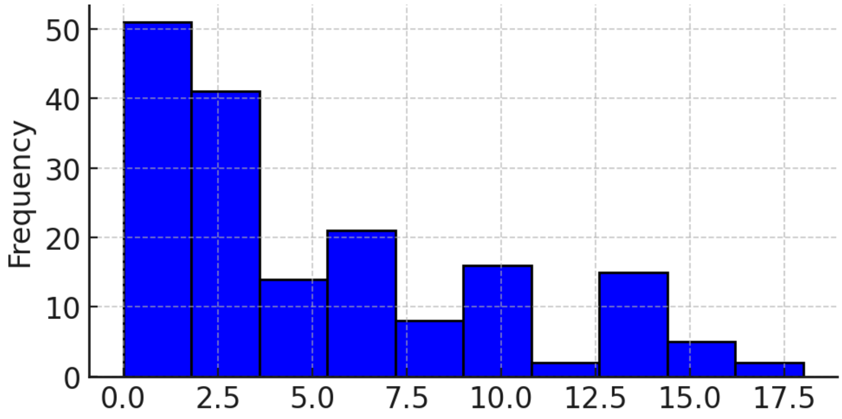


Figure 4. 9: Histogram of Standard Deviation

Only 37 students responded to the Percentile question, revealing a mean score of 1.16 out of a possible 3, which is below the expected average of 1.5. This suggests that the majority of students struggled with this concept, as indicated by the below-average performance. The standard deviation of 1.01 reflects moderate variability around the mean. The skewness value of -0.17 denotes a slight leftward skew, indicating that more students scored at the higher end of the scale, though the overall performance remains suboptimal. The kurtosis value of -1.77 signifies a flatter-than-normal distribution, with fewer extreme scores observed, meaning more scores are concentrated on the higher end. The kurtosis of -1.77 indicates fewer extreme scores and a flatter curve compared to a normal distribution. Overall, the data is relatively spread out of standard deviation of 1.01 with most scores likely falling between 0.15 and 2.17. The slight leftward skew and flat distribution with fewer outliers. A histogram showing the distribution of students' performance on the achievement test on percentile is presented in Figure 4.4

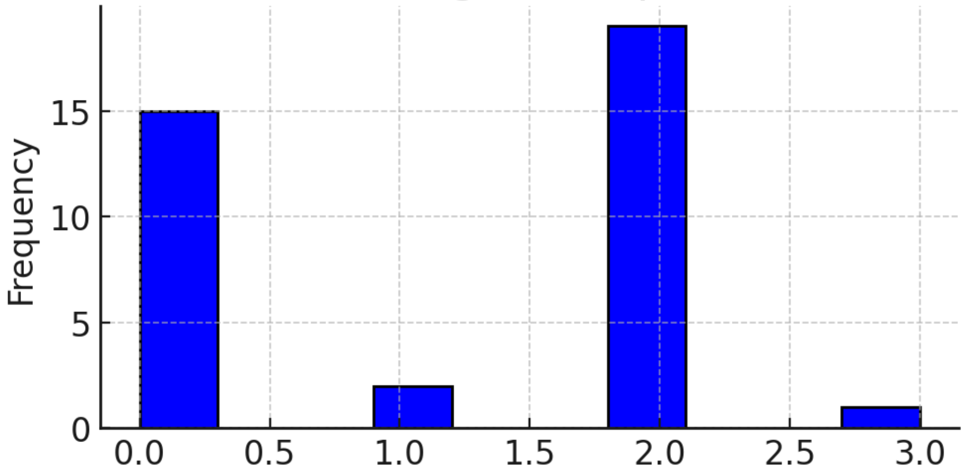


Figure 4. 10: Histogram of Percentile

From Table 4.1, the score distribution of 34 students who attempted quartile questions reveals significant variability, with a mean of 3.68 and a standard deviation of 4.75. The distribution is slightly skewed to the right at 0.81 and relatively flat at -1.21, indicating fewer extreme scores. This result suggests most students struggled or did not attempt the question. A histogram showing the distribution of students' performance on the achievement test on quartiles is presented in Figure 4.11.

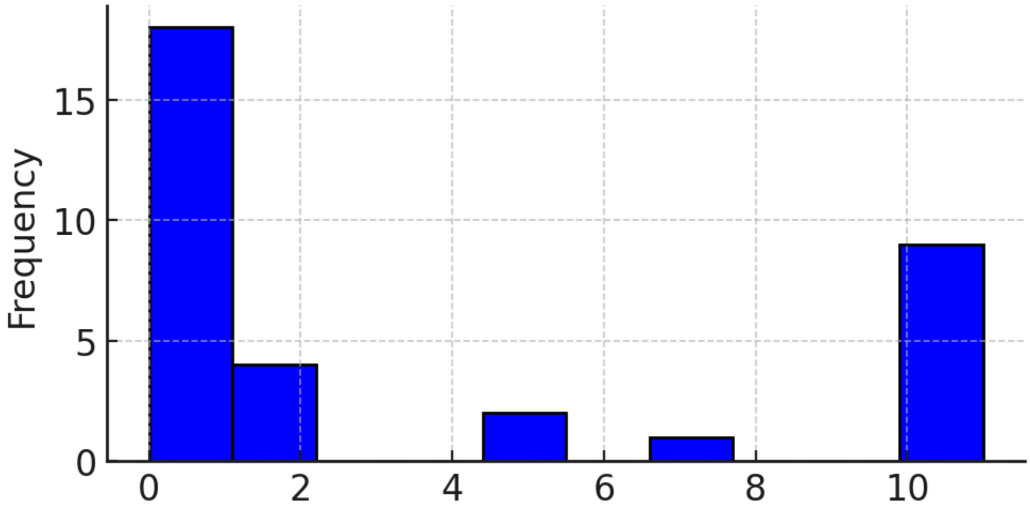


Figure 4. 11: Histogram of Quartile

The score distribution of 55 students who responded to Ogive's question with a maximum score of 5. The scores showed a mean of 3.1, indicating average performance. The standard deviation of 1.81 indicates variability. The skewness of -0.64 indicates a slight negative skew, meaning more students scored higher. The kurtosis of -0.75 indicates a relatively flat distribution. A histogram showing the distribution of students' performance on the achievement test on Ogive is presented in Figure 4.6

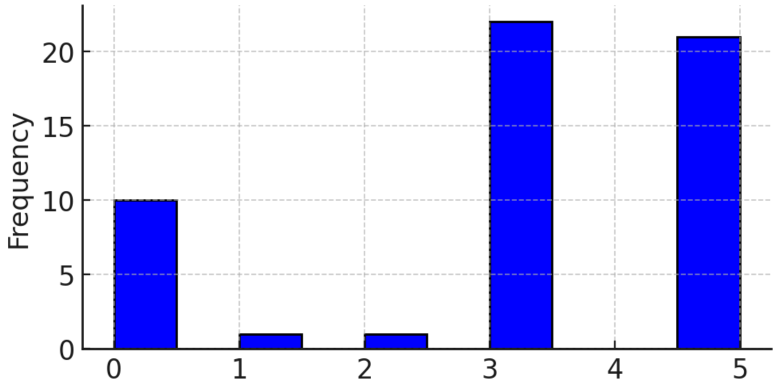


Figure 4.12: Histogram of Ogive

The score distribution of 30 students who attempted the histogram question with a maximum score of 1.23 indicates extremely low performance. The standard deviation of 2.05 indicates significant variability, with scores ranging widely. A positive skewness of 0.81 reveals a disproportionate number of students scoring low. The kurtosis of -0.27 indicates a relatively flat distribution, lacking extreme scores. A histogram showing the distribution of students' performance on the achievement test on the histogram is presented in Figure 4.7.

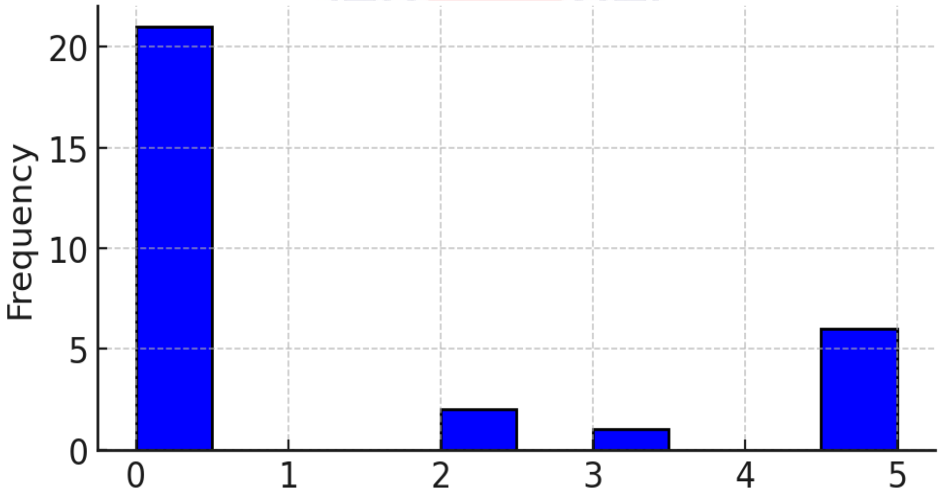


Figure 4. 13: Histogram of Histogram

The Quartile question, attempted by 39 students, produced a mean score of 3.68, slightly above the expected average of 3.5, indicating moderate performance. However, the high standard deviation of 4.75 highlights significant variability in students' understanding of quartiles. The positive skewness of 0.81 suggests that a larger proportion of students scored below the mean, with only a few achieving higher marks. The kurtosis value of -1.21 indicates a flat distribution, with fewer extreme scores, suggesting a lack of exceptional performance in this area. The mean of 0.41, of the maximum, indicates extremely poor performance, indicating students struggled profoundly with the question. The high standard deviation of 0.82 highlights significant variability among students, with scores ranging from very low to relatively high. The strongly positive skewness of 1.52 reveals a disproportionate number of students scoring very low struggling to achieve even half of the maximum score. This indicates instructional gaps or misconceptions likely to exist, and students lack essential knowledge or skills. Furthermore, the distribution indicated by a kurtosis of 0.32, points to the presence of outliers. A histogram showing the distribution of students' performance on the achievement test on the application of standard deviation (Q17) is presented in Figure 4.14.

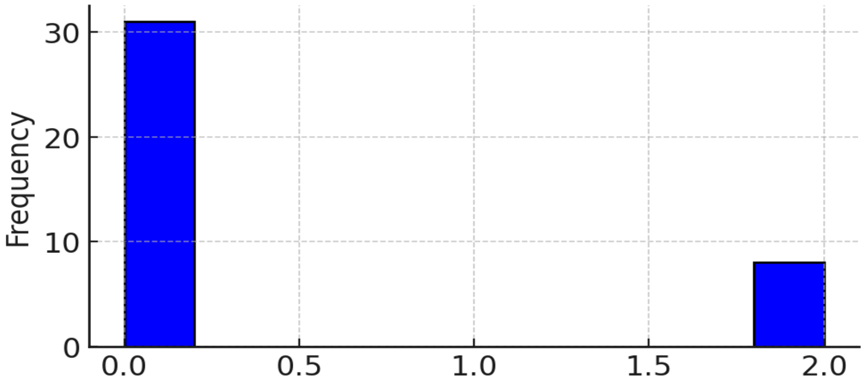


Figure 4.14: Histogram of Standard Deviation

The score distribution of 51 students who attempted the application of statistics questions with a maximum score of 4 paints a concerning picture. With a mean score of merely 1 of the maximums, it's clear that the student struggled profoundly with the question. This extremely low performance indicates significant knowledge gaps of misconceptions. The standard deviation of 1.15 indicates variability among students, but the slight positive skewness of 0.45 reveals that more students clustered at the lower end of the scale. This skewness suggests that a larger proportion of students scored closer to 0 to 4, underscoring the prevalence of student struggling students. Furthermore, the kurtosis of 0.72 indicates a relatively peaked distribution, with score clustering around the mean. A histogram showing the distribution of students' performance on the achievement test on the application of statistics (19) is presented in Figure 4.14.

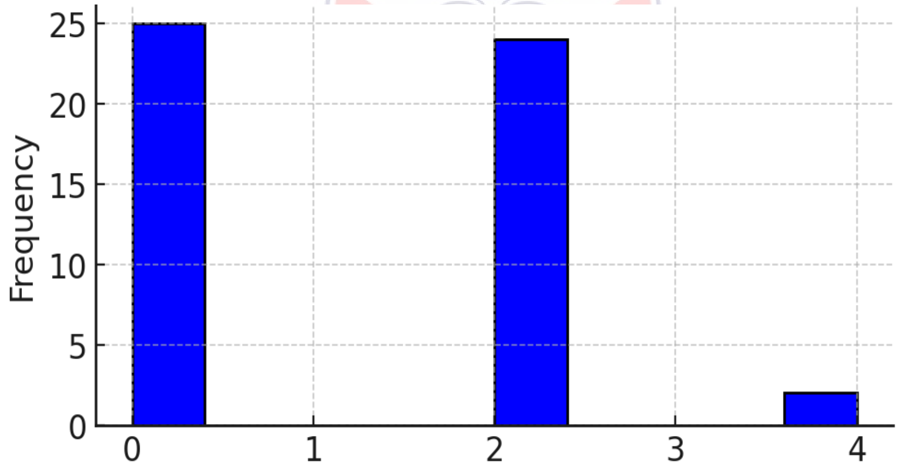


Figure 4.15:Histogram of Application of Statistics

In summary, an average total score of 13.57 out of 52, the overall performance level indicates that many students faced challenges in comprehending various statistical concepts. This below-expected performance (compared to the benchmark score of 26) suggests a need for more targeted instructional support. The wide spread in standard deviations, with a total SD of 10.51, further emphasizes the disparity in student abilities, underscoring the importance of tailored teaching strategies to accommodate diverse levels of understanding. Standard deviation quantifies the extent to which scores deviate from the mean, indicating the level of dispersion or spread. Based on the data presented in the table, a standard deviation of 1.74 suggests a mid-range variability in the range of scores. The average scores among students exhibit a significant diversity, as indicated by a standard deviation of 3.01. A standard deviation of 4.63 indicates substantial variability in the extent to which students' scores differ from the average. The standard deviation of the total score, which is 10.51, indicates a significant variation in all students' performance.

Conversely, other students demonstrated a wider distribution of performance. The extensive range of standard deviation, spanning from 0 to 18, indicates a substantial disparity in the consistency of pupils' performance. The total scores exhibit a broad range, spanning from 0 to 52, which signifies a notable disparity in overall ability. The majority of students achieve scores between 6 and 19.

The histograms provide a graphical depiction of the distribution of data for each statistic. The distribution of scores is slightly tilted to the left, with the majority of students performing in the middle to higher ranges. The data has a moderate central tendency with a tiny positive skew, suggesting that the majority of students have average scores while a few have higher scores. The Standard Deviation (SDV) indicates a distribution that is skewed to the right, indicating that while the majority of students have modest standard deviations, a small number have very high values. Both Percentile and Quartiles exhibit highly skewed distributions, characterized by a significant clustering of pupils inside specific percentiles or quartiles. The distribution patterns of Ogive, Histogram, application of standard deviation (Q17), and application of statistics (Q19) exhibit diversity, with some displaying a more pronounced right-skewness while others appear rather flat. The distribution of total scores is right-skewed, showing a greater frequency of students with lower total scores but a small number with very higher totals.

Skewness indicates the asymmetry in the data distribution. A positive skew suggests a longer right tail, while a negative skew suggests a longer left tail. From Table 1, a skewness of 1.02 for the total score indicates that while most students score below the average, a few significantly higher scores pull the distribution to the right.

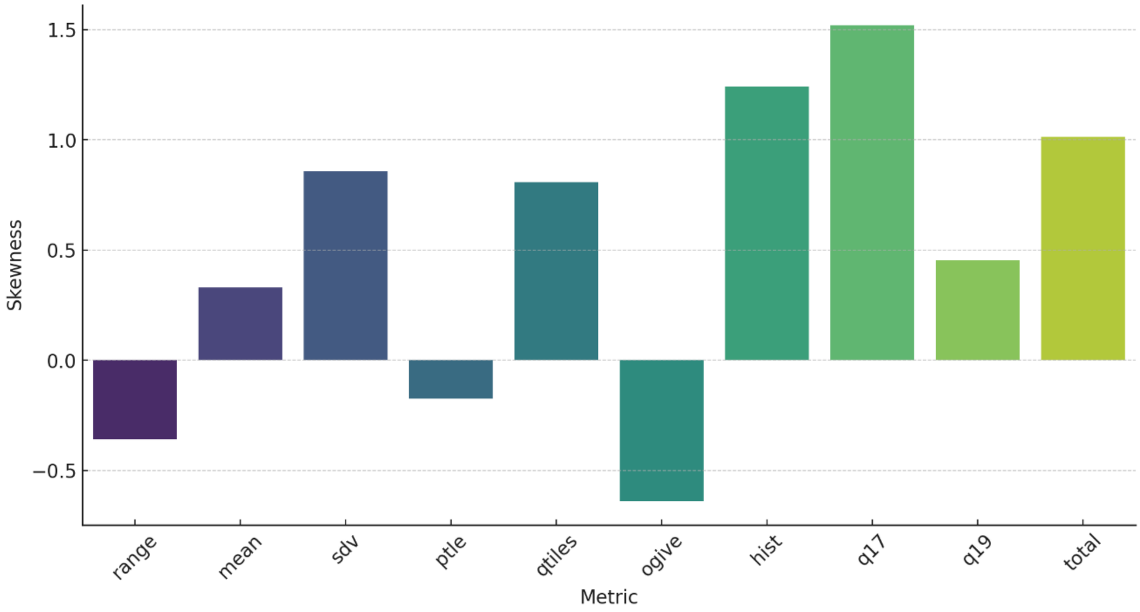


Figure 4.16: Histogram of Skewness

This graph figure 4.16 shows how each metric is skewed. Metrics like standard deviation and application of standard deviation (q17) show positive skewness, indicating that most students performed below average with a few outliers performing better.

The study analysed the Kurtosis, a measure of the "tailedness" of the data distribution. Positive kurtosis indicates heavy tails and sharp peaks, while negative kurtosis indicates lighter tails and flatter distributions. With a kurtosis of -0.65, the mean scores are also somewhat flatly distributed with fewer extreme values. The slight negative kurtosis of -0.33 for standard deviation implies a flatter distribution of how students' scores vary from the mean. The total score's kurtosis of 1.32 suggests a distribution with heavier tails, meaning there are more extreme scores (both high and low) than would be expected in a normal distribution. Most metrics have negative kurtosis, indicating flatter distributions compared to a normal distribution, except for the application of standard deviation (q17) and total, which show slightly higher peaks.

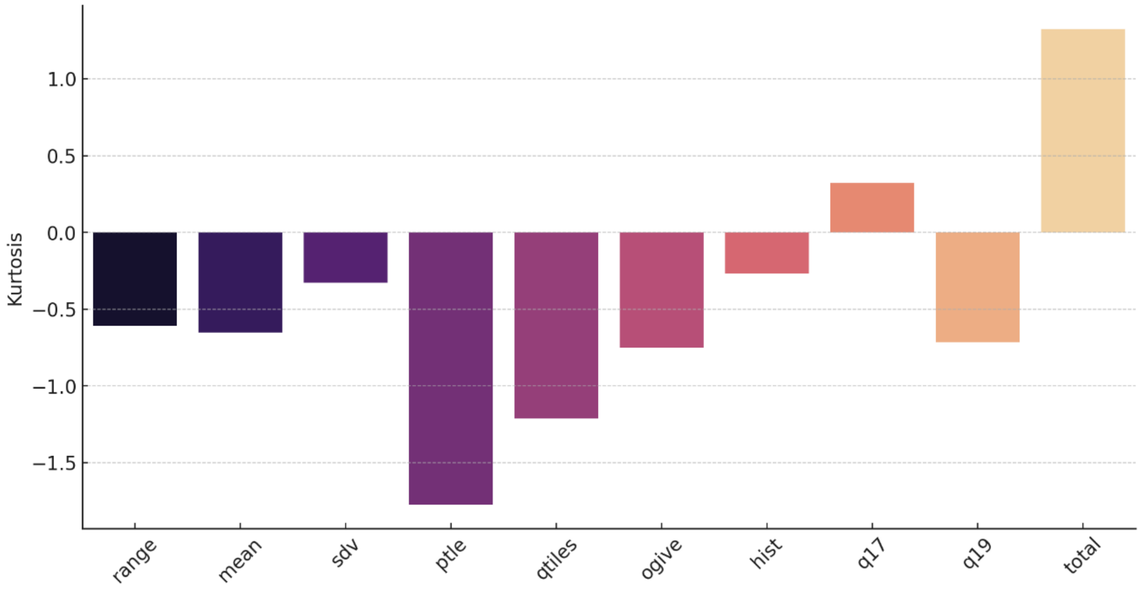


Figure 4. 17: Kurtosis of Each Metric

From the figure 4.17 graph, most metrics have negative kurtosis, suggesting a flatter distribution, except total, which shows a slight positive kurtosis, indicating more outliers or extreme values. The analysis of the test performance data for SHS 3 students in different statistical areas shows some significant observations on the variation and spread of their results. The analysis revealed that the majority of measures have a wide range, suggesting considerable diversity among pupils. For instance, the total scores span from 0 to 52, indicating that while some students achieved very low scores, others demonstrated remarkable performance. Furthermore, the standard deviation also emphasizes significant variability within each metric. The average standard deviation (SDV) is 5.03, with certain measurements exhibiting significantly greater variability, such as the SDV itself, which can go up to 18. This suggests that the performance of students is not concentrated around the average, but instead is dispersed, indicating varying levels of understanding among students. Furthermore, the metrics application of standard deviation (q17) and histogram demonstrate positive skewness, indicating that most students obtain scores below the mean, while a smaller number of students achieve significantly higher scores.